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Chapter 10
Estimation of the maximum load power in periodic WPT systems

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The article presents results for a proposed analytical solution to the analysis of the periodic Wireless Power Transfer (WPT) system. The model consists of transmitting and receiving plane coils. The analysis takes into account different variants geometry like distance between the transmitting and receiving coils and the number of turns. The influence of variable system geometry and the frequency on system efficiency is observed. The obtained results indicate at which system parameters wireless energy transfer is possible. The proposed systems can be used for wireless charging of mobile devices, and to shape the distribution of the magnetic field. The given solution allows for studying the influence of coil geometry on power transmission.

Index terms: wireless power transfer (WPT), magnetic field, analytical methods, inductive elements.

Introduction

The demand for energy in wireless devices continues to increase. Along with the growing demand for such energy, problems arise regarding the requirements for: batteries with increased capacity or limited charging time. It is becoming more common to power mobile devices by charging with the use of wireless power transfer (WPT) [1–8].

WPT is increasingly used, among others in the automotive industry, in solutions for hybrid and electric cars [4–9]. In order to charge the batteries while driving, distributed coils along the path of the car were used. Thanks to such use of WPT, it is possible to use electric vehicles, e.g. in factories.

The WPT system is an alternative method of charging wireless devices, where a pair of coils [9–13] or an array of coils [14, 15] is used. The WPT system is also considered in the systems of beacons [10], medical implants in human body [11] and inside buildings with sensors [24–25].

WPT systems are still analyzed by using various solutions and configurations of this type of systems [16–20]. Every solution requires a multi-variant analysis and verification of the results. In order to avoid early prototyping and performing a number of analyzes, it is possible to apply analytical and numerical methods at the design stage.

In the article, the WPT system, in which a set of several coils participating in energy transfer was replaced with surfaces made of periodically distributed planar coils, was analyzed. Adjacent segments containing a pair of coils (transmit and receive) between which energy is exchanged, can be used to power multiple independent loads or replace conventional WPT systems. The developed analytical method for solving WPT systems was presented. Its target is to quickly determine the parameters of the system (e.g. power, efficiency) without the need to make complex models and solve them using numerical methods.

Calculations were performed to determine the parameters of WPT systems, i.e. equivalent properties, source power, receiver power and efficiency. The analysis was multi-variant – the number of turns, their parameters and the distance between coils were changed.

The article presents the approach and results of the analytical method. By an appropriate selection of load resistance, it was possible to calculate the maximum power of the receiver of the WPT system. The proposed method may be an alternative to the use of experimental prototypes applied in WPT systems.

Analyzed model

This article presents the periodic wireless power transfer system with many inductive elements. The considered system is composed of many pairs of transmitters and receivers, which constitute a WPT cell with outer dimensions $d \times d$ (Fig. 10.1).

The coils are identical with parameters: radius (r), where $d \approx 2r$ and number of turns (n). The windings are wound around a dielectric carcass with additional compensating capacitors. The transmitting and receiving coils are placed at a distance (h). The analyzed periodic distribution of WPT cells give transmitting and receiving surfaces, where between them the energy transmission occurs. The transmitting surface consist of coils which are connected parallel to a sinusoidal voltage source (U).

Planar spiral coils were wound by several dozen of turns, made of ultra-thin wires with diameter w and insulated from each other by an electrical insulator of thickness i. The compensating capacitor can be modeled as an element with lumped capacity C. A voltage source (U) with specified frequency (f) is connected with the transmitting coil, while load Z is connected with the receiving coil.

