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# Chapter 4
# Decomposition of the load's current supplied from a sinusoidal and asymmetrical voltage source in accordance with the Currents' Physical Components (CPC) Theory

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The correct description of circuits supplied from an asymmetrical and sinusoidal voltage source in which the load can be asymmetric or unbalanced necessitates a well-defined approach. The present development of electrical engineering allows for describing three-phase three-wire systems with asymmetric sinusoidal or non-sinusoidal supply. In addition, it is possible in that kind of systems to balance the load and compensate for reactive power.

The subject of this article is to exhibit the possibility of describing three-phase circuits powered from an asymmetric sinusoidal voltage source. Determining the physical components of the current associated closely with specific physical phenomena makes it possible to distinguish the components of unbalanced active and reactive components. Each of these currents, excluding the active current, contributes to the power of unbalanced and reactive power, which should be minimized or completely removed from the circuit.

**Index terms:** Current's Physical Components (CPC), power definitions, power theory, asymmetric voltage source

## Introduction

Transmission of electricity from sources to loads through power systems, where it is transformed adequately to the needs of the consumer, is described by power theories [1, 2, 3, 4, 5, 6, 7, 17]. Over 100 years of energy transmission, many different approaches have been created.

The description of power theory is divided into two domains, i.e. time and frequency. The characterization within the time domain, in view of the speed of calculation, is primarily used to control semiconductor devices in active or hybrid power filters [4, 5, 11, 12, 13, 14, 15, 16, 17, 18]. The most common time domain methods are [4, 5, 6, 17]. The description in the frequency domain [1, 3, 25], using Fourier transform, causes delays in the measuring length; however, the methods based on frequency description are more precise and are also used to generate the reference current of the active power filter [8, 9, 10]. In power theories quoted above, the mathematical description and, as a result, the obtained results are correct, on the assumption that the voltage source is symmetry.

In publications [19, 20, 21] a description of asymmetrical sinusoidal three-phase three-wire power circuits is presented. In addition, in these articles, the possibility of building an unbalanced and reactive power compensator has been demonstrated.

This publication proposes an expansion of the Currents' Physical Components (CPC) Theory for asymmetrical sinusoidal three-phase four-wire power systems, i.e. circuits with a neutral conductor (N) and/or a protective earth conductor (PE).

## Currents' physical components in three-phase four-wire systems at sinusoidal and asymmetric voltage supply

The source voltage of the distribution system can be referred to as a three-phase vector, where its elements are the voltages at terminals R, S, T, namely $\boldsymbol{e} = [e_R\ e_S\ e_T]$. This voltage as asymmetrical can have symmetrical components of the positive sequence $-\boldsymbol{e}^p$, negative sequence $-\boldsymbol{e}^n$ and zero sequence $-\boldsymbol{e}^z$. It should also be mentioned that in the event of asymmetrical power supply, the internal impedance of the power source should be considered; therefore, the ideal source should not be used – which is a departure from pure theory. In connection with the above, the line voltage is defined by vector $\boldsymbol{u} = [u_R\ u_S\ u_T]^T$ that has three components, i.e.: $\boldsymbol{u} = \boldsymbol{u}^p + \boldsymbol{u}^n + \boldsymbol{u}^z$ [24].

An unbalanced linear time-invariant (LTI) load supplied by a sinusoidal but asymmetric voltage source is shown in Figure 4.1.

Symbols $\boldsymbol{u}$ and $\boldsymbol{i}$ denote voltage and current vectors:

$$\begin{aligned} \boldsymbol{u}(t) &= \left[u_R(t)\quad u_S(t)\quad u_T(t)\right]^T, \\ \boldsymbol{i}(t) &= \left[i_R(t)\quad i_S(t)\quad i_T(t)\right]^T. \end{aligned} \tag{4.1}$$

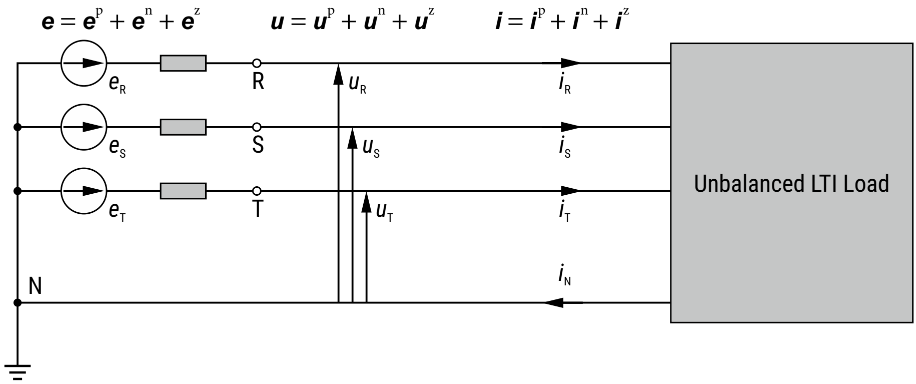

FIGURE 4.1. LTI load supplied by a four-wire line

The sinusoidal voltage supplied linear time-invariant (LTI) unbalanced load can be presented in the form of:

$$\boldsymbol{u}(t)=\begin{bmatrix}u_{\mathrm{R}}(t)\\ u_{\mathrm{S}}(t)\\ u_{\mathrm{T}}(t)\end{bmatrix}=\sqrt{2}Re\begin{bmatrix}U_{\mathrm{R}}\\ U_{\mathrm{S}}\\ U_{\mathrm{T}}\end{bmatrix}e^{j\omega t}=\sqrt{2}Re\{\boldsymbol{U}e^{j\omega t}\} \tag{4.2}$$

The line current can be depicted identically, namely:

$$\boldsymbol{i}(t)=\begin{bmatrix}i_{\mathrm{R}}(t)\\ i_{\mathrm{S}}(t)\\ i_{\mathrm{T}}(t)\end{bmatrix}=\sqrt{2}Re\begin{bmatrix}I_{\mathrm{R}}\\ I_{\mathrm{S}}\\ I_{\mathrm{T}}\end{bmatrix}e^{j\omega t}=\sqrt{2}Re\{\boldsymbol{I}e^{j\omega t}\} \tag{4.3}$$

The supply voltage $\boldsymbol{u}$ could be asymmetrical – Figure 4.1; therefore, it is possible to present it as the sum of the voltages of the positive, negative and zero sequences:

$$\boldsymbol{u}=\boldsymbol{u}^{\mathrm{p}}+\boldsymbol{u}^{\mathrm{n}}+\boldsymbol{u}^{\mathrm{z}}=\sqrt{2}Re\{(\boldsymbol{U}^{\mathrm{p}}+\boldsymbol{U}^{\mathrm{n}}+\boldsymbol{U}^{\mathrm{z}})e^{j\omega t}\}=\sqrt{2}Re\{(\boldsymbol{1}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}}+\boldsymbol{1}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{z}})e^{j\omega t}\}, \tag{4.4}$$

where $U^{\mathrm{p}}$, $U^{\mathrm{n}}$ and $U^{\mathrm{z}}$ are the complex rms (crms) values of the symmetrical components of the positive, negative and zero sequences, described by the Fortesque Transformation [7, 21]:

$$\begin{bmatrix}U^{\mathrm{p}}\\ U^{\mathrm{n}}\\ U^{\mathrm{z}}\end{bmatrix}=\frac{1}{3}\begin{bmatrix}1 & \alpha & \alpha^{*}\\ 1 & \alpha^{*} & \alpha\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}U_{\mathrm{R}}\\ U_{\mathrm{S}}\\ U_{\mathrm{T}}\end{bmatrix}, \tag{4.5}$$

where $\alpha = 1e^{j120°}$, $\alpha^{*} = 1e^{-j120°}$, and symbols

$$\boldsymbol{1}^{\mathrm{p}}=\begin{bmatrix}1\\ \alpha^{*}\\ \alpha\end{bmatrix}=\begin{bmatrix}1\\ 1e^{-j120°}\\ 1e^{j120°}\end{bmatrix},\ \boldsymbol{1}^{\mathrm{n}}=\begin{bmatrix}1\\ \alpha\\ \alpha^{*}\end{bmatrix}=\begin{bmatrix}1\\ 1e^{j120°}\\ 1e^{-j120°}\end{bmatrix},\ \boldsymbol{1}^{\mathrm{z}}=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix} \tag{4.6}$$

denote unit symmetrical three-phase coefficients of the positive – $\mathbf{1}^{\mathrm{p}}$, negative – $\mathbf{1}^{\mathrm{n}}$, and zero sequences – $\mathbf{1}^{\mathrm{z}}$, shown in Figure 4.2.

