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7. FRACTIONAL-ORDER LTI SYSTEM IDENTIFICATION USING INTEGER-ORDER STATE-SPACE MODEL

The optimal input signal design is a technique of generating an informative excitation signal to estimate the model parameters with maximum accuracy. In the paper, a novel optimal input formulation and a numerical scheme for fractional-order LTI system identification are presented. The Oustaloup recursive approximation (ORA) method is used to determine the fractional-order differentiation in an integer-order state-space form. Then, the proposed method is used to obtain an optimal input signal for fractional-order system parameter estimation from the interval $0.5 \leq \alpha \leq 2.0$. The methodology presented in this paper has been verified using numerical examples, and the experiment results have been discussed.

7.1. INTRODUCTION

Fractional-order calculus has received a lot of attention in different scientific fields, including precise system modeling and automatic control problems [1, 2]. It has been shown that fractional-order models guarantee a more exact system dynamics depiction because real-life processes appear to be of non-integer order [3, 4]. The fractional-order calculus is the generalization of integration and differentiation where the power is of fractional-order [5]. Many reports have been devoted to study the accuracy of the non-integer calculus in application to different domains, e.g.: bioengineering [6], physics [7, 8], chaos theory [9], control systems [10, 11] and fractional signal processing [12, 13].

It is clear that the rise of interest in the fractional-order calculus domain has a relationship with the increasing availability of high-performance computational packages. Adaptation of the methods of the fractional-order estimation to real-life industrial problems should bring about quality improvement and cost minimization. Moreover, fractional-order approximation methods used for automatic control purposes

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results in the improvement of control loops. Fractional-order calculus is often applied to robotics and automation for the system identification and automatic control purposes [2]. The control performance evaluation has a great impact on the economic condition of the real-life processes. In contrast to the fractional-order controllers, a conventional PID controller has been shown to be unsatisfactory for industrial applications due to its limited tuning flexibility [14].

The optimal excitation signal design task concerns the optimal control methods for linear and nonlinear integer-order systems. The main goal of this paper is to introduce a novel optimal input design formulation and the numerical scheme for fractional-order system identification. The methodology is presented using the LTI inertial model. The Oustaloup recursive approximation (ORA) method has been used to exact approximation of the non-integer order operator, which is then transformed into a zero-pole transfer function [15]. The estimation results are then used for a transfer function conversion into an integer-order state-space form. The problem appears for fractional-orders ($\alpha > 1$) during the transfer function conversion into the state-space form, when the order of the numerator is equal to the order of the denominator. This issue has been solved by augmenting a fractional-order system dynamics with one extra state. An optimal input design task for non-integer order linear time-invariant system identification has been verified by numerical examples in an order range from the interval (0.5, 2.0). The problems of the optimal input design, in the context of the integer-order system identification, are considered in earlier works of the author [16, 17, 18].

7.2. FRACTIONAL-ORDER CONTROL PROBLEM

The problem of the fractional-order calculus is a generalization of integral and differential operators to a non-integer operator ${}_aD_t^\alpha$. The continuous operator of the fractional-order α is given by

$$ {}_aD_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt^\alpha} & \Re(\alpha) > 0, \\ 1 & \Re(\alpha) = 0, \\ \displaystyle\int_a^t (d\tau)^{-\alpha} & \Re(\alpha) < 0, \end{cases} \tag{7.1} $$

where: a, t - denote the limits of the process and α is the set for all complex numbers. The fractional-order calculus is the special case of a classical integer-order differential equations task. Linear fractional-order continuous-time SISO dynamic system is comensature-order if all powers of a derivative are integer multiples of the order q in such a way that $\alpha_k, \beta_k = kq, q \in R^+$, and is given by the following equation [1, 2]

$$ \sum_{k=0}^{n} a_k D_t^{\alpha_i} y(t) = \sum_{k=0}^{m} b_k D_t^{\beta_i} u(t), \tag{7.2} $$

where: a_k, b_k are model constant coefficients. The discrete-time formulation for different orders can be discovered from [19]. The LTI model is of rational-order if $q = r^{-1}$, and $q \in R^{+}$. Using the Laplace transformation to equation (2), and applying zero initial conditions to the input-output specification of the fractional-order model, the transfer function formulation can be written as

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}}. \tag{7.3}$$

The continuous-time system of commensurate-order q can be modified to obtain the pseudo-rational transfer function formula $H(\lambda)$ in the form

$$H(\lambda) = \frac{\sum_{k=0}^{m} b_k \lambda^k}{\sum_{k=0}^{n} a_k \lambda^k}, \tag{7.4}$$

where $\lambda = s^q$. On the basis of this conception, pseudo-rational description of the fractional-order linear time-invariant model can be formulated by a state-space equation given by

$$\begin{aligned} {}_0D_t^{\alpha} x(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t). \end{aligned} \tag{7.5}$$

For the model parameters estimation purposes, the difference equation representing input-output dynamics of the system is more useful than the state-space formulation. However, the state-space model description provides multiple input and multiple output (MIMO) fractional-order systems representation.

