PRACTICAL PROBLEMS FOR INTRODUCTORY ELECTRICAL ENGINEERING

Politechnika Białostocka

PRACTICAL PROBLEMS FOR INTRODUCTORY ELECTRICAL ENGINEERING

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Introduction

"That is a good problem!" How often do you hear an exclamation like this? If you work in industry, you probably hear it quite seldom. In industry problems are bad, not good. As academics, we look for *good* problems. Problems that are good for research, for learning or for both. A good problem is a task that lets you discover your limitations and gives you an opportunity to overcome them.

The authors behind this text, work with teaching electrical engineering at university. We are often called *lecturers,* a term that indicates a position behind a lectern, where we lecture, i.e. convey our wisdom to an interested and attentive audience. Experience from 150 years of electrical engineering education has shown that lecturing is never sufficient for teaching and learning the trade. Lectures have always been accompanied with exercises and lab work. *Learning by doing* has always been a part of acquiring the skills of an electrical engineer.

After WW2, the invention radar and the utilization of atomic energy, there was a worldwide shift in engineering education towards heavier emphasis on theoretical fundamentals. This was, no doubt, a necessary development. During the last decades, however, a growing realization has emerged that, perhaps, the education in our field has become *too* theoretical. The proverbial pendulum has now turned, and a wave of more project based and problem based, or *student active* learning types is now guiding teaching and learning in all kinds of university departments, not only in engineering.

So, what makes up *a good problem?* We will not attempt to give a complete answer to that question. A partial answer is that, for engineering education, it is desirable if a problem in some sense is *practical*. It should, if possible, relate to the students' previous experience, connect to applications that the student knows or is about to learn about. Such experienced relevance of the problem might be the spark that inspires an otherwise marginally motivated learner.

When selecting problems for this booklet, we have chosen such ones that in our experience have served us well. Several of them have been developed and revised in collaboration with students and learning assistants and have been tried out in courses at our respective institutions. Even though intended learning objectives may be common, the actual syllabus and course construction must always be adapted to local culture, tradition and administrative requirements. A reader may find that some of our suggestions fit well with his or her own way of teaching, others not. Our hope is that most teachers in our field will find some inspiration in *Practical Problems For Introductory Electrical Engineering.* Feel free to use parts or all, adapt, subtract and add to any of the suggested tasks.

A bit about the content

Some problems are given without solutions, but then you will only find the answer. Most of exercises are meant to be solved together (in order) and are placed under a central heading with a common solution sheet. In fact, each whole chapter is best analysed when all the exercises are solved in order from the top, although it is not needed. The solution sheets are called "Solution(s)" and are available by QR code. Also many exercises have several correct solutions, not just the ones shown in the attached files.

This digital book contains active links to suggested information provided in the form of videos, documents and simulations. If it happens that some of the links cease to be active, we suggest replacing them with others available online.

In this book most of problems were created by the authors, however we drew inspiration also from various sources. We especially recommend the book available on the Internet (free downloaded as a PDF) *Basic Engineering Circuit Analysis* by J. David Irwin, R. Mark Nelms in which you may find the theoretical basis for chapters 1 to 5.

We appreciate and thank the many learning assistants who in the course Introduction to Analog and Digital Electronics (ADE) have come with important feedback that has influenced the development of the material to a great degree.

Only the great natures are the least free from error
Dionysius of Halicarnassus – 1st century BC

CHAPTER 1. Basic concepts. Energy and Power

You can find the necessary theoretical information in *Basic Engineering Circuit Analysis* by J. David Irwin, R. Mark Nelms on pp. 2-9, 13-17, 25-32, 40-45.

Problem 1.1
A portable refrigerator is powered by a battery with the DC voltage U_n = 12 V. The nominal current of the refrigerator related to this voltage is I_n = 5 A. Calculate the voltage drop on the power line with a length of l = 10 m when it is made of copper (Cu) and aluminium (Al), and its cross-section is:
a) 1.5 mm^2,
b) 2.5 mm^2,
c) 4.0 mm^2.

The electrical resistivity of copper at 20°C is: ρ_{Cu} = 1.72·10^{-8} Ω·m, and the electrical resistivity of aluminium is: ρ_{Al} = 2.82·10^{-8} Ω·m.
Answer:
a) U_{Cu} = 1.15 V, U_{Al} = 1.88 V, b) U_{Cu} = 0.69 V, U_{Al} = 1.13 V, c) U_{Cu} = 0.43 V, U_{Al} = 0.71 V.

Problem 1.2
A copper cable YDY 5 × 2.5 mm^2 is sold as a spool. One spool contains l = 500 m of cable. Calculate how many spools can be taken to a car trailer with a load capacity of 300 kg. Assume that the weight of the cable insulation is 10% of the weight of pure copper. Copper density is ρ_{Cu} = 8933 kg /m^3.
Answer: You can take 4 spools of cable to the car trailer.