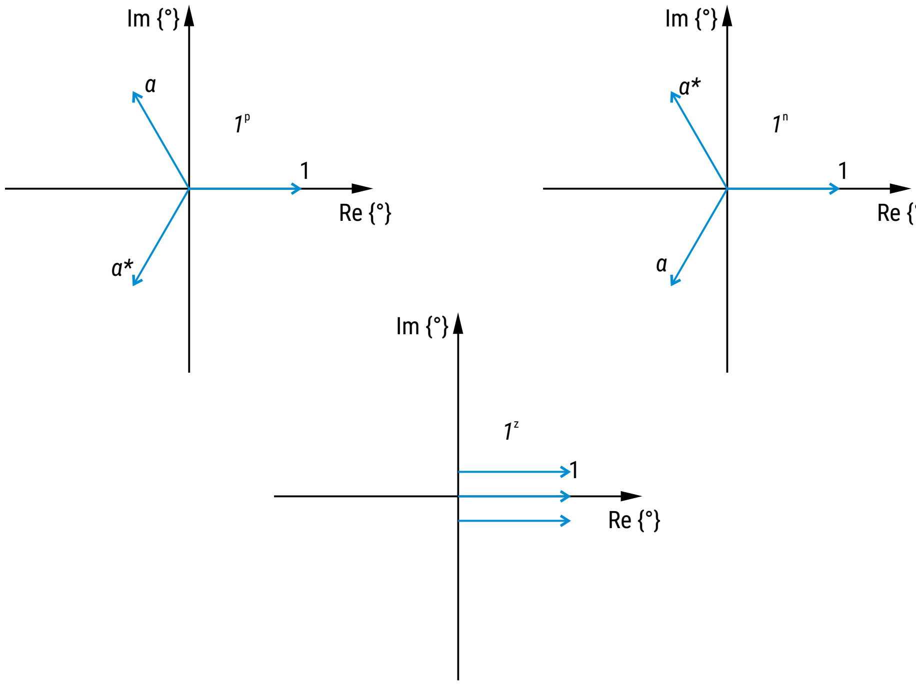


FIGURE 4.2. **Unit symmetrical vectors**

In publications [19, 20, 21], the difference between the apparent power module, apparent power and complex power, which is marked as ***C***, will be used later in the article.

For a linear balanced and time-invariant load supplied by the four-wire system is considered the load powered by sinusoidal symmetric voltage, despite the current asymmetry. For each unbalanced load, it is possible to find a balanced load, equivalent to it for active power *P* and reactive power *Q*. In order for it to be equivalent to the original circuit due to active power *P* and reactive power *Q*, its phase admittance takes the value of:

$$\boldsymbol{Y}_{\mathrm{b}} = G_{\mathrm{b}} + jB_{\mathrm{b}} = \frac{P - jQ}{\|\boldsymbol{u}\|^2} = \frac{\boldsymbol{C}^*}{\|\boldsymbol{u}\|^2}, \tag{4.7}$$

where $\|\boldsymbol{u}\|$ denotes three-phase rms value of the voltage supply, equal to:

$$\|\boldsymbol{u}\| = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{u}^{\mathrm{T}}(t)\,\boldsymbol{u}(t)\,dt} = \sqrt{U_{\mathrm{R}}^2 + U_{\mathrm{S}}^2 + U_{\mathrm{T}}^2}, \tag{4.8}$$

and the $\boldsymbol{Y}_{\mathrm{b}}$ admittance defined by (7) will be described as the **equivalent admittance of the balanced load**, which is presented in Figure 4.3.

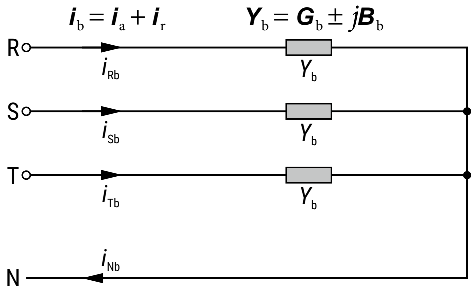

FIGURE 4.3. Balanced load, equivalent to the original load due to active and reactive power

The balanced load, shown in Figure 4.3, loads the supply source by the balanced current $\boldsymbol{i}_{\mathrm{b}}$, which has an active component:

$$\boldsymbol{i}_{\mathrm{a}}=\mathrm{G}_{\mathrm{b}}\,\boldsymbol{u}=\sqrt{2}Re\left\{\mathrm{G}_{\mathrm{b}}\left(\boldsymbol{U}^{\mathrm{p}}+\boldsymbol{U}^{\mathrm{n}}+\boldsymbol{U}^{\mathrm{z}}\right)e^{j\omega t}\right\}=\sqrt{2}Re\left\{\mathrm{G}_{\mathrm{b}}\left(\boldsymbol{1}^{\mathrm{p}}U^{\mathrm{p}}+\boldsymbol{1}^{\mathrm{n}}U^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{z}}U^{\mathrm{z}}\right)e^{j\omega t}\right\} \tag{4.9}$$

and a reactive component:

$$\boldsymbol{i}_{\mathrm{r}}=B_{\mathrm{b}}\boldsymbol{u}\left(t+T/4\right)=\sqrt{2}Re\left\{jB_{\mathrm{b}}\left(\boldsymbol{U}^{\mathrm{p}}+\boldsymbol{U}^{\mathrm{n}}+\boldsymbol{U}^{\mathrm{z}}\right)e^{j\omega t}\right\}=$$
$$=\sqrt{2}Re\left\{jB_{\mathrm{b}}\left(\boldsymbol{1}^{\mathrm{p}}U^{\mathrm{p}}+\boldsymbol{1}^{\mathrm{n}}U^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{z}}U^{\mathrm{z}}\right)e^{j\omega t}\right\} \tag{4.10}$$

Three-phase rms value of the active current $\boldsymbol{i}_{\mathrm{a}}$ and reactive current $\boldsymbol{i}_{\mathrm{r}}$ are respectively:

$$\|\boldsymbol{i}_{\mathrm{a}}\|=G_{\mathrm{b}}\,\|\boldsymbol{u}\|=\frac{\mathrm{P}}{\|\boldsymbol{u}\|}\,, \tag{4.11}$$

$$\|\boldsymbol{i}_{\mathrm{r}}\|=|B_{\mathrm{b}}|\,\|\boldsymbol{u}\|=\frac{|Q|}{\|\boldsymbol{u}\|}\,. \tag{4.12}$$