7.3. FRACTIONAL-ORDER OPERATOR APPROXYMATION

The problem of approximating the fractional-order system by an integer-order one has been presented in [1]. The Oustaloup recursive approximation (ORA) method, which has a very good fitting to the fractional-order transfer functions is widely used in practice. We focus our attention on the Oustaloup recursive approximation algorithm during the experiments. Choosing the appropriate frequency fitting range, the problem of a fractional differentiator or a fractional integrator estimation can be solved using following formulas

$$s^{\alpha} \approx K \prod_{k=1}^{N} \frac{s + \omega_k'}{s + \omega_k}, \tag{7.6}$$

where poles, zeros and a gain of the filter can be obtained from

$$\omega_k' = \omega_b \cdot \omega_u^{(2k-1-\alpha)/N}, \tag{7.7}$$

$$\omega_k = \omega_b \cdot \omega_u^{(2k-1+\alpha)/N}, \tag{7.8}$$

$$K = \omega_h^{\alpha}, \tag{7.9}$$

$$\omega_u = \sqrt{\frac{\omega_h}{\omega_b}}. \tag{7.10}$$

Where N is the order of the approximation, and ω_b, ω_h are the selected frequency fitting range. The order of the approximation is $2N + 1$, and considering higher orders of N the approximation results should be more accurate.

The Oustaloup filter provides an exact fitting to a fractional operator in a chosen frequency interval, and a orders range [4]. Thus, for the fractional-order operators, where $\alpha \geq 1$ one should separate a fractional order using the following formula

$$s^{\alpha} = s^n s^{\gamma}, \tag{7.11}$$

where $n = \alpha - \gamma$ is the integer part of α and s^{γ} is solved according to (7.6) using Oustaloup recursive approximation. The transfer function obtained from ORA filter has been used to transform the external model form into the integer-order internal state-space representation. In general, the n-th order transfer function obtained from the pole-zero formula is as follows

$$H(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}, \tag{7.12}$$

where a and b are the factors of the polynomials in descending powers of s, and $a_0 = 1$. Then, it is possible to determine an optimal input signal for the fractional-order system identification using integer state-space equation [15].

Since the choice of the state coefficients can differ, the transfer function representation can also be different. Referring to publication [20], the fractional order operator ${}_{t_0}D_t^{\alpha}$ has the following form

$${}_{t_0}D_t^{\alpha} x(t) \approx \begin{Bmatrix} \dot{z} = A_F z + B_F u \\ x = C_F z + D_F u \end{Bmatrix}, \tag{7.13}$$

where the corresponding matrices are as follows

$$A_F = \begin{bmatrix} -a_{n-1} & -a_{n-2} & \cdots & -a_1 & -a_0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, \tag{7.14}$$

$$B_F = [1 \quad 0 \quad 0 \quad \dots \quad 0]^T, \tag{7.15}$$

$$C_F = \begin{bmatrix} (b_n - a_n b_0)(b_{n-1} - a_{n-1} b_0) \cdots \\ \cdots (b_2 - a_2 b_0)(b_1 - a_1 b_0) \end{bmatrix}, \tag{7.16}$$

$$D_F = b_0 = d. \tag{7.17}$$

To solve the problem of the optimal input design for fractional-order model identification, there is a need to approximate the fractional-order operator, and transform this problem, to be solved using one of the available software packages for optimal control.