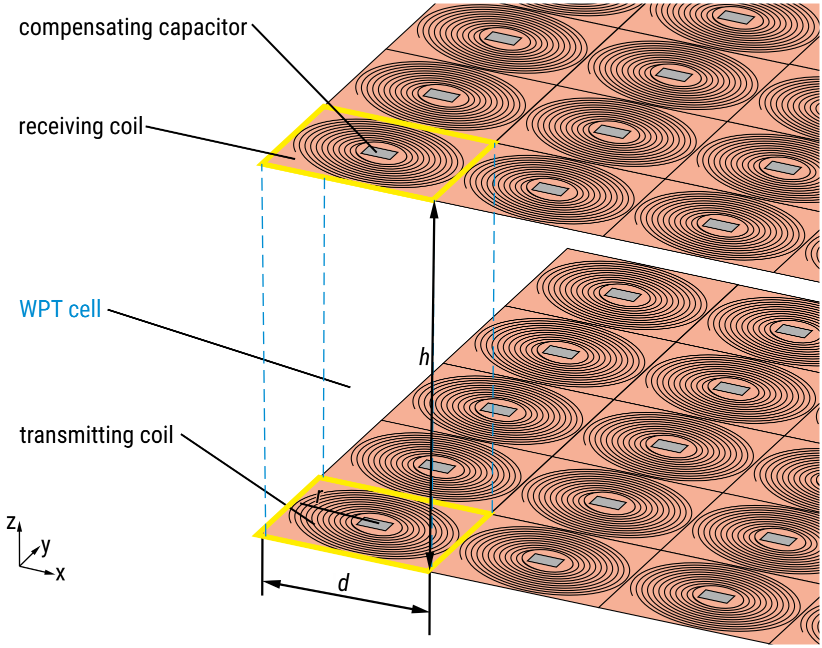

FIGURE 10.1. **Periodic wireless power transfer system**

Analytical approach

The implementation of the numerical model is more difficult than the analytical one due to, inter alia, the appropriate selection of boundary conditions and many necessary simplifications imposed by the adopted numerical method. Therefore, in many cases a simpler model is desirable, providing a faster initial solution.

The article presents the developed analytical solution for the model of periodic WPT system (Fig. 10.1). The analysis of the infinitely extensive periodic network was reduced to the case of a single WPT cell. The solution of the analytical model in the frequency domain can be performed using methods of circuit analysis (Fig. 10.2). The main problem in this type of analysis is to determine the values of the lumped parameters, taking into account the influence of adjacent segments on the equivalent inductances of the transmitting coil L_{tr} and the receiving coil L_{re} and their mutual inductance M_{tr}.

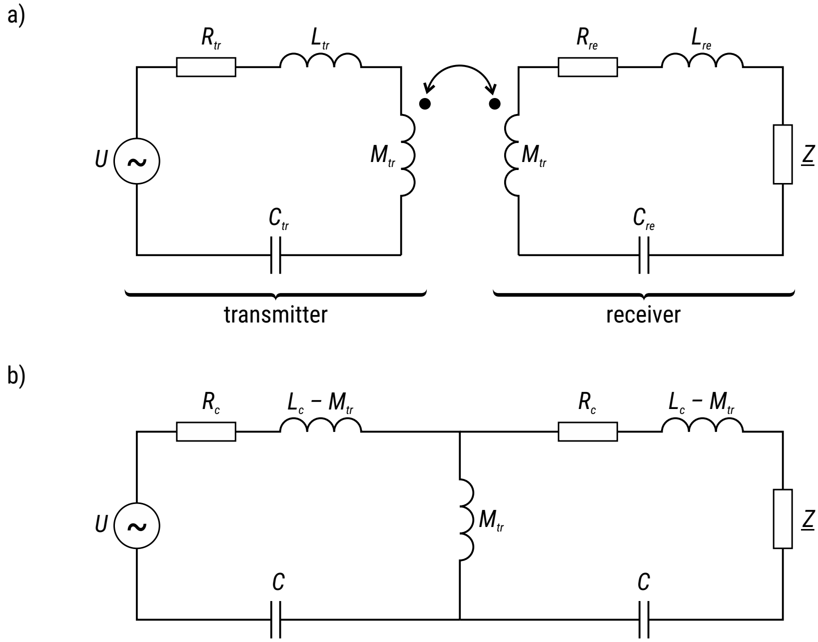

FIGURE 10.2. Analytical model of the WPT cell: (a) overall model of periodic cell, (b) replacement model of the cell for identical transmitting and receiving coils