Problem 1.3
A boiler for heating water with a nominal power of P_n = 400 W is supplied with voltage U_n = 24 V DC. The boiler is connected to a power source (battery) with a 2 × 2.5 mm^2 copper cable with a length of l = 5 m. Calculate the voltage drop on the supply cable. By how much will the voltage drop increase when the cable is placed in the air and its temperature increased from 20°C to 70°C under the influence of the flowing current and the sunlight. The electrical resistivity of copper at 20°C is: ρ_{Cu} = 1.72·10^{-8} Ω·m.

The dependence of the conductor resistance on temperature is described by the formula:

$$R_T = R_{20}\left[1 + \alpha\left(T - T_{20}\right)\right]$$

where:
R_T – conductor resistance at T temperature,
R_{20} – conductor resistance at $T_{20} = 20°C$,
α – temperature coefficient of resistance (for copper $\alpha = 4.3 \cdot 10^{-3}\ °C^{-1}$).
Answer: The voltage drop on the power supply cable at 20°C will be 1.147 V, and at 70°C will be 1.393 V. The voltage drop will increase by 21.5%.

Problem 1.4
Resistance values of the resistors available on the market are standardized and belong to one of the numerical series. When designing electronic circuits, it is often necessary to use a resistor with a non-existent resistance. The solution to this problem can be the use of existing resistors connected in series or in parallel. Having resistors: $R_1 = 10\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 15\ \Omega$, calculate the equivalent resistances that can be obtained by connecting these three resistors in any way (in series, in parallel, mixed).
Answer: 3.75 Ω, 7.14 Ω, 8.57 Ω, 16 Ω, 20 Ω, 35 Ω.

Problem 1.5
A car is equipped with a fully charged battery with a nominal voltage of $U_n = 12$ V and a nominal capacity K = 44 Ah. When no load is turned on in the car, the current drawn from the battery is approx. $I = 30$ mA (engine controller memory, radio memory, central locking, alarm). Assuming that the car is at a constant temperature of 25°C (e.g. in a garage), calculate how long it takes before the battery will be fully discharged. What will be the time of complete discharge of the battery when the car is left at the parking lights (4 × W5W bulbs, $P_n = 5$ W)?

Answer: In a car without loads turned on, the battery will be discharged after approx. 61 days, and with parking lights on – after approx. 25.9 hours.

Problem 1.6
A portable hard drive is powered from the USB 2.0 port with the output voltage $U_n = 5$ V. The drive works properly with the maximum voltage drop on the power cable not exceeding 5% of the supply voltage ($U_{max} = 0.25$ V). Check if the drive will work properly if it is powered by USB 24 AWG cable (conductor cross-section $S_{24} = 0.205\ mm^2$) or by USB 28 AWG cable (conductor cross-section $S_{28} = 0.081\ mm^2$) with the following lengths:
a) 0.5 m,
b) 1.0 m,
c) 1.5 m.

In the case of a USB 2.0 port, the maximum current consumed by the connected device is I_{max} = 500 mA. The electrical resistivity of copper at 20°C is ρ_{Cu} = 1.72·10^{-8} Ω·m. Consider the presence of contact resistance, which for one new contact is approx. R_k = 10 mΩ, and for one used contact it is approx. R_k = 30 mΩ.

Answer: Assuming the most unfavourable case (I_{max} = 500 mA, R_k = 30 mΩ), the drive can be connected with a USB 24 AWG cable with lengths of 0.5 m; 1.0 m; 1.5 m and a USB 28 AWG cable with a length of 0.5 m.

Problem 1.7

There are 4 bulbs installed in the rear lamp of a car:
- indicator – type: P21W, rated power: P_1 = 21 W, rated voltage: U_n = 12 V,
- break light – type: W16W, rated power: P_2 = 16 W, rated voltage: U_n = 12 V,
- parking light – type: W5W, rated power: P_3 = 5 W, rated voltage: U_n = 12 V,
- reversing light – type: W16W, rated power: P_4 = 16 W, rated voltage: U_n = 12 V.

What should be the nominal current of the fuse protecting the car's lamp circuit? Standard automotive fuses have the nominal currents of I_n = 1 A; 2 A; 3 A; 4 A; 5 A; 7.5 A; 10 A; 15 A; 20 A; 30 A; 40 A.
Answer: I = 5 A.

Problem 1.8

An electric heater for water heating consists of three heating coils with a power of 300 W each. The heater is powered with 24 V DC. The coils can work independently. What should be the nominal current of the current-limiting circuit-breaker protecting the electric heater? Standard nominal currents of the current-limiting circuit-breakers are I_n = 6 A, 10 A, 16 A, 20 A, 25 A, 32 A, 40 A, 50 A, 63 A, 80 A and 125 A.
Answer: I_n = 40 A.