7.4. OPTIMAL INPUT GENERATION PROBLEM

To illustrate the efficacy of this technique to fractional-order system parameter estimation, using the ORA filter, we have selected Riots_95 toolbox, which has been developed to solve the optimal control problems [21]. The optimal excitation signal design for fractional-order system identification that minimizes objective function is as follows

$$J = g\left(C_F z(t_0) + D_F u(t_0), C_F z(t_f) + D_F u(t_f)\right) + \int_{t_0}^{t_f} l(C_F z + D_F u, u, t) dt, \tag{7.18}$$

subject to the system dynamics

$$\dot{z}(t) = A_F z + B_F\big(h(C_F z + D_F u, u, t)\big), \tag{7.19}$$

with respect to the initial conditions

$$z(t_0) = \frac{x_{t_0} T}{C_F T}. \tag{7.20}$$

The real state-space variable $x(t)$ formulation is given by

$$x(t) = C_F z(t) + D_F u(t), \tag{7.21}$$

where x is the state-space vector, $t \in [t_0, t_f]$ is time duration. The potential set of constraints is as follows

$$u(t) \in \langle u_{min}(t), u_{max}(t) \rangle, \tag{7.22}$$

$$\big(C_F z(t_0) + D_F u(t_0)\big) \in \langle u_{min}(t_0), u_{max}(t_0) \rangle. \tag{7.23}$$

The convergence of the optimization is related with the vector T selection. Regarding to vector B_F, which is described by the matrix (7.15), vector T is given by

$$T = [1 \quad 0 \quad 0 \quad \dots \quad 0]^T. \tag{7.24}$$

The angular pulsation for the Oustaloup approximation has been selected as [0.01, 100] rad/s. The selection of the frequency range depends on the discretization of the control duration corresponding to the software algorithm for solving OCP problems, a wide fitting range increases the computational effort. The final time has been chosen as $t_f = 1.5\ s$. The selection of the Oustaloup filter order N has been based on the below rule

$$N = log(\omega_h) - log(\omega_b). \tag{7.25}$$

The frequency range selection for the ORA method is a very important step because a narrow bandwidth results in a lack of the fit.

7.5. FRACTIONAL ORDER SYSTEM IDENTIFICATION

The problem of an optimal input design for the fractional-order LTI system identification is presented in this chapter. The optimal control method for fractional-order model approximation in the state-space form has been presented in [20]. The main purpose of this method is to represent the optimal input design problem using the Lagrange form with the chosen set of constraints. To verify the suitability of this method to the system parameter identification purposes, a fractional inertial model has been selected

$$G(s) = \frac{k}{s^{\alpha}T + 1}, 0.5 \leq \alpha \leq 2.0, \tag{7.26}$$

where $k = 1$ is the gain of the model, and $T = a_1/a_0 = 1$ is the time constant. The fractional -order LTI system should be presented by the state-space equation given by

$$\begin{aligned} {}_0D_t^{\alpha}x(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t) + v(t), \end{aligned} \tag{7.27}$$

where $u(t)$, $y(t)$ are the input and output vectors, $x(t)$ is the state vector, A, B, C, D are the state-space matrices describing the system dynamics, and $v(t)$ is a stationary random Gaussian noise process. The main objective of the system parameter identification is to maximize the sensitivity of the state variable to the unknown model parameter [22]. The Cramer-Rao definition provides a lower bound for the variance of an unbiased parameter to be identified. Applying the above definition to input design purposes, one can obtain the parameter estimate which is lowered, for optimal inputs

$$cov(A, B, C, D) \geq M^{-\mathbf{1}}. \tag{7.28}$$

The optimal input signal problem for fractional-order inertial system identification is verified in this paper. According to the Cramer-Rao rule, the sensitivity of the state variable $x(t, d)$ to the parameter d (i.e. the gain of the open-loop system) has been maximized. The objective function formulated based on [22] is as follows

$$J_\alpha(u) = \int_0^{t_f} x_d^2(t,d)\,dt, \tag{7.29}$$

the sensitivity of the state variable is

$$x_d(t,d) = \frac{\partial x(t,d)}{\partial d}, \tag{7.30}$$

subject to input energy

$$\int_0^{t_f} u(t)^T u(t)\,dt \le E. \tag{7.31}$$

The presented method is appropriate only for models with the fractional order values $\alpha \le 1.0$. For the fractional order values α from the interval (1.0, 2.0), it is necessary to extend the state-space equations by one extra state. The problem formulation for solving this task would be presented in the further part of the current section. In general case, an optimal input signal design to the fractional-order inertial LTI system identification is formulated by the state-space model

$$\begin{gathered} {}_0D_t^\alpha x = Ax + Bu, \\ y = Cx, \end{gathered} \tag{7.32}$$

where: $A = -1$, $B = 1$, and $C = 1$ are model parameters (according to (7.32)), with the initial condition

$$x(0) = 5. \tag{7.33}$$

The reformulated performance criterion to be maximized has the following form

$$J_\alpha(u) = \int_0^{t_f} (C_F x_d(t) + u)^2 dt, \tag{7.34}$$

with respect to the constraints

$$\begin{gathered} -1 \le u(t) \le 1, t \in [0, t_f], \\ \int_0^{t_f} (t_f - t)^{2(1-\alpha)} u(t)^T u(t) dt \le 1, t \in [0, t_f]. \end{gathered} \tag{7.35}$$