The resistance of a coil is determined by replacing the spiral structure of windings converging circles of equal width *w+i*. The whole length of all circles is represented by:

$$l_{sum} = 2\pi n\left[r-(n-1)(w+i)\right]. \tag{10.1}$$

The equation determining the resistance of a conductor is described by:

$$R_c = \frac{l_{sum}}{\sigma\pi\frac{w^2}{4}} \tag{10.2}$$

The self-inductance of a spiral planar coil is calculated using equation [14]:

$$L_{self} = \frac{\mu_0 c_1 d_m n^2}{2}\left[\ln\left(\frac{c_2}{wsp}\right)+c_3 wsp + c_4 wsp^2\right], \tag{10.3}$$

where d_m – diameter and *wsp* is a fill factor;

$$d_m = \frac{2r+2\left[r-n(w+i)\right]}{2}, \tag{10.4}$$

$$wsp = \frac{2r-2\left[r-n(w+i)\right]}{2r+2\left[r-n(w+i)\right]}. \tag{10.5}$$

Coefficients c_1, c_2, c_3, c_4 used in the equation (10.3) depend on the shape of a coil [14]. For identical transmitting and receiving coils the calculated inductances are equal to $L_c = L_{tr} = L_{re}$ (Figure 10.2). Mutual inductance M_{pe}, which directly results from the periodic distribution of coils arranged on the surface, is a sum of all mutual inductances [8, 21]

$$M_{pe} = \sum_i \sum_j \left(M_{x+i,y+j} \right) - M_{x,y}, \tag{10.6}$$

where:
$M_{x+i,y+j}$ – mutual inductance between the coil and the coil at i-th column and j-th row,
$M_{x,y} = L_{self}$ is self-inductance.
Taking into account the assumptions that the system is periodic and symmetrical ($M_{x+i,y+j} = M_{x-i,y-j}$), equation (10.6) is simplified to the formula:

$$M_{pe} = 8M_{x,y+1}, \tag{10.7}$$

where:
$M_{x,y+1}$ is mutual inductance between the coil and an edge of the adjacent coil (Figure 10.2) and is calculated on the basis of the formula [8, 14, 21]

$$M_{x,y+1} = \frac{\mu_0 p^2}{4\pi} \int_{\Phi_i}^{\Phi_o} \int_{\Phi_i}^{\Phi_o} \frac{\left[(1+\phi_1\phi_2)\cos(\phi_2 - \phi_1) - (\phi_2 - \phi_1)\sin(\phi_2 - \phi_1) \right] d\phi_1 d\phi_2}{\sqrt{\left(d_m + p\phi_2 \cos\phi_2 - p\phi_1 \cos\phi_1\right)^2 + \left(p\phi_2 \sin\phi_2 - p\phi_1 \sin\phi_1\right)^2}} \tag{10.8}$$

where:
$p = (w+i)/(2\pi)$,
$\Phi_i = [r - (w+i)n]/p$,
$\Phi_o = r/p$.
Assuming equation (10.7), the inductance of the considered coil in the segment $A_{x,y}$ takes a form of:

$$L_c = L_{self} + M_{pe} = L_{self} + 8M_{x,y+1}. \tag{10.9}$$

Compensating capacity C at definite frequency is represented by:

$$C(f) = \frac{1}{4\pi^2 f^2 L_c} = \frac{1}{4\pi^2 f^2 \left(L_{self} + M_{pe}\right)} = \frac{1}{4\pi^2 f^2 \left(L_{self} + 8M_{x,y+1}\right)} \tag{10.10}$$