Problem 1.9

Electric water heater (boiler) with a heater power of P_n = 120 W is powered by U_{bat} = 12 V DC battery. The heater operating voltage ranges from 10.8 V to 14.6 V. When the specified range is exceeded, the heater automatically turns off. A copper cable with a cross-section area of S = 2.5 mm^2 is used to connect the heater. Check that after connecting the heater with a cable with a length:
a) 5 m,
b) 10 m,
c) 15 m,
the heater voltage is not too low. For simplicity, assume that the heater's power is constant. The electrical resistivity of copper at 20°C is: ρ_{Cu} = 1.72·10^{-8} Ω·m. The electrical circuit of the analysed system is presented in the figure below.
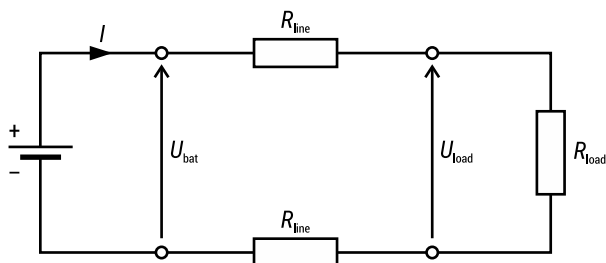
FIG. 1.9. The electrical circuit for problem 1.9

Answer: In the analysed circuit, the correct value of the supply voltage will be reached after connecting the heater with a cable of length $l = 5$ m.

Problem 1.10

An alkaline battery is an example of a real voltage source. As the battery discharges, the value of its electromotive force E decreases, while the value of the internal resistance R_{int} increases. The measuring system shown in Fig. 1.10 was used to determine the electromotive force and internal resistance of the used AA alkaline battery with a nominal voltage of 1.5 V. Resistance R_1 was connected to the battery and the voltage drop U_1 was measured (Fig. 1.10a). In the same way, the resistance R_2 was connected and the voltage drop U_2 was measured (Fig. 1.10b). Then, using a digital multimeter, the values of the resistances R_1 and R_2 were measured. The following measurement results were obtained: $R_1 = 10.119\ \Omega$, $U_1 = 1.362$ V, $R_2 = 20.118\ \Omega$, $U_2 = 1.383$ V. Based on the measurement results, calculate the electromotive force E and the internal resistance R_{int} of the battery.
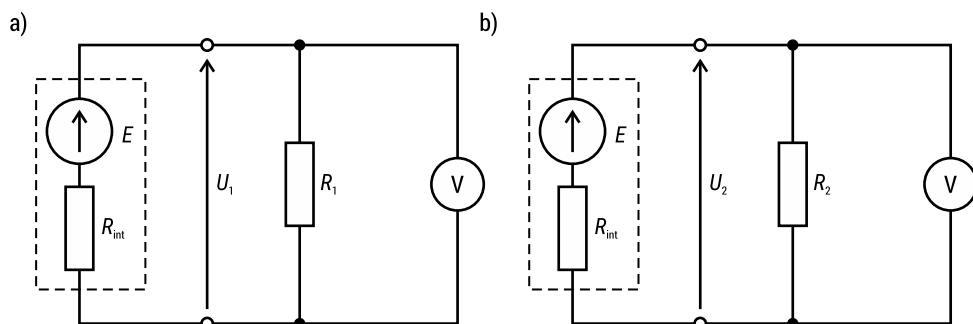

FIG. 1.10. The electrical circuits for problem 1.10

Answer: $R_{int} = 0.319\ \Omega$, $E = 1.405$ V.

Problem 1.11

The purpose of the microprocessor system is to control the battery voltage. Due to the difference in the operating voltage levels of the battery and the microprocessor system, an interface circuit, voltage divider, should be designed (Fig. 1.11a).

The maximum output voltage of the battery can be $U_{bat} = 15$ V, and the corresponding input voltage of the microprocessor system should then have the value of $U_{mp} = 5$ V. Determine the values of resistance R_1 and R_2 assuming that the system should consume no more than $I = 1$ mA. Check how the output voltage will change when a microprocessor system with internal resistance $R_{mp} = 1$ MΩ is connected to the divider (Fig. 1.11b)

a)   b)
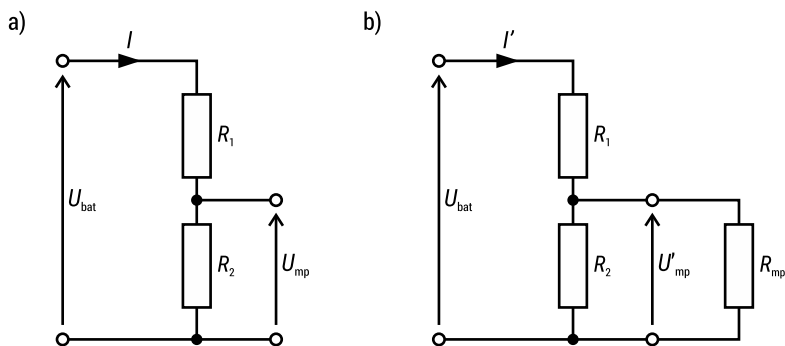
FIG. 1.11. The electrical circuits for problem 1.11

Answer: $R_1 = 15$ kΩ, $R_2 = 7.5$ kΩ, $U'_{mp} = 4.975$ V.