With respect to the fact that these currents are proportional to the voltage and supply voltage shifted in time by a quarter period, they, therefore, have exactly the same degree of asymmetry as the supply voltage. However, the current of the original load $\boldsymbol{i}$, as a consequence of its being unbalanced, does not have the same asymmetry as the supply voltage $\boldsymbol{u}$. The current of the imbalanced load may, therefore, have an unbalanced component:

$$\boldsymbol{i}-\left(\boldsymbol{i}_{\mathrm{a}}+\boldsymbol{i}_{\mathrm{r}}\right)=\boldsymbol{i}-\boldsymbol{i}_{\mathrm{b}}=\boldsymbol{i}_{\mathrm{u}}\,, \tag{4.13}$$

where its waveform is as follows:

$$\boldsymbol{i}_{\mathrm{u}}=\sqrt{2}Re\left\{\boldsymbol{I}_{\mathrm{u}}e^{j\omega t}\right\}=\sqrt{2}Re\left\{\left(\boldsymbol{I}-\boldsymbol{I}_{\mathrm{b}}\right)e^{j\omega t}\right\}=\sqrt{2}Re\left\{\left(\boldsymbol{I}-\boldsymbol{Y}_{\mathrm{b}}\boldsymbol{U}\right)e^{j\omega t}\right\}. \tag{4.14}$$

Upon rearrangement (13), the load current can be expressed as follows:

$$\boldsymbol{i}\ \breve{}\ \boldsymbol{i}_{\smile}\quad \boldsymbol{i}\quad \boldsymbol{i}\ , \tag{4.15}$$

which means that the current of the load is the sum of the active current, reactive current and unbalanced current. Each component is associated with a different physical phenomenon. The active current $\boldsymbol{i}_{\mathrm{a}}$ is related to the permanent flow of energy from the source to the load. The reactive current $\boldsymbol{i}_{\mathrm{r}}$ is related to the displacement of the load current in relation to the supply voltage. The unbalanced current $\boldsymbol{i}_{\mathrm{u}}$ is the result of the asymmetry of the currents caused by the unbalanced condition of the load. On the grounds of the unambiguous connection of these currents with separate physical phenomena in the circuit, they can be treated as Currents' Physical Components of the load current.

The three-phase rms value of the load's current supplied by the sinusoidal asymmetrical voltage source is equal to:

$$\|\boldsymbol{i}\|^2=\|\boldsymbol{i}_{\mathrm{a}}\|^2+\|\boldsymbol{i}_{\mathrm{r}}\|^2+\|\boldsymbol{i}_{\mathrm{u}}\|^2. \tag{4.16}$$

Equation (16) is true provided that the current components of the load are mutually orthogonal. Orthogonality is presented in Appendix A.

In order to demonstrate the possibility of building a balancing compensator of the negative and zero sequences and the reactive power compensator, the components of the unbalanced current should be decomposed.

Equivalent admittance of the balanced load (7) can be expressed by the parameters of the load and the supply voltage as follows:

$$\boldsymbol{Y}_{\mathrm{b}}=G_{\mathrm{b}}+jB_{\mathrm{b}}=\frac{P-jQ}{\|\boldsymbol{u}\|^2}=\frac{\boldsymbol{Y}_{\mathrm{R}}U_{\mathrm{R}}^2+\boldsymbol{Y}_{\mathrm{S}}U_{\mathrm{S}}^2+\boldsymbol{Y}_{\mathrm{T}}U_{\mathrm{T}}^2}{\|\boldsymbol{u}\|^2}. \tag{4.17}$$

At the symmetry condition of the supply voltage, i.e.: $U_{\mathrm{R}}=U_{\mathrm{S}}=U_{\mathrm{T}}=\|\boldsymbol{u}\|$, equivalent admittance of the balanced load assumes the form:

$$\boldsymbol{Y}_{\mathrm{b}}=\frac{1}{3}\left(\boldsymbol{Y}_{\mathrm{R}}+\boldsymbol{Y}_{\mathrm{S}}+\boldsymbol{Y}_{\mathrm{T}}\right)=\boldsymbol{Y}_{\mathrm{e}} \tag{4.18}$$

and is called **the equivalent admittance of the load supplied by the symmetric voltage source**.

The difference between admittances expressed in (4.17) and (4.18) is:

$$\boldsymbol{Y}_{\mathrm{d}}=G_{\mathrm{d}}+jB_{\mathrm{d}}=\boldsymbol{Y}_{\mathrm{e}}-\boldsymbol{Y}_{\mathrm{b}}=\frac{1}{3}\left(\boldsymbol{Y}_{\mathrm{R}}+\boldsymbol{Y}_{\mathrm{S}}+\boldsymbol{Y}_{\mathrm{T}}\right)-\frac{\boldsymbol{Y}_{\mathrm{R}}\,\mathrm{U}_{\mathrm{R}}^2+\boldsymbol{Y}_{\mathrm{S}}\,\mathrm{U}_{\mathrm{S}}^2+\boldsymbol{Y}_{\mathrm{T}}\,\mathrm{U}_{\mathrm{T}}^2}{\|\boldsymbol{u}\|^2} \tag{4.19}$$

and denotes **the voltage asymmetry dependent admittance**.

According to [22, 23] in four-wire systems supplied from a source of sinusoidal voltage, the two admittances respond to the imbalanced form of the load, i.e.: **the unbalanced admittance of the negative sequence**, described in the relationship:

$$\boldsymbol{A}^{\mathrm{n}} = \frac{1}{3}\left(\boldsymbol{Y}_{\mathrm{R}} + \alpha \boldsymbol{Y}_{\mathrm{S}} + \alpha^{*} \boldsymbol{Y}_{\mathrm{T}}\right) \tag{4.20}$$

and **the unbalanced admittance of the zero sequence**:

$$\boldsymbol{A}^{\mathrm{z}} = \frac{1}{3}\left(\boldsymbol{Y}_{\mathrm{R}} + \alpha^{*} \boldsymbol{Y}_{\mathrm{S}} + \alpha \boldsymbol{Y}_{\mathrm{T}}\right). \tag{4.21}$$

Based on (4.17), (4.18) and (4.19), the crms current value can be represented as follows:

$$\begin{aligned}\boldsymbol{I}_{\mathrm{R}} = \boldsymbol{Y}_{\mathrm{R}}\boldsymbol{U}_{\mathrm{R}} &= \frac{1}{3}\left(\boldsymbol{Y}_{\mathrm{R}} + \boldsymbol{Y}_{\mathrm{S}} + \boldsymbol{Y}_{\mathrm{T}}\right)\boldsymbol{U}_{\mathrm{R}} + \left(\frac{1}{3}\begin{bmatrix}1\\ \alpha\\ \alpha^{*}\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{Y}_{\mathrm{R}}\\ \boldsymbol{Y}_{\mathrm{S}}\\ \boldsymbol{Y}_{\mathrm{T}}\end{bmatrix} + \frac{1}{3}\begin{bmatrix}1\\ \alpha^{*}\\ \alpha\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{Y}_{\mathrm{R}}\\ \boldsymbol{Y}_{\mathrm{S}}\\ \boldsymbol{Y}_{\mathrm{T}}\end{bmatrix}\right)\boldsymbol{U}_{\mathrm{R}} = \\ &= \boldsymbol{Y}_{\mathrm{e}}\boldsymbol{U}_{\mathrm{R}} + \left(\boldsymbol{1}^{\mathrm{n}^{\mathrm{T}}}\boldsymbol{Y}_{\mathrm{e}} + \boldsymbol{1}^{\mathrm{p}^{\mathrm{T}}}\boldsymbol{Y}_{\mathrm{e}}\right)\boldsymbol{U}_{\mathrm{R}}\end{aligned} \tag{4.22}$$

and can be transformed using (4.20) and (4.21) into:

$$\boldsymbol{I}_{\mathrm{R}} = \boldsymbol{Y}_{\mathrm{e}}\boldsymbol{U}_{\mathrm{R}} + \left(\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{R}}^{\mathrm{p}} + \left(\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{R}}^{\mathrm{n}} + \left(\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{R}}^{\mathrm{z}}. \tag{4.23}$$