Mutual inductance M_{tr} between the transmitter and the receiver is presented by

$$M_{tr} = \frac{\mu_0 p^2}{4\pi} \int_{\Phi_i}^{\Phi_o} \int_{\Phi_i}^{\Phi_o} \frac{\left[(1+\phi_1\phi_2)\cos(\phi_2 - \phi_1) - (\phi_2 - \phi_1)\sin(\phi_2 - \phi_1) \right] d\phi_1 d\phi_2}{\sqrt{h^2 + p^2\phi_1^2 + p^2\phi_2^2 - 2p^2 \phi_1 {}_2 \cos(\phi_2 - \phi_1)}} \tag{10.11}$$

Assumptions of the analysis

Using the proposed analytical method, many variants of the periodic WPT systems were analyzed (Table 1). With the same coil radius, the number of turns and the distance between the coils (transmitter and receiver) were changed.

TABLE 10.1. Geometrical parameters for analyzed models

r [mm]	*n*	*h* [mm]
25	40	12.5 (*r* /2) and 25 (*r*)
25	50	12.5 (*r* /2) and 25 (*r*)
25	60	12.5 (*r* /2) and 25 (*r*)
25	70	12.5 (*r* /2) and 25 (*r*)
25	80	12.5 (*r* /2) and 25 (*r*)
25	90	12.5 (*r* /2) and 25 (*r*)

The values, which were used in the analysis, are presented in Table 10.2.

TABLE 10.2. Values used in the analysis

parameter	symbol	value
wire with a diameter	*w*	200 μm
conductivity of wire	*σ*	$5.6 \cdot 10^7$ S/m
source with an effective value	*U*	1 V
thickness of wire insulation	*i*	5 μm

The analysis is connected with the frequency domain from f_{min} = 100 kHz to f_{max} = 1000 kHz. The value of coefficients used in equation (10.3) are $c_1 = 1$, $c_2 = 2.5$, $c_3 = 0$, $c_4 = 0.2$.

On the basis of the obtained results for several exemplary periodic WPT systems, the correctness of the proposed analytical model was verified by comparing the active power of the receiver:

$$P_o = Z\left|\underline{I}_{re}\right|^2, \tag{10.12}$$

where $\underline{I}_{re}$ is a current flowing through the receiving coil.

Transmitter power is represented by:

$$P_z = U\underline{I}_{tr}, \tag{10.13}$$

where $\underline{I}_{tr}$ is a current flowing through the transmitting coil.

Using equations (12) and (13), the power transfer efficiency was described by:

$$\eta = \frac{P_o}{P_z} 100\%. \tag{10.14}$$

Additionally, the results were based on the correct selection of Z_p (optimal load impedance) to make the maximum power transfer:

$$Z_p = R_c + \frac{\omega^2 M_{tr}^{\ 2}}{R_c}. \tag{10.15}$$

Results of the analysis

The results of the analysis of the WPT system were obtained by the analytical method. The results were related to the maximum load power. In order to determine the maximum power transmitted to the receiver, the values of the load impedance were calculated, taking into account the number of turns and the distance between the coils. The transmitter power (P_z) (Figs. 3, 6, 9, 12), the receiver power (P_o) (Figs. 4, 7, 10, 13), power transfer efficiency (η) (Figs. 5, 8, 11, 14) were presented on this basis.

The characteristics for the model, where the distance between the coils was half the radius (h = 12.5 mm) are shown in Figs. 10.3–10.8. Figures 10.9–10.14 show the characteristics for the model, where the distance between the coils was equal to the radius (h = 25 mm).

The efficiency of the system for n = 40 ÷ 60 with h = 12.5 mm reaches 50% with P_o in the range of 62÷84 mW. However, for a larger number of turns, with a lower receiver power (50÷56 mW), a higher efficiency of the WPT system is achieved.