Problem 1.12

Fig. 1.12 shows a simplified circuit of the wiring in a car. The circuit includes a battery with voltage $E_b = 12.8$ V DC and internal resistance $R_b = 10$ mΩ and an alternator with DC voltage $E_a = 14.4$ V and internal resistance $R_a = 0.1$ Ω. This circuit supplies a load in the form of a car refrigerator with nominal voltage $U_n = 12$ V and nominal power $P_n = 40$ W. Calculate battery (I_b), alternator (I_a) and refrigerator (I_l) currents with the car and the refrigerator running.

FIG. 1.12. The electrical circuit for problem 1.12

Answer: $I_b = -11.024$ A, $I_a = 14.898$ A, $I_l = 3.873$ A.

Problem 1.13

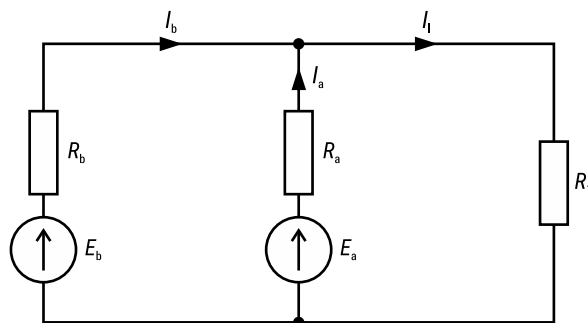An electronic clock, which also functions as an alarm clock, is powered by two batteries with nominal voltage $U_n = 1.5$ V connected in parallel (Fig. 1.13). As the battery discharges, the clock display becomes weaker and weaker. The supply voltage drop below 1.4 V causes the illegibility of the displayed data. The clock is powered by two batteries – one new and one used. The electromotive force of the first battery is $E_1 = 1.5$ V, and its internal resistance is $R_1 = 0.3$ Ω. In the case of the second battery, these parameters are: $E_2 = 1.35$ V, $R_2 = 0.5$ Ω. Check if the clock will work properly after using the above batteries. The nominal value of the current consumed by the clock is $I_n = 90$ mA.
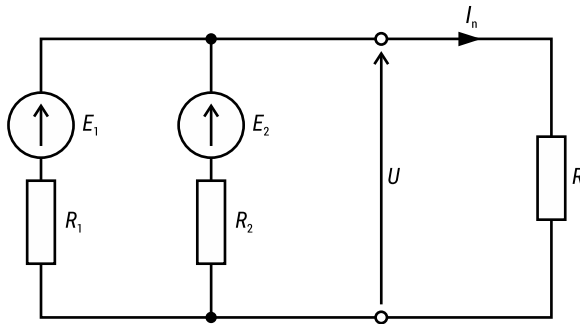
FIG. 1.13. **The electrical circuit for problem 1.13**

Answer: The voltage on the clock terminals is sufficient for its correct operation

Problem 1.14

Popular power banks are usually built from lithium-ion accumulators. To ensure that you can charge your mobile phone anywhere, design a power bank from the available in the table below cells allowing you to charge a phone with a battery capacity of 5000 mAh five times from 0% to 100%. Assume 80% efficiency of the power bank. Draw a diagram of the connection of the battery cells. Make the power bank at the lowest possible cost. Compare the cost of construction with the cost of buying a power bank with these parameters. The video attached to the task shows an example of the construction of a power bank.

- *https://www.youtube.com/watch?v = wmDbkxIVZzM&fbclid = IwAR00Cffs2GgQ 60Mr4x_3j_vTMwSSA4iLiQxUkZ7M-67GcWY4v6k3zakfuZY*

The prices in the table are from January 2023. If they have changed, include the new prices in the calculations.

Size	18650
Nominal Capacity	2500 mAh
Nominal Voltage	3.6 V
Continuous Discharge	20 A
Protected	No
Rechargeable	Yes
Approximate Weight	43.8 g
Price	$ 4.99

https://www.18650batterystore.com/ collections/18650-batteries/products/ samsung-25r-18650

Size	18650
Nominal Capacity	3000 mAh
Nominal Voltage	3.6 V
Continuous Discharge Rating	15 A
Protected	No
Rechargeable	Yes
Approximate Weight	48 g
Price	$ 5.99

https://www.18650batterystore.com/ collections/18650-batteries/products/ samsung-30q

Size	18650
Nominal Capacity	3500 mAh
Nominal Voltage	3.6 V
Continuous Discharge	8 A
Protected	No
Rechargeable	Yes
Approximate Weight	50 g
Price	$ 7.99

https://www.18650batterystore.com/ collections/18650-batteries/products/ samsung-35e

Size	18650
Nominal Capacity	3500 mAh
Nominal Voltage	3.6 V
Continuous Discharge	10 A
Protected	No
Rechargeable	Yes
Approximate Weight	48 g
Price	$ 6.99

https://www.18650batterystore.com/ collections/18650-batteries/products/ molicel-m35a-18650-battery

Answer: It takes 9 batteries with a price of 6.99 $; cost = 62.91 $ – capacity 31.5 Ah.