Then (23) is further transformed and, using the $\boldsymbol{Y}_{\mathrm{e}} = \boldsymbol{Y}_{\mathrm{b}} + \boldsymbol{Y}_{\mathrm{d}}$ condition, the current in the R-line can be expressed:

$$\boldsymbol{I}_{\mathrm{R}} = \left(\boldsymbol{Y}_{\mathrm{b}} + \boldsymbol{Y}_{\mathrm{d}}\right)\boldsymbol{U}_{\mathrm{R}} + \left(\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{R}}^{\mathrm{p}} + \left(\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{R}}^{\mathrm{n}} + \left(\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{R}}^{\mathrm{z}}. \tag{4.24}$$

The crms current values of the current in the S-line can be expressed in the same way, i.e.:

$$\boldsymbol{I}_{\mathrm{S}} = \left(\boldsymbol{Y}_{\mathrm{b}} + \boldsymbol{Y}_{\mathrm{d}}\right)\boldsymbol{U}_{\mathrm{S}} + \left(\alpha^{*}\boldsymbol{A}^{\mathrm{n}} + \alpha\boldsymbol{A}^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{S}}^{\mathrm{p}} + \left(\alpha^{*}\boldsymbol{A}^{\mathrm{n}} + \alpha\boldsymbol{A}^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{S}}^{\mathrm{n}} + \left(\alpha^{*}\boldsymbol{A}^{\mathrm{n}} + \alpha\boldsymbol{A}^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{S}}^{\mathrm{z}} \tag{4.25}$$

and the T-line current as:

$$\boldsymbol{I}_{\mathrm{T}} = \left(\boldsymbol{Y}_{\mathrm{b}} + \boldsymbol{Y}_{\mathrm{d}}\right)\boldsymbol{U}_{\mathrm{T}} + \left(\alpha\boldsymbol{A}^{\mathrm{n}} + \alpha^{*}\boldsymbol{A}^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{T}}^{\mathrm{p}} + \left(\alpha\boldsymbol{A}^{\mathrm{n}} + \alpha^{*}\boldsymbol{A}^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{T}}^{\mathrm{n}} + \left(\alpha\boldsymbol{A}^{\mathrm{n}} + \alpha^{*}\boldsymbol{A}^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{T}}^{\mathrm{z}}. \tag{4.26}$$

Combining (4.24), (4.25) and (4.26) into one equation, we get a vector of three-phase crms values of the line currents of the load:

$$\boldsymbol{I} = \begin{bmatrix} \boldsymbol{I}_{\mathrm{R}} \\ \boldsymbol{I}_{\mathrm{S}} \\ \boldsymbol{I}_{\mathrm{T}} \end{bmatrix} = \boldsymbol{Y}_{\mathrm{b}}\boldsymbol{U} + \boldsymbol{Y}_{\mathrm{d}}\boldsymbol{U} + \boldsymbol{1}^{\mathrm{p}}\left(\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{z}} + \boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{n}}\right) +$$

$$+ \boldsymbol{1}^{\mathrm{n}}\left(\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{z}}\right) + \boldsymbol{1}^{\mathrm{z}}\left(\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{p}}\right), \tag{4.27}$$

where the unbalanced current $\boldsymbol{I}_{\mathrm{u}}$ is equal to:

$$\boldsymbol{I}_{\mathrm{u}} = \boldsymbol{Y}_{\mathrm{d}}\boldsymbol{U} + \boldsymbol{1}^{\mathrm{p}}\left(\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{z}} + \boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{n}}\right) + \boldsymbol{1}^{\mathrm{n}}\left(\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{z}}\right) + \boldsymbol{1}^{\mathrm{z}}\left(\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{p}}\right) =$$

$$= \boldsymbol{Y}_{\mathrm{d}}\boldsymbol{U} + \boldsymbol{J}^{\mathrm{p}} + \boldsymbol{J}^{\mathrm{n}} + \boldsymbol{J}^{\mathrm{z}}, \tag{4.28}$$

where current sources $\boldsymbol{J}^{\mathrm{p}}$, $\boldsymbol{J}^{\mathrm{n}}$, $\boldsymbol{J}^{\mathrm{z}}$ represent the currents of the positive, negative and zero sequences, proportional respectively:

1) the current source of the positive sequence $\boldsymbol{J}^{\mathrm{p}}$ – relative to the symmetrical component of the supply voltage of the negative sequence $\boldsymbol{U}^{\mathrm{n}}$ and zero sequence $\boldsymbol{U}^{\mathrm{z}}$,
2) the current source of the negative sequence $\boldsymbol{J}^{\mathrm{n}}$ – relative to the symmetrical component of the supply voltage of the positive sequence $\boldsymbol{U}^{\mathrm{p}}$ and zero sequence $\boldsymbol{U}^{\mathrm{z}}$,
3) the current of the zero sequence $\boldsymbol{J}^{\mathrm{z}}$ – relative to the symmetrical component of the supply voltage of the positive sequence $\boldsymbol{U}^{\mathrm{p}}$ and negative sequence $\boldsymbol{U}^{\mathrm{n}}$.

and their vectors of the cmrs values are:

$$\boldsymbol{J}^{\mathrm{p}} = \boldsymbol{1}^{\mathrm{p}}\left(\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{z}} + \boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{n}}\right), \tag{4.29}$$

$$\boldsymbol{J}^{\mathrm{n}} = \boldsymbol{1}^{\mathrm{n}}\left(\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{z}}\right), \tag{4.30}$$

$$\boldsymbol{J}^{\mathrm{z}} = \boldsymbol{1}^{\mathrm{z}}\left(\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{p}}\right). \tag{4.31}$$

Schematically, current sources (29), (30) and (31), distortion admittance (19) and equivalent admittance of the balanced load (7) are shown in Figure 4.4.

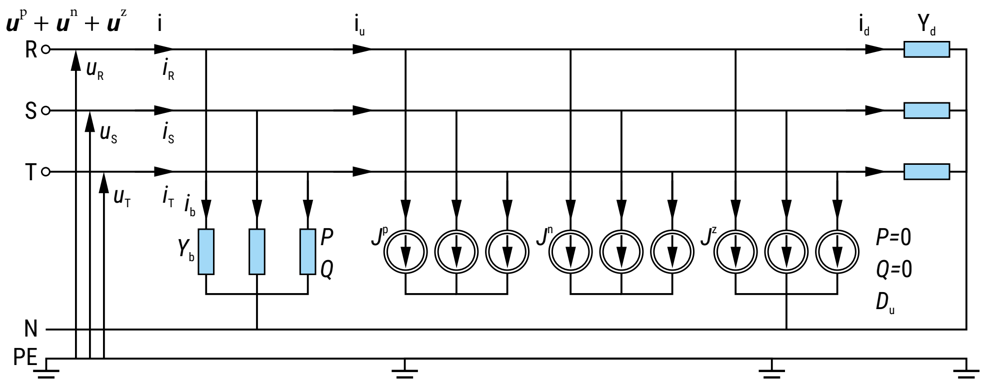


FIGURE 4.4. The equivalent circuit of the unbalanced load with an asymmetrical sinusoidal supply voltage