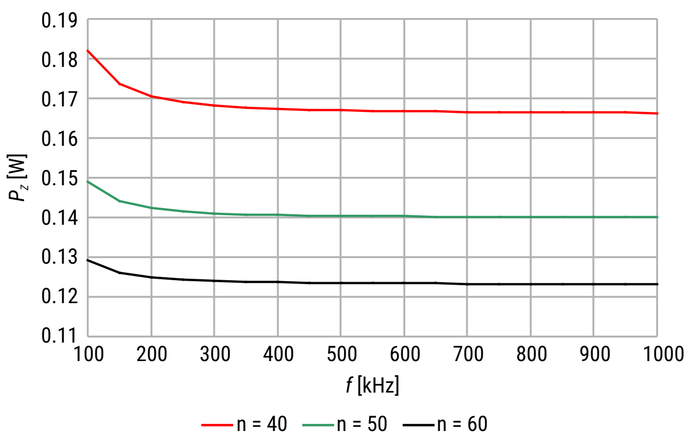

FIGURE 10.3. Results of transmitter power (P_z) dependent on the number of turns (n=40÷60) at distance h=12.5 mm

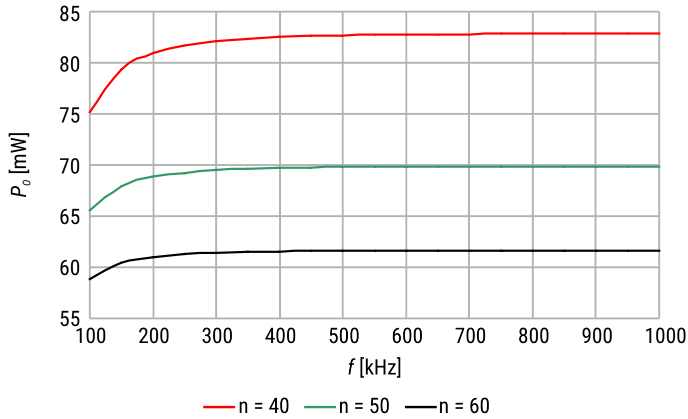

FIGURE 10.4. Results of receiver power (P_o) dependent on the number of turns (n=40÷60) at distance h=12.5 mm

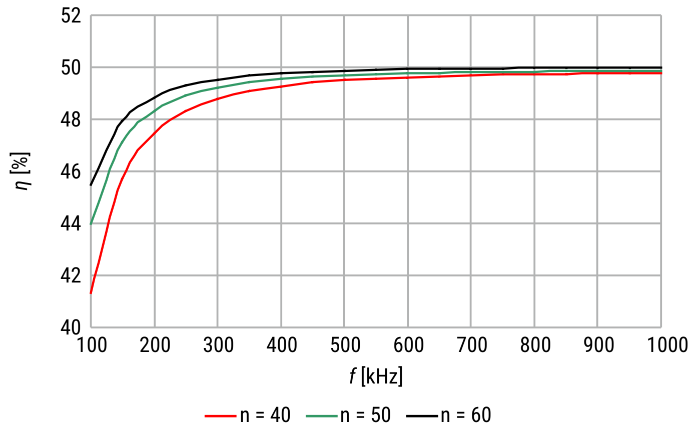

FIGURE 10.5. Results of power transfer efficiency dependent on the number of turns (n=40÷60) at distance h=12.5 mm

Doubling the distance causes the transmitter power to have higher values than at h = 12.5 mm by even 70% (Figs. 10.3, 10.9).

Doubling the distance between the coils results in lower efficiency system values. For n = 40÷60, the efficiency does not reach 50% (Fig. 10.11). At 500 kHz and twice the distance between the coils, the efficiency is 10% lower than for the half-radius distance (Figs. 10.5, 10.11). A similar relationship is for a larger number of turns (n = 70÷90). For h = 12.5 mm, the efficiency reaches 50% already at f = 200÷400 kHz (Fig. 10.8). However, for twice the distance between the coils, the system reaches 50% efficiency only at 1000 kHz (Fig. 10.14).