Problem 1.15

Nowadays, we often neglect to unplug the charger from the socket. However, it is worth remembering that when the charger is idle, it also consumes energy. Assuming that a mobile phone is charged for 3 hours every day, calculate how much electricity (E) is 'wasted' by one charger plugged into a socket during the remaining time of day. Assuming that 5 people in the flat charge their phones every day, calculate the annual idle cost of the chargers, assuming that the charger consumes 100 mW when idle. Assume an electricity cost of 0.14 € per 1 kWh.

Answer: E = 2.1 Wh Annual cost = 0.54 €

Problem 1.16

It is necessary to select a suitable heater for the construction of the thermoelectric heater. Nickel-Chromium 80/20 resistance wire is often used for this purpose because it has a high resistivity of 1.09 $\frac{\Omega \cdot \text{mm}^2}{\text{m}}$. Using the available diameters and their prices listed below (Table 1.16), determine the most cost-effective cross section to make a 2000 W heater. Assume supply voltage of 230 V.

TABLE 1.16. The Nichrome wire prices

Diameter, mm	Price per meter, $
0.15	3.23
0.20	3.24
0.30	3.36
0.50	3.99

Answer: The cheapest wire to buy is 0.15 mm with total cost = 46.22 $

Analysis of a speaker system » *keywords: Energy*

Speakers are devices that use a changing current to move air, thus making sound![1] We have connected two speakers to a voltage source using cables (see figure below). For our purposes we can model the two speakers as two resistors with the same value R_1. But when modeling the system, it is important to remem ber that physical cables also have a resistance. We therefore model the cables as resistors of value R_2. The circuit is shown in the figure below:
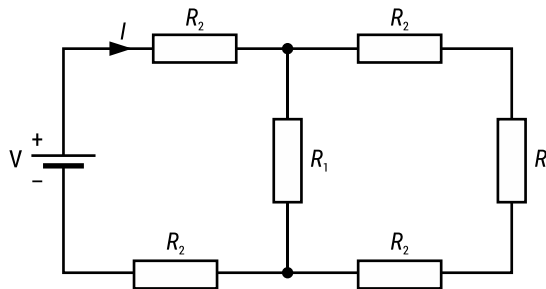

FIG. 1.16. The equivalent circuit with two speakers

Since we are only interested in the current I for now, we can replac eall of the resistors with a single equivalent resistor:

$$R = 2R_2 + R_1 \, || \left(R_1 + 2R_2 \right).$$

Problem 1.17 Extra Challenge
Show that the equation can be written as:

$$R = \frac{R_1^2 + 6R_1 R_2 + 4R_2^2}{2\left(R_1 + R_2 \right)}.$$

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[1] Curious? Look here: *An Introduction to Audio Electronics: Sound, Microphones, Speakers, and Amplifiers – Technical Articles (allaboutcircuits.com)*

Problem 1.18

Find an expression for the power delivered by the voltage source. Notice: The symbol *V* in the figure indicates that the voltage source has an *effective value* (RMS) of *V* volt. But you can solve the problem as if it is a DC voltage of value *V*.

In the original situation we thought of the two speakers as being identical. Now we will look at another situation. The speaker closest to the voltage source is still modelled with a resistance R_1 while the other speaker is replaced by another speaker with the resistance R_3. Problem 1.19-1.22 will look at this new situation.

Problem 1.19

Draw the circuit model for the new situation and find an expression for the equivalent resistance (as seen from the voltage source). Use the symbol $\|$ to indicate resistors in parallel (to avoid getting a very complicated expression).

Problem 1.20 Extra Challenge

Show that the equivalent resistance, as seen from the voltage source, can be expressed as:

$$R = \frac{4R_1R_2 + R_1R_3 + 4R_2^2 + 2R_2R_3}{R_1 + 2R_2 + R_3}.$$

Problem 1.21

Look at the situation where $R_1 = 0$. Draw the circuit for this situation. What is the equivalent resistance as seen from the voltage source now? (Can you imagine what has to happen to the speaker system to make $R_1 = 0$?).

Problem 1.22

Look at the situation where $R_1 \to \infty$. Draw the circuit for this situation. What is the equivalent resistance as seen from the voltage source? Does this agree with the expression under "Problem 1.20"? (Can you imagine what has to happened to the speaker system to make $R_1 \to \infty$?).

Problem 1.23 Extra Challenge

Now look at the original situation with identical speakers again. The effective value of the voltage source is $V = 1$ V, and the resistors have the values: $R_1 = 8\,\Omega$ and $R_2 = 0.5\,\Omega$.

We previously looked at the total power delivered by the voltage source but how much of this is delivered to each of the speakers?