The relationship (4.28) can be further transformed and then the vector of crms values is as follows:

$$\boldsymbol{I}_{\rm u} = \boldsymbol{1}^{\rm p} \boldsymbol{Y}_{\rm d} \boldsymbol{U}^{\rm p} + \boldsymbol{1}^{\rm n} \boldsymbol{Y}_{\rm d} \boldsymbol{U}^{\rm n} + \boldsymbol{1}^{\rm z} \boldsymbol{Y}_{\rm d} \boldsymbol{U}^{\rm z} + \boldsymbol{1}^{\rm p}\left(\boldsymbol{A}^{\rm n}\boldsymbol{U}^{\rm z} + \boldsymbol{A}^{\rm z}\boldsymbol{U}^{\rm n}\right) + \\ + \boldsymbol{1}^{\rm n}\left(\boldsymbol{A}^{\rm n}\boldsymbol{U}^{\rm p} + \boldsymbol{A}^{\rm z}\boldsymbol{U}^{\rm z}\right) + \boldsymbol{1}^{\rm z}\left(\boldsymbol{A}^{\rm n}\boldsymbol{U}^{\rm n} + \boldsymbol{A}^{\rm z}\boldsymbol{U}^{\rm p}\right) = \boldsymbol{I}_{\rm u}^{\rm p} + \boldsymbol{I}_{\rm u}^{\rm n} + \boldsymbol{I}_{\rm u}^{\rm z}, \tag{4.32}$$

,

- the waveform of the unbalanced current of the positive sequence $\boldsymbol{I}_{\rm u}^{\rm p}$ is:

$$\boldsymbol{i}_{\rm u}^{\rm p} = \sqrt{2} Re\left\{\boldsymbol{I}_{\rm u}^{\rm p} e^{j\omega t}\right\} = \sqrt{2} Re\left\{\boldsymbol{1}^{\rm p}\left(\boldsymbol{Y}_{\rm d}\boldsymbol{U}^{\rm p} + \boldsymbol{A}^{\rm n}\boldsymbol{U}^{\rm z} + \boldsymbol{A}^{\rm z}\boldsymbol{U}^{\rm n}\right) e^{j\omega t}\right\} \tag{4.33}$$

and the crms value is equal to:

$$\boldsymbol{I}_{\rm u}^{\rm p} = \boldsymbol{1}^{\rm p}\left(\boldsymbol{Y}_{\rm d}\boldsymbol{U}^{\rm p} + \boldsymbol{A}^{\rm n}\boldsymbol{U}^{\rm z} + \boldsymbol{A}^{\rm z}\boldsymbol{U}^{\rm n}\right), \tag{4.34}$$

- the waveform of the unbalanced current of the negative sequence $\boldsymbol{I}_{\rm u}^{\rm n}$ is:

$$\boldsymbol{i}_{\rm u}^{\rm n} = \sqrt{2} Re\left\{\boldsymbol{I}_{\rm u}^{\rm n} e^{j\omega t}\right\} = \sqrt{2} Re\left\{\boldsymbol{1}^{\rm n}\left(\boldsymbol{Y}_{\rm d}\boldsymbol{U}^{\rm n} + \boldsymbol{A}^{\rm n}\boldsymbol{U}^{\rm p} + \boldsymbol{A}^{\rm z}\boldsymbol{U}^{\rm z}\right) e^{j\omega t}\right\} \tag{4.35}$$

and the crms value is equal to:

$$\boldsymbol{I}_{\rm u}^{\rm n} = \boldsymbol{1}^{\rm n}\left(\boldsymbol{Y}_{\rm d}\boldsymbol{U}^{\rm n} + \boldsymbol{A}^{\rm n}\boldsymbol{U}^{\rm p} + \boldsymbol{A}^{\rm z}\boldsymbol{U}^{\rm z}\right), \tag{4.36}$$

- the waveform of the unbalanced current of the zero sequence $\boldsymbol{I}_{\rm u}^{\rm z}$ is:

$$\boldsymbol{i}_{\rm u}^{\rm z} = \sqrt{2} Re\left\{\boldsymbol{I}_{\rm u}^{\rm z} e^{j\omega t}\right\} = \sqrt{2} Re\left\{\boldsymbol{1}^{\rm z}\left(\boldsymbol{Y}_{\rm d}\boldsymbol{U}^{\rm z} + \boldsymbol{A}^{\rm n}\boldsymbol{U}^{\rm n} + \boldsymbol{A}^{\rm z}\boldsymbol{U}^{\rm p}\right) e^{j\omega t}\right\} \tag{4.37}$$

and the crms value is equal to:

$$\boldsymbol{I}_{\rm u}^{\rm z} = \boldsymbol{1}^{\rm z}\left(\boldsymbol{Y}_{\rm d}\boldsymbol{U}^{\rm z} + \boldsymbol{A}^{\rm n}\boldsymbol{U}^{\rm n} + \boldsymbol{A}^{\rm z}\boldsymbol{U}^{\rm p}\right). \tag{4.38}$$

The load's current has five physical components:

$$\boldsymbol{i} = \boldsymbol{i}_{\rm a} + \boldsymbol{i}_{\rm r} + \boldsymbol{i}_{\rm u}^{\rm p} + \boldsymbol{i}_{\rm u}^{\rm n} + \boldsymbol{i}_{\rm u}^{\rm z}. \tag{4.39}$$

With this respect all current components are mutually orthogonal (appendix A), the three-phase rms values of the load's current are:

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_{\rm a}\|^2 + \|\boldsymbol{i}_{\rm r}\|^2 + \|\boldsymbol{i}_{\rm u}^{\rm p}\|^2 + \|\boldsymbol{i}_{\rm u}^{\rm n}\|^2 + \|\boldsymbol{i}_{\rm u}^{\rm z}\|^2. \tag{4.40}$$

By multiplying (40) by the square of the rms value of the three-phase voltage $\|\boldsymbol{u}\|^2$, we obtain the power equation of the load supplied from an asymmetrical sinusoidal voltage source:

$$S^2 = P^2 + Q^2 + D_{\rm u}^{\rm p2} + D_{\rm u}^{\rm n2} + D_{\rm u}^{\rm z2}, \tag{4.41}$$

where individual powers are

- the apparent power:

$$S = \|\boldsymbol{u}\|\|\boldsymbol{i}\|, \tag{4.42}$$

- the active power:

$$P = \|\boldsymbol{u}\|\|\boldsymbol{i}_{\mathrm{a}}\|, \tag{4.43}$$

- the reactive power:

$$Q = \|\boldsymbol{u}\|\|\boldsymbol{i}_{\mathrm{r}}\|, \tag{4.44}$$

- the unbalanced power of the positive sequence:

$$D_{\mathrm{u}}^{\mathrm{p}} = \|\boldsymbol{u}\|\|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}\|, \tag{4.45}$$

- the unbalanced power of the negative sequence:

$$D_{\mathrm{u}}^{\mathrm{n}} = \|\boldsymbol{u}\|\|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}\|, \tag{4.46}$$

- the unbalanced power of the zero sequence:

$$D_{\mathrm{u}}^{\mathrm{z}} = \|\boldsymbol{u}\|\|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}\|. \tag{4.47}$$

The power factor $\lambda$ of that circuit is equal to:

$$\lambda = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2 + D_{\mathrm{u}}^{\mathrm{p}2} + D_{\mathrm{u}}^{\mathrm{n}2} + D_{\mathrm{u}}^{\mathrm{z}2}}} = \frac{\|\boldsymbol{i}_{\mathrm{a}}\|}{\|\boldsymbol{i}\|} = \frac{\|\boldsymbol{i}_{\mathrm{a}}\|}{\sqrt{\|\boldsymbol{i}_{\mathrm{a}}\|^2 + \|\boldsymbol{i}_{\mathrm{r}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}\|^2}}. \tag{4.48}$$

## Three-Phase Four-Wire Circuit Supplied by Sinusoidal and Asymmetrical Voltages – The Measuring Example

The authors of the article built a three-phase four-wire system in order to verify the CPC theory for circuits supplied from an asymmetric sinusoidal voltage source (VSI). The circuit is shown in Figure 4.5.