The controllability Gramian [13] of fractional order α is used to the energy cost minimization purposes. The term $(t_f - t)^{2(1-\alpha)}$ under the integral (7.35) is designated to neutralize the singularity at $t = t_f$. This component is also used to ensure the convergence of the integral. The reformulated system dynamics has the following form

$$\dot{z} = A_F z + B_F(-(C_F z + D_F u) + u), \tag{7.36}$$

with the initial conditions

$$z(0) = [5 \quad 0 \quad \cdots \quad 0]^T. \tag{7.37}$$

The equation (7.32) can be reformulated to solve the fractional-order model identification for non-integer orders $\alpha > 1.0$. For this purpose, the system dynamics should be extended by another state variable. The cost function (7.34) is maximized for order values from the interval ($1.0 < \alpha \leq 2.0$) considering the following dynamics

$$\begin{aligned} \dot{x}_1 &= C_F x_2 + D_F u, \\ {}_0D_t^{\beta} x_2 &= A_F x_2 + B_F(-(C_F x_2 + D_F u) + u), \end{aligned} \tag{7.38}$$

it was assumed that $\beta = \alpha - 1$, and the initial conditions are: $x_1(0) = 5,\ x_2(0) = 0$. The previous problem can be presented by the state equation given below

$$\begin{aligned} \dot{x} &= C_F z + D_F u, \\ \dot{z} &= A_F z + B_F(-(C_F z + D_F u) + u), \end{aligned} \tag{7.39}$$

subject to the constraints, and the initial conditions described by equations (7.35), and (7.37). The fractional-order optimal input design problem is to be solved using the Runge-Kutta method.

7.6. EXPERIMENTAL RESULTS

The frequencies for the Oustaloup method has been chosen from the interval $[10^{-2}, 10^{2}]$ rad/s. The order of the ORA filter has been obtained using equation (7.25) with $N = 4$. The Oustaloup filter frequencies have been chosen to fit in with the discretization of the integration method used by Riots_95 [21]. This toolbox should be included in Matlab kit as a separate library, and allows to solve optimal control problems containing fixed, and the free final time tasks.

The optimal input design problem for fractional LTI system identification is then generated for the arbitrarily selected parameters (7.32) $A = -1$, $B = 1$, $C = 1$, and the chosen time period $t = [0, 1.0]$ seconds, using the sequential quadratic programming (SQP) algorithm. The final extended period of the time t_f would certainly cause notable computational effort. The fractional-order model initial conditions have been selected according to (7.37), and the initial condition of the input signal has been fixed on $u(0) = 1$. The optimal input signal trajectory $u(t)$ has been limited to the range of motion $[-1, +1]$ in order to prevent sudden changes of the input signal. The optimal input signals were obtained using the 4th order Runge-Kutta method with grid interval of 0.01 sec.

The optimal input signals and the state variables to fractional-order inertial LTI system generated for different orders of the state-space model (7.32) (i.e. $0.5 \leq \alpha \leq 1.0$) are shown in Fig. 7.1. As seen, the input signals are considerably different, while the value of the order of α decreases. For the state-space model orders of $\alpha \leq 1.0$, the input signal transition reduces its duration, while the control signal obtained for $\alpha = 0.5$ is substantially a step input signal (i.e. yellow solid line).