SOLUTIONS

Powering up a speaker (1) » *keywords: Energy*

Problem 1.24

A speaker R_L is connected to the circuit (look at the figure below). You can assume that the speaker acts an ideal resistor, and that it performs it's best at 100 mW. Find the minimum voltage that the source V can have for the speaker to perform its best. The resistor values are: R_p = 15 Ω, R_g = 7.5 Ω, R_1 = 10 Ω, R_2 = 20 Ω and R_L = 14 Ω.
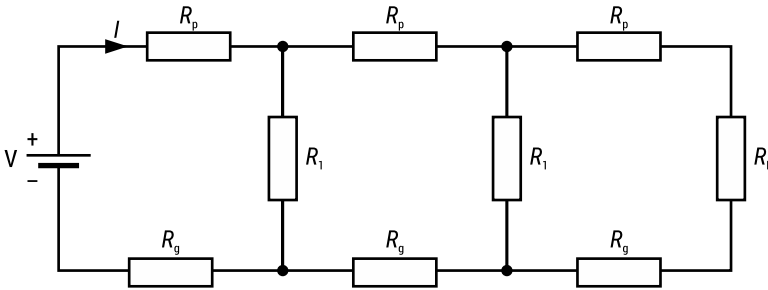

FIG. 1.24. The circuit for supply the speaker

SOLUTION

Powering up a speaker (2) » *keywords: Energy*

Problem 1.25

We want to provide power to our beloved speaker. We have an amplifier that has an output resistance of 1 Ω, and that delivers a voltage with an effective value of 5 V when nothing is connected to the output. But we of course want to connectour speaker to the output. Our speaker has a resistance of 8 Ω. How much power does it receive when connected to the amplifier? **Hint:** Start with drawing a model of the amplifier, then add the speaker to it's output.

SOLUTION

CHAPTER 2. DC Circuits in Transient State

If you need the introduction to the theory of circuits in transient state see the videos (prepared by author):
- *https://box.biaman.pl/f/84288e1a58b24985a87f/*
- *https://box.biaman.pl/f/e13aea3114e9438494ee/*

    Before you start to solve these problems look at
- *https://youtu.be/hWpzEgPp_Lg* or
- *https://youtu.be/XXTNBSIDAkY* or another web side concerning *Transient and Steady State Response*, to find some good examples with explained solution and a review of theory.

Problem 2.1

In the circuit presented below the capacitor presents the model of a sensor that starts working when its voltage has a minimum 4 V. To save the energy the circuit is supplied only if the measured quantity changes. Find the time after switching the supplying source when the sensor is ready for use.
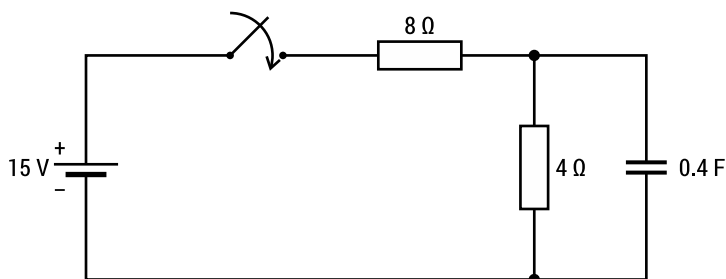

FIG. 2.1. Circuit with a capacitive sensor

Problem 2.2

The switch of the circuit shown below has been closed for a long time and is opened at $t = 0$. Find the current $i(t)$ and voltage $v(t)$ of the capacitor for $t \geq 0$.
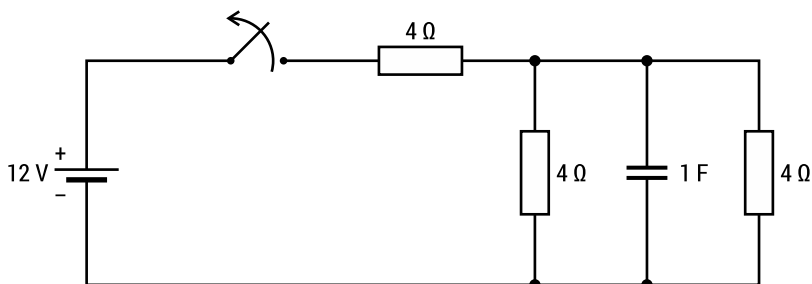
FIG. 2.2. Circuit with a capacitance in transient state

Problem 2.3

The switch of the circuit shown below has been closed for a long time and is opened at $t = 0$. Find the current $i(t)$ of the capacitor and voltage $v(t)$ of the resistor 3 Ω for $t \geq 0$.
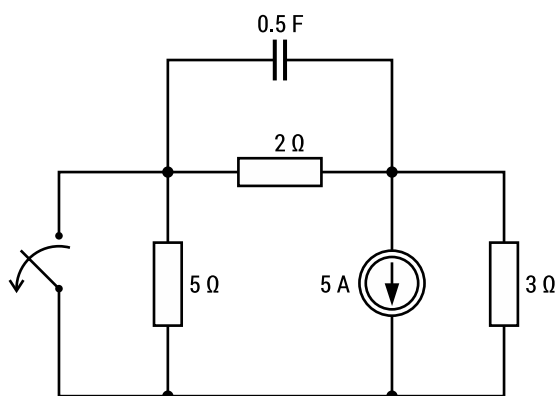