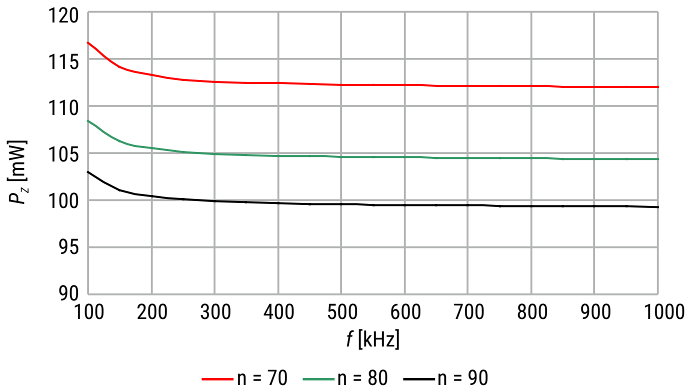

FIGURE 10.6. Results of transmitter power (P_z) dependent on the number of turns (n=70÷90) at distance h=12.5 mm

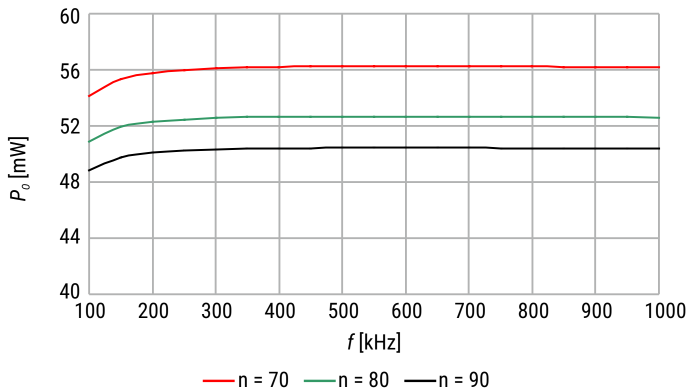

FIGURE 10.7. Results of receiver power (P_o) dependent on the number of turns (n=70÷90) at distance h=12.5 mm

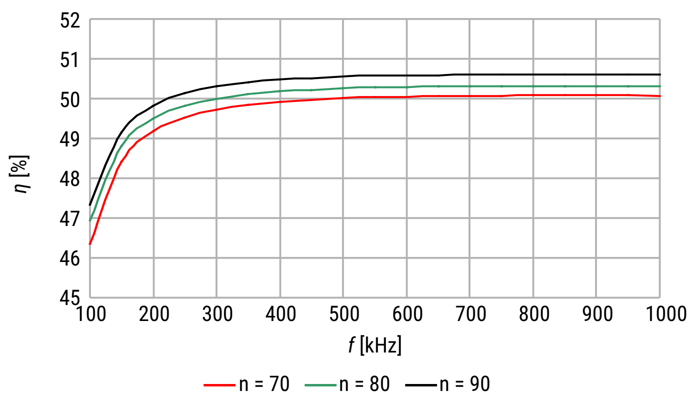

FIGURE 10.8. Results of power transfer efficiency dependent on the number of turns (n=70÷90) at distance h=12.5 mm

At $h = 25$ mm, the efficiency of the system has similar values for a different number of turns ($n = 70 \div 90$) (Fig. 10.14), although the receiver power has different values (Fig. 10.13). The efficiency is 50% at 700 kHz when $n = 40 \div 60$ and the distance between the coils is half the radius (12.5 mm) (Fig. 10.5). Whereas for $n = 70 \div 90$, efficiency is higher than 50% already at 300 kHz (Fig. 10.8).