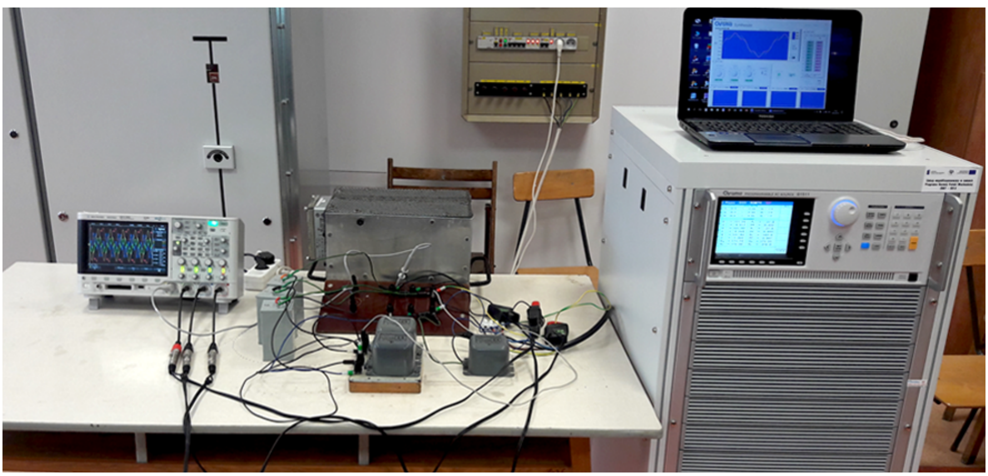

FIGURE 4.5. View of the three-phase four-wire system with the asymmetrical sinusoidal voltage source

According to the assumptions, the load shown in Figure 4.5 is linear and time-invariant (LTI). It was built of resistors, reactors, and capacitors. The parameters of individual elements are listed in Table 4.1.

TABLE 4.1. **Parameters of elements used in the measurement system**

| Parameter | Phase R | Phase S | Phase T |
|---|---|---|---|
| Resistance | 63 Ω | 68 Ω | 65 Ω |
| Inductance | 460 mH | 460 mH | 460 mH |
| Capacity | 9.8 μF | 9.8 μF | 9.8 μF |
| Frequency | 50 Hz | | |

Table 4.2 lists the measured values of asymmetric sinusoidal supply voltage and line currents.

TABLE 4.2. **The values of the measured supply voltage and line currents**

| Parameter | Phase R | Phase S | Phase T |
|---|---|---|---|
| Voltage | $79.89e^{j0^\circ}$ V | $120.02e^{j230^\circ}$ V | $100.06e^{j130^\circ}$ V |
| Current | $0.319e^{-j81.7^\circ}$ V | $1.886e^{j109.3^\circ}$ V | $0.373e^{j84^\circ}$ V |

Figure 4.6 shows the circuit diagram from Figure 4.5.

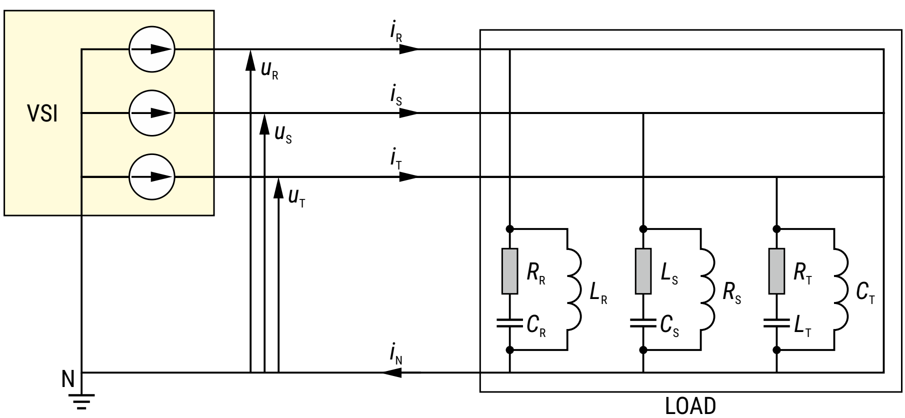


FIGURE 4.6. **Diagram of the measurement circuit from Figure 4.5**

On the basis of (8) three-phase rms values of supply voltage and line current are:

$$\|\boldsymbol{u}\| = 175.497 \text{ V}, \ \|\boldsymbol{i}\| = 1.949 \text{ A}$$

Figure 4.7 presents the waveform of the instantaneous value of the phase voltage (2).

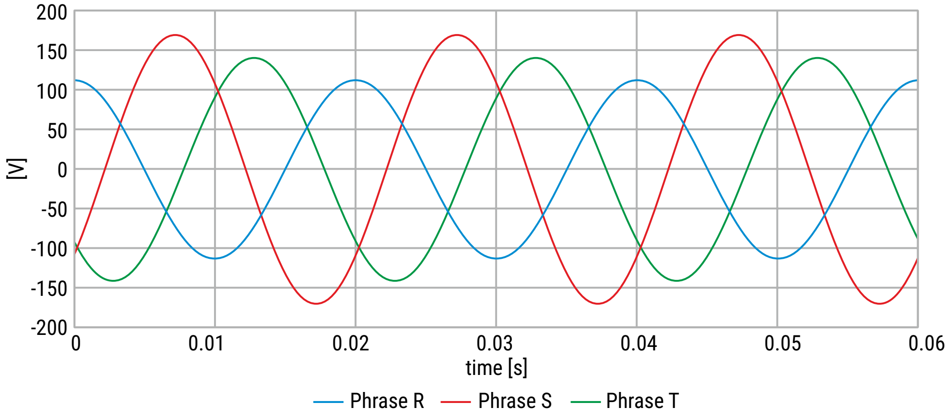


FIGURE 4.7. **The waveform of the instantaneous value of the phase voltage**

Figure 4.8 shows the waveform of the instantaneous current value of the line (3).

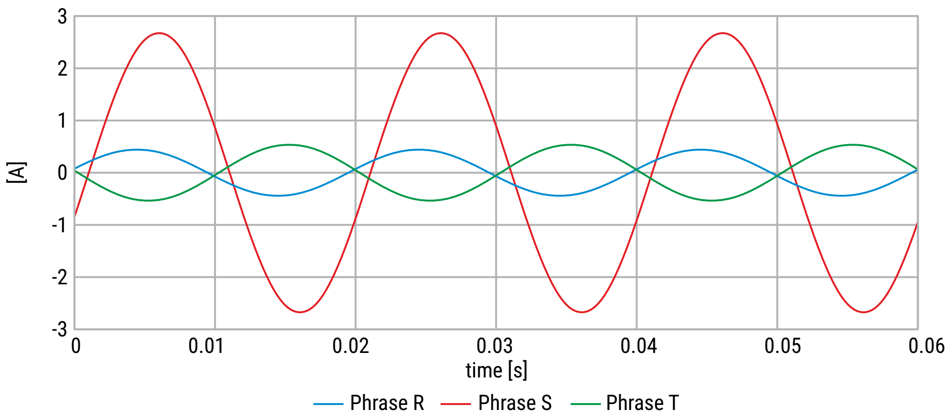


FIGURE 4.8. **The waveform of the instantaneous current value of the line**

According to (5) the Symmetric Fortesque Components are equal to:

$$\begin{bmatrix} \boldsymbol{U}^{\mathrm{p}} \\ \boldsymbol{U}^{\mathrm{n}} \\ \boldsymbol{U}^{\mathrm{z}} \end{bmatrix} = \begin{bmatrix} 98.882e^{-j0.67^{\circ}} \\ 6.439e^{j76.17^{\circ}} \\ 21.148e^{-j166.05^{\circ}} \end{bmatrix} \mathrm{V}\,.$$