FIG. 2.3. Circuit with a current source in transient state

Problem 2.4

The switches of the circuit have been moved instantaneously to new positions at $t = 0$. Find the time it takes to discharging the capacitor (when 5% of initial energy remains).
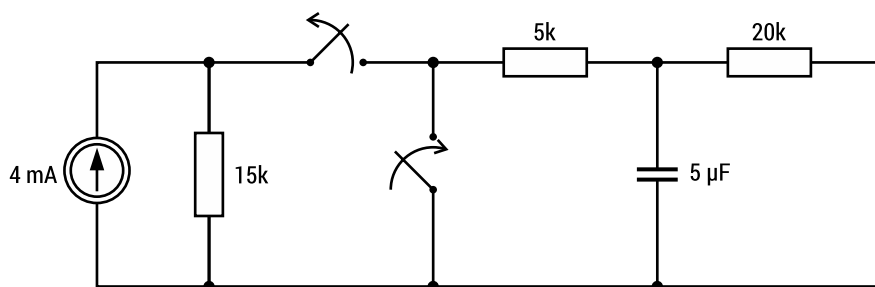

FIG. 2.4. Circuits with a charging and discharging capacitor

Problem 2.5

The switch of the circuit shown below has been opened for a long time and is closed at *t = 0*. Find the current *i(t)* and voltage *v(t)* of the inductor for *t ≥ 0*.

FIG. 2.5. Circuit with short-circuit resistor
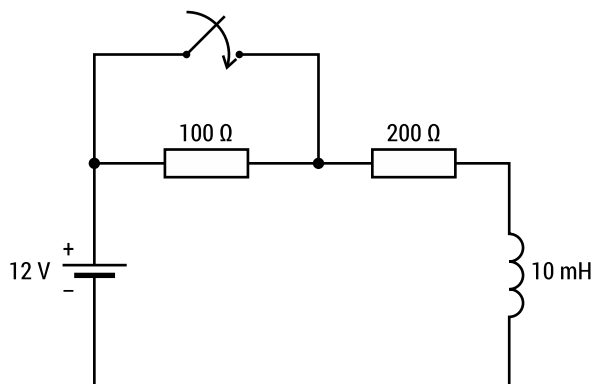
Problem 2.6

The switch of the circuit shown in Fig. 2.6 has been closed for a long time and is opened at *t = 0*. Find the current and the voltage of the magnetic coil. How long will it take for the coil current to reach the 95% of its final value (when t = ∞)? Consider the two following cases:
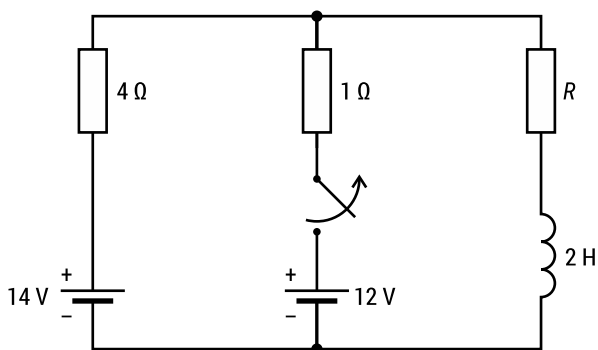a) $R = 1\ \Omega$,
b) $R = 10\ \Omega$.

FIG. 2.6. Circuit with disconnected source

Problem 2.7

In this circuit the model of an inductive sensor is presented. It works if its current is minimum 50 mA. To save energy the sensor is supplied only if the measured quantity changes. Find the time after switching the supplying source (the model of changing the quantity) when the sensor is ready for use.
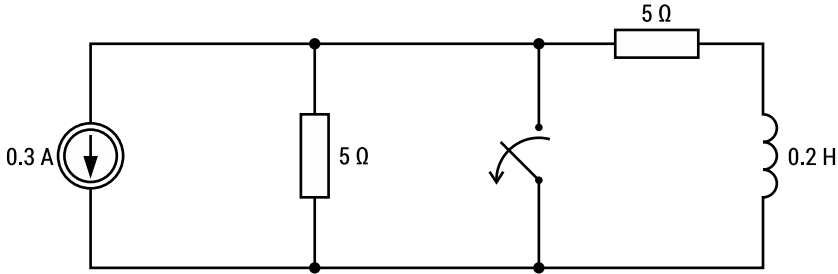
FIG. 2.7. Circuit with inductive sensor

Problem 2.8

The switch of the circuit shown below has been opened for a long time and is closed at t = 0. Find the current of the battery and voltage over the inductive coil for t ≥ 0.