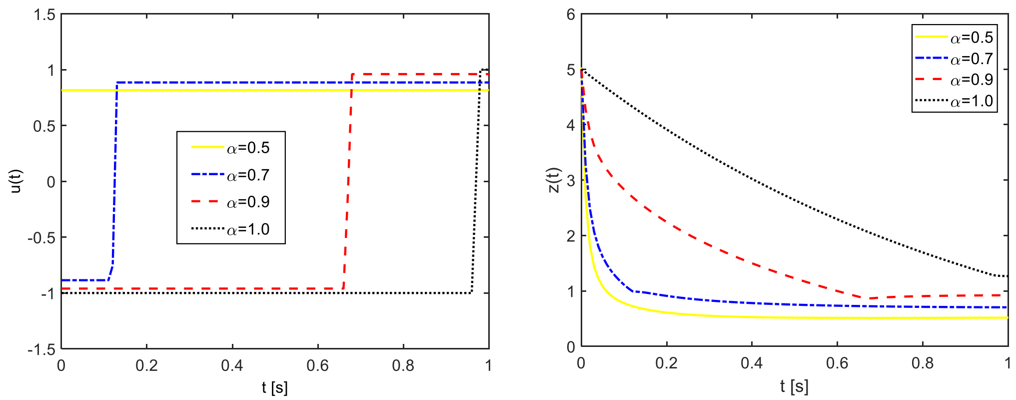

Fig. 7.1. The optimal excitation signal $u(t)$ and the state variable $z(t)$ to the fractional inertial system as function of time t for orders from the interval $0.5 \leq \alpha \leq 1.0$

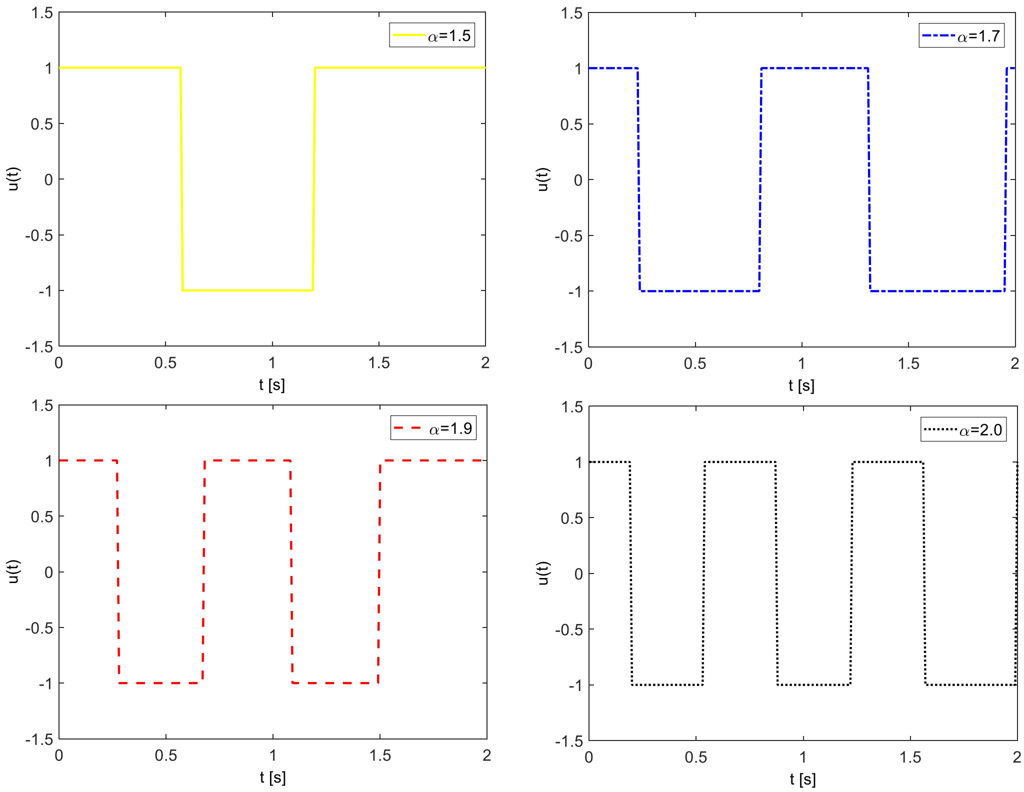

Fig. 7.2. The optimal input signals $u(t)$ to the fractional-order inertial system as function of time t for orders from the interval $1.0 < \alpha \leq 2.0$

The plot shown on the right side of Fig. 7.1 presents the reformulated state variables $z(t)$ to the fractional-order inertial system as a function of time for various orders of α from the interval $0.5 \leq \alpha \leq 1.0$. As it has already been indicated, the sensitivity of the state variable $x(t, d)$ to the parameter d (i.e. the gain of the open-loop system) has been maximized. The parameter d is the gain of the non-integer model (7.27) after the Oustaloup approximation operation. The imprecise estimation of the gain can lead to instability of the open-loop system especially using rapidly changing input signals. The examples of the optimal inputs to the fractional-order inertial system identification as a function of time for different orders of α from the interval $1.5 \leq \alpha \leq 2.0$ are shown in Fig. 7.2. The fractional-order system identification for $\alpha \geq 1.0$ requires the extension of the state-space equation by an additional state subject to (7.39). It can be noted that increasing the value of the system order, the input signal is characterized by the increased number of the oscillations.