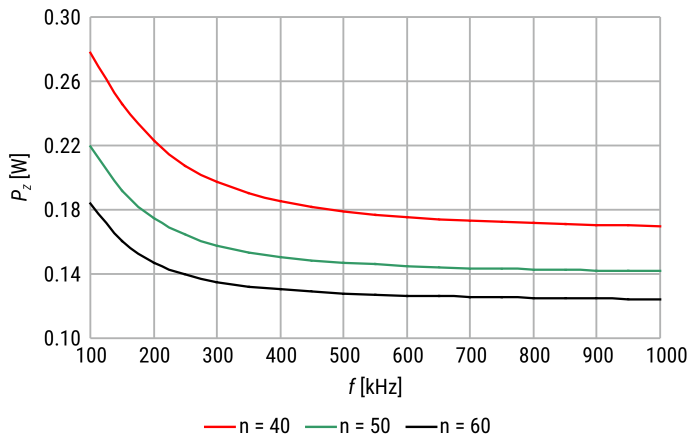

FIGURE 10.9. Results of transmitter power (P_z) dependent on the number of turns (n=40÷60) at distance h=25 mm

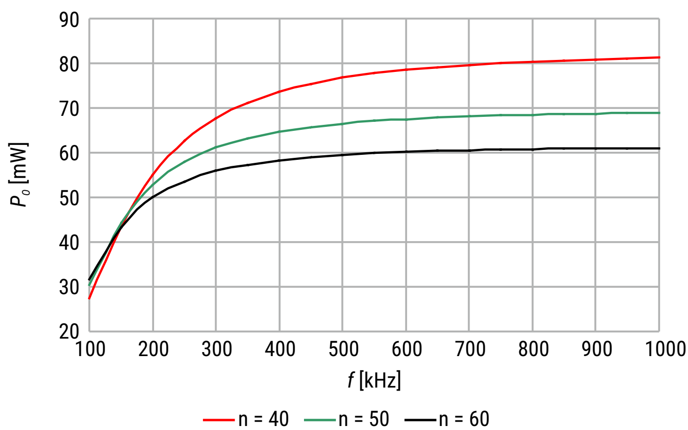

FIGURE 10.10. Results of receiver power (P_o) dependent on the number of turns (n=40÷60) at distance h=25 mm

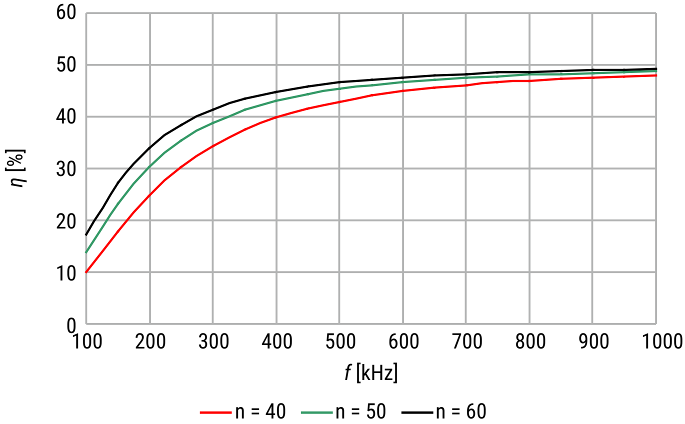

FIGURE 10.11. Results of power transfer efficiency dependent on the number of turns (*n*=40÷60) at distance *h*=25 mm.

Due to the fact that the characteristics of receiver power were "stabilizing" along with the efficiency tending to 50%, by increasing *n* it was possible to compensate this effect and broaden the frequency range of the maximum load power.