On the basis of (4.17–21) the admittances are equal:

- $\boldsymbol{Y}_{\mathrm{b}} = 7.891e^{j6.6^{\circ}}$ mS,
- $\boldsymbol{Y}_{\mathrm{e}} = 5.968e^{-j3.5^{\circ}}$ mS,
- $\boldsymbol{Y}_{\mathrm{d}} = 2.268e^{-j146.1^{\circ}}$ mS,
- $\boldsymbol{A}^{\mathrm{n}} = 5.343e^{j161.4^{\circ}}$ mS,
- $\boldsymbol{A}^{\mathrm{z}} = 5.4303e^{-j93.4^{\circ}}$ mS.

According to (11), (12), (34), (36) and (38) the rms values of the load's currents are:

- $\|\boldsymbol{i}_{\mathrm{a}}\| = 1.376$ A,
- $\|\boldsymbol{i}_{\mathrm{r}}\| = 0.159$ A,
- $\|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}\| = 0.257$ A,
- $\|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}\| = 1.007$ A,
- $\|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}\| = 0.896$ A.

The waveform of the instantaneous value of the active current (9) is shown in Figure 4.9.

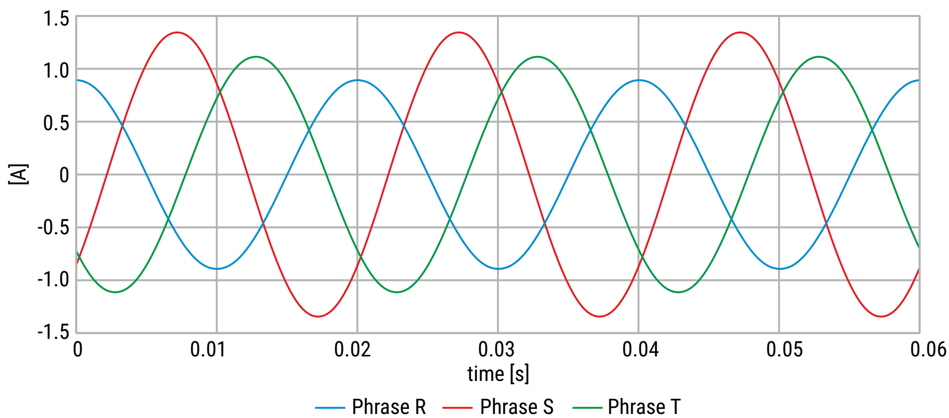


FIGURE 4.9. The waveform of the instantaneous value of the active current

Figure 4.10 presents the waveform of the instantaneous value of the reactive current (10).

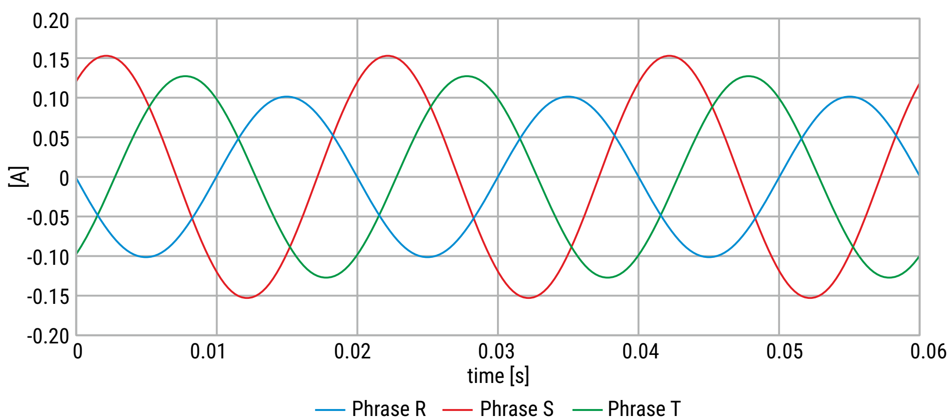


FIGURE 4.10. The waveform of the instantaneous value of the reactive current

Figure 11 shows the waveform of the instantaneous value of the unbalanced current of the positive sequence (33).

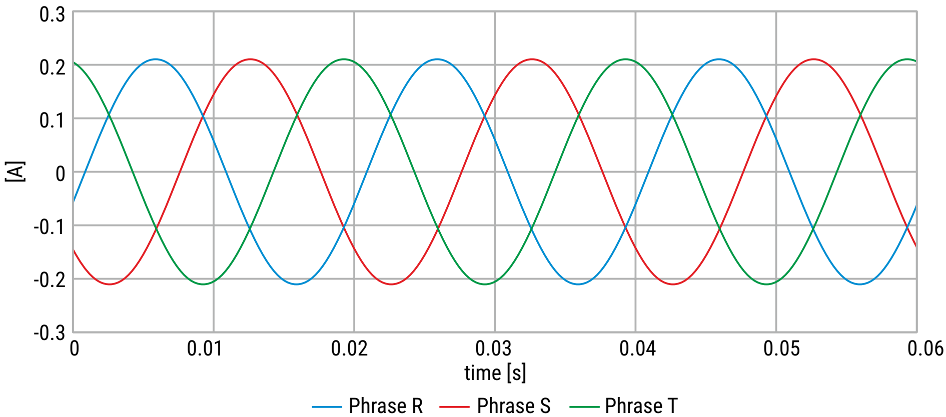


FIGURE 4.11. **The waveform of the instantaneous value of the unbalanced currentof the positive sequence**

The waveform of the instantaneous value of the unbalanced current of the negative sequence (35) is shown in Figure 4.12.

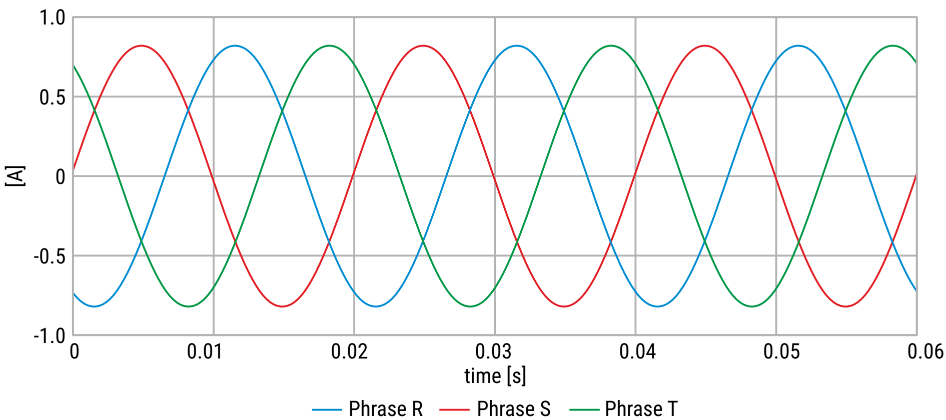


FIGURE 4.12. **The waveform of the instantaneous value of the unbalanced current of the negative sequence**

Figure 4.13 presents the waveform of the instantaneous value of the unbalanced current of the zero sequence (37).