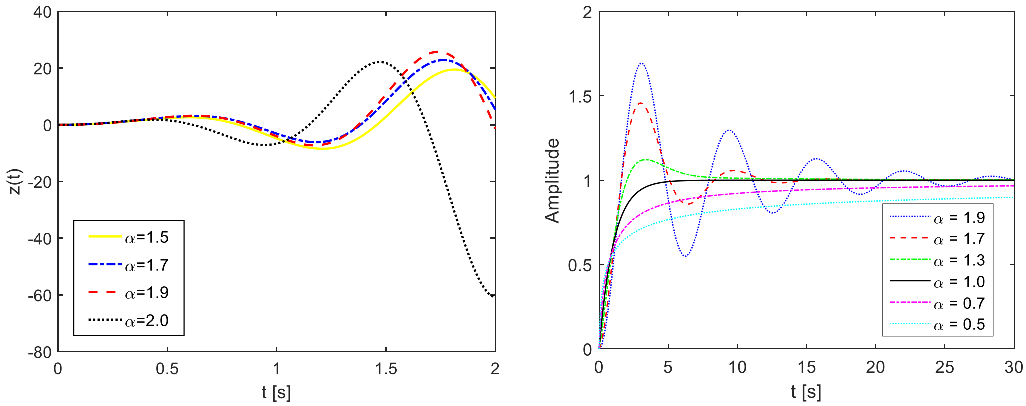

Fig. 7.3. The state-space variable $z(t)$ to the fractional-order inertial system as function of time t, and the step responses comparison for different order values

As it has been shown in Fig. 7.3, since increasing the order of the fractional system from the interval of $1.5 \leq \alpha \leq 2.0$, the model's response starts to become oscillatory. The left panel of Fig. 7.3 shows the waveforms of the state variable $z(t)$ of the fractional-order inertial system as a function of the time. The comparison of the step responses to the inertial system using various values of α is shown on the right panel of the Fig. 7.3. It can be noticed that the step responses for orders $\alpha \leq 1$ are aperiodic, however conventional first-order inertial system step response can be observed for $\alpha = 1$. The step responses have oscillatory form for the fractional system orders $\alpha \geq 1$. Finally, it should be stated that presented methodology cannot be used for fractional-order systems, where $\alpha > 1$. This inconvenience is related to the fact that the approximated transfer function numerator has the higher order than the order of the denominator. Consequently, it is impossible to convert a zero-pole transfer function to the state-space form. This problem can be solved by augmenting the fractional-order system dynamics with one extra state.

7.7. CONCLUSIONS

In this paper, we have proposed a novel optimal input design formulation for the fractional-order system identification. The methodology for finding an optimal input solution is verified using the numerical examples. The model design is based on accurate Oustaloup recursive approximation process and has been then used for the fractional-order operator estimation in the integer-order transfer function form. If the transfer function numerator order is equal to the denominator order, the conversion of a transfer function to a state-space form is allowed. The technique presented in this paper enables the solution of the optimal input signal for fractional-order system identification. The problem solution is based on the state variable sensitivity to the fractional-order system parameter d (i.e. the gain of the state-space system) minimization, subject to a set of constraints imposed on input signal. Increasing the gain of the system makes the system underdamped, and in an extreme case, leads to instability of the open-loop system. Consequently, a precise gain value approximation is a significant task during the fractional-order system identification. It has been noticed that the most significant loss in the objective function value has been obtained for α value from 0.9 to 1.0. This loss of the performance is the consequence of the fractional-order differentiator conversion into the integer-order form.

The most significant step has been to reformulate our optimal input design problem, represented by the Lagrange formulation with the set of constraints, into a twin fractional-order input design. Then it is possible to solve the optimal input signal problem using one of the available toolboxes for solving optimal control problems. The numerical simulations confirm that the result obtained for the fractional-order case study (i.e. for $\alpha = 1$) is the same as the one received from the integer-order input design problem. The numerical examples also confirm that for fractional order values $\alpha > 1$, there is need to augment the state-space equations by one extra state. Moreover, the selection of appropriate frequencies for the Oustaloup recursive technique is also a very important design step.

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