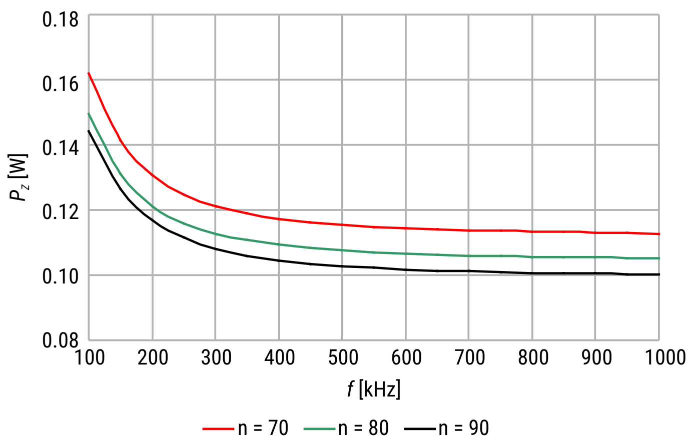

FIGURE 10.12. Results of transmitter power (P_z) dependent on the number of turns (*n*=70÷90) at distance *h*=25 mm

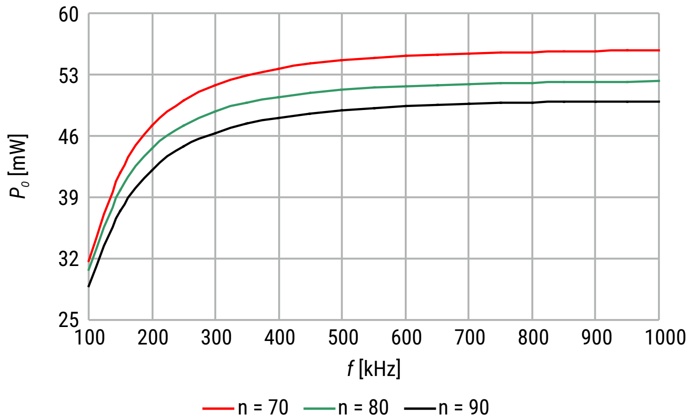

FIGURE 10.13. Results of receiver power (P_o) dependent on the number of turns (n=70÷90) at distance h=25 mm

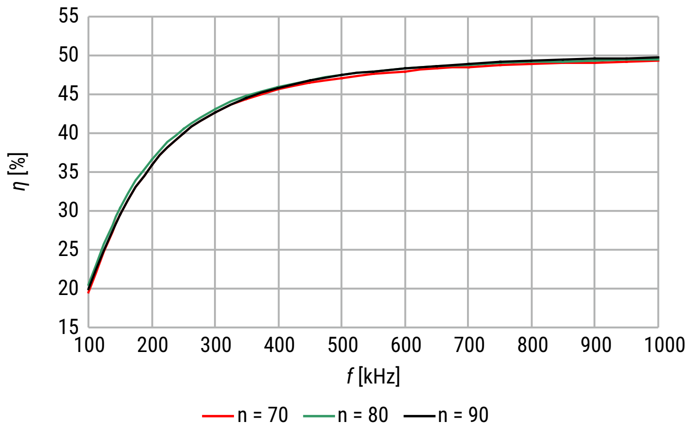

FIGURE 10.14. Results of power transfer efficiency dependent on the number of turns (n=70÷90) at distance h=25 mm

Conclusion

The maximum load power in periodic WPT systems was estimated based on exemplary structures with many magnetic couplings between constituent inductors. The models used for solving considered structures, arranged of many spiral planar coils, were developed. The purpose was to quickly determine the output parameters (e.g. power, efficiency) by using analytical equations. The utilization of the electrical circuit, representing a single cell of the WPT system, eliminated the need to make very complex models, which are solved using numerical methods.

The given solutions, adopted in electrical models, allow for studying the influence of coil geometry and a distance between the transmitter and the receiver on power transmission. Calculations were performed over a wide frequency range. The analysis concerned the influence of geometrical parameters of coils in the WPT cell (e.g. number of turns) on the efficiency of the system and the power of the transmitter and the receiver. By simply adjusting the number of turns and increasing the frequency of a current, it was possible to obtain high power transmission for the supplied loads using the proposed system, without the use of intermediate coils or iron cores. By an appropriate selection of load impedance it was possible to determine the power transferred to the receiver and corresponding efficiency.

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