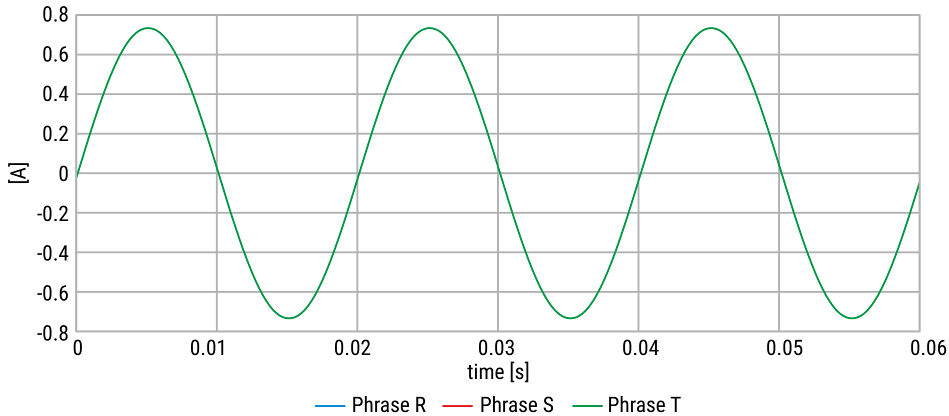

FIGURE 4.13. The waveform of the instantaneous value of the unbalanced current of the zero sequence

Summing up the instantaneous waveforms of the component currents of the load shown in Figures 4.9–4.13, we obtain the instantaneous waveform identical to that shown in Figure 4.8. In addition, according to (40), the three-phase rms value of the line current is equal to the three-phase rms value expressed by (8).

## Conclusion

The article shows that the physical components of the load's current are associated with specific physical phenomena and are possible to describe in the asymmetry of the supply voltage.

Chapter III presents the measurements of electrical quantities, i.e. asymmetrical sinusoidal supply voltage and the line current of the unbalanced load. As it has been proved, the distribution of the measured line current by dependencies derived in Chapter II makes it possible to determine the physical components of the current of the load. In addition, each of the current components is mutually orthogonal, which means the opportunity to calculate the balancing reactance compensator parameters contributing to the compensation of reactive power and balancing the load.

The equations shown in this article allow for describing three-phase four-wire systems with the asymmetrical sinusoidal supply voltage. This is another step in the development of the Currents' Physical Components Theory, which was until then described only with symmetrical power supply.

In addition, this is the commencement of the description of the possibility to determine the parameters of the balancing reactance compensator in asymmetrical four-wire power systems supplied from a sinusoidal or non-sinusoidal voltage source.

## Appendix – Orthogonality

Vectors are mutually orthogonal when the three-phase scalar product [1,2] is equal to zero:

$$(\boldsymbol{x},\boldsymbol{y})=\frac{1}{\mathrm{T}}\int_0^T \boldsymbol{x}(t)^{\mathrm{T}}\cdot\boldsymbol{y}(t)dt=0. \tag{4.A1}$$

If three-phase quantities are expressed in the form of complex rms values, namely:

$$\boldsymbol{x}=\sqrt{2}Re\{\boldsymbol{X}e^{j\omega t}\},\ \boldsymbol{y}=\sqrt{2}Re\{\boldsymbol{Y}e^{j\omega t}\}, \tag{4.A2}$$

then their scalar product is equal to:

$$(\boldsymbol{x},\boldsymbol{y})=\frac{2}{\mathrm{T}}\int_0^T Re\{\boldsymbol{X}^{\mathrm{T}}e^{j\omega t}\}Re\{\boldsymbol{Y}e^{j\omega t}\}dt=Re\{\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Y}^*\}. \tag{4.A3}$$

The active and reactive currents are mutually orthogonal since they are shifted by a quarter of a period. There remains the question of the orthogonality of the sum of these currents relative to the unbalanced current. The value of the scalar product of these currents is:

$$(\boldsymbol{i}_{\mathrm{b}},\boldsymbol{i}_{\mathrm{u}})=Re\{\boldsymbol{I}_{\mathrm{b}}^{\mathrm{T}}\boldsymbol{I}_{\mathrm{u}}^*\}=Re\{\boldsymbol{I}_{\mathrm{b}}^{\mathrm{T}}(\boldsymbol{I}-\boldsymbol{I}_{\mathrm{b}})^*\}=Re\{Y_{\mathrm{b}}\boldsymbol{U}^{\mathrm{T}}(\boldsymbol{I}-Y_{\mathrm{b}}\boldsymbol{U})^*\}=$$
$$=Re\{Y_{\mathrm{b}}\boldsymbol{U}^{\mathrm{T}}\boldsymbol{I}^*-Y_{\mathrm{b}}\boldsymbol{U}^{\mathrm{T}}Y_{\mathrm{b}}^*\boldsymbol{U}^*\}=Re\{Y_{\mathrm{b}}(\boldsymbol{U}^{\mathrm{T}}\boldsymbol{I}^*-\boldsymbol{U}^{\mathrm{T}}Y_{\mathrm{b}}^*\boldsymbol{U}^*)\}=Re\{Y_{\mathrm{b}}(\boldsymbol{C}-\boldsymbol{C}_{\mathrm{b}})\}=0.$$

The currents of the positive and negative sequence are mutually orthogonal because of the opposite of their sequence. The scalar product of the unbalanced current of the positive and zero sequence $(\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}},\ \boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}})$ is calculated as follows:

$$(\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}},\ \boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}})=Re\{\boldsymbol{I}_{\mathrm{u}}^{\mathrm{p}^{\mathrm{T}}}\boldsymbol{I}_{\mathrm{u}}^{\mathrm{z}^*}\}=$$
$$=Re\{(\boldsymbol{1}^{\mathrm{p}}(Y_{\mathrm{d}}\boldsymbol{U}^{\mathrm{p}}+\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{z}}+\boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{n}}))^{\mathrm{T}}\cdot(\boldsymbol{1}^{\mathrm{z}}(Y_{\mathrm{d}}\boldsymbol{U}^{\mathrm{z}}+\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{p}}))^*\}=$$
$$=Re\{(Y_{\mathrm{d}}\boldsymbol{U}^{\mathrm{p}}+\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{z}}+\boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{n}})^{\mathrm{T}}\cdot(Y_{\mathrm{d}}\boldsymbol{U}^{\mathrm{z}}+\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{p}})^*(1+\alpha^*+\alpha)\}=0$$

and the unbalanced current of the negative and zero sequence $(\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}},\ \boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}})$:

$$(\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}},\ \boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}})=Re\{\boldsymbol{I}_{\mathrm{u}}^{\mathrm{n}^{\mathrm{T}}}\boldsymbol{I}_{\mathrm{u}}^{\mathrm{z}^*}\}=$$
$$=Re\{(\boldsymbol{1}^{\mathrm{n}}(Y_{\mathrm{d}}\boldsymbol{U}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{p}}+\boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{z}}))^{\mathrm{T}}\cdot(\boldsymbol{1}^{\mathrm{z}}(Y_{\mathrm{d}}\boldsymbol{U}^{\mathrm{z}}+\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{p}}))^*\}$$
$$=Re\{(Y_{\mathrm{d}}\boldsymbol{U}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{p}}+\boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{z}})^{\mathrm{T}}\cdot(Y_{\mathrm{d}}\boldsymbol{U}^{\mathrm{z}}+\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}\boldsymbol{U}^{\mathrm{p}})^*(1+\alpha+\alpha^*)\}=0.$$

With respect to the fact that all currents are mutually orthogonal, relationship (40) is accomplished.

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