FIG. 2.8. Circuit with an additional connected resistor

Problem 2.9

Calculate the current in the magnetic coil if the switch has been opened at t = 0.

$$U_1 = U_2 = 10 \text{ V}, R_0 = 2.5 \ \Omega, R = 0.2 \ \Omega, C = 1 \text{ F}, L = 0.25 \text{ H}$$
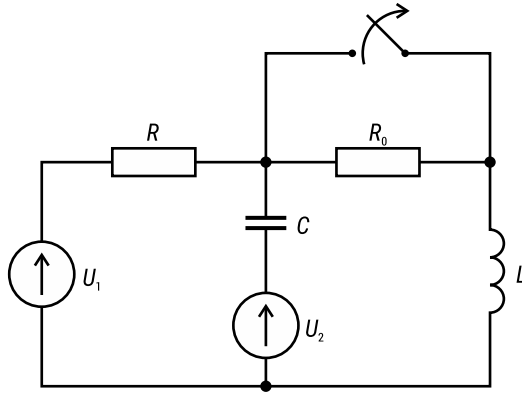
FIG. 2.9. **The RLC circuit with two sources**

Problem 2.10
Calculate the voltage $u_c(t)$ at capacitor and the inductor current $i_L(t)$ if the circuit breaker is opened at time t = 0.

$$J = 10 \text{ A}, R = R_1 = R_2 = 10 \ \Omega, L = 1 \text{ H}, C = 10 \text{ mF}.$$
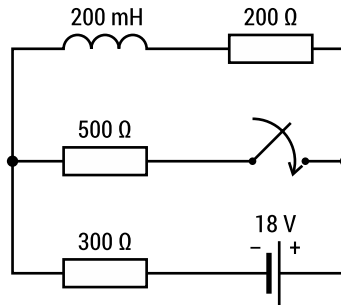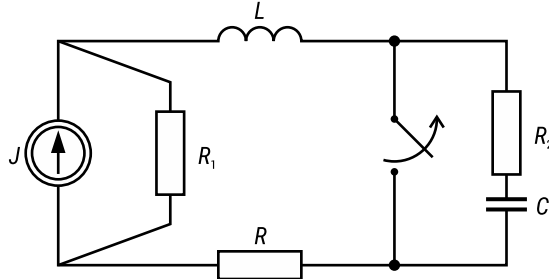

FIG. 2.10. **The circuit for problem 2.10**

Problem 2.11
In the circuit shown in Fig. 2.11, the switch was closed at t = 0. Calculate the waveforms of voltage u_C and current i_L. Before commutation, the circuit was in a steady state.

$$R = R_1 = R_2 = 10 \ \Omega, L = 1 \text{ H}, C = 10 \text{ mF}, j(t) = 14.1\sin10t \text{ A}$$
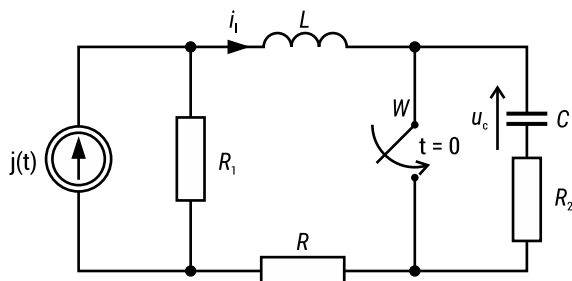
FIG. 2.11. Circuit with shorted RC circuit branch

SOLUTIONS

Problem 2.12

When you open the car door at night the lamp inside is switched on. When you close the door, it subsides and dies after certain time.

For this purpose design a simple RC circuit supplied by the real DC voltage source to reach the adjusted "die time".

Confirm your theoretical solution by simulation in MatLab or MicroCap or PSPICE.

Problem 2.13

The circuit presented in Fig. 2.13 can be a model of a security alarm for a house door. The open switch represents the open door and the resistor of 7 Ω is the alarm indicator, which is activated when its voltage decreases by twice the initial value. Find the time between opening the door and starting the alarm.

FIG. 2.13. The security alarm equivalent circuit

Problem 2.14

Describe the functions of voltage $v_C(t)$ and the current $i_C(t)$ over the capacitor (Fig. 2.14) for all time t > 0.
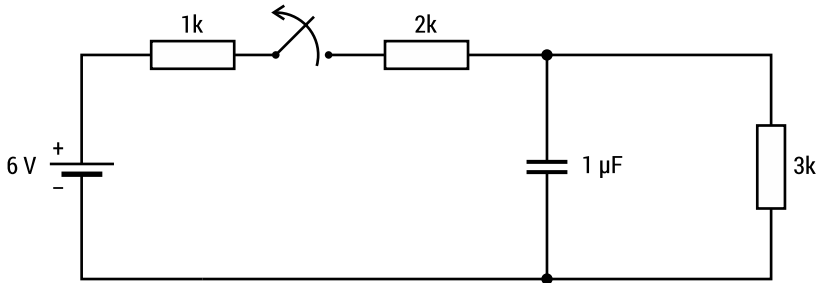
FIG. 2.14. The circuit for problem 2.14

Problem 2.15

The voltage source V_S in the circuit in the Fig. 2.15 is nominally 5 V. A power surge at time t = 0 causes it to instantaneously jump to 10 V, and then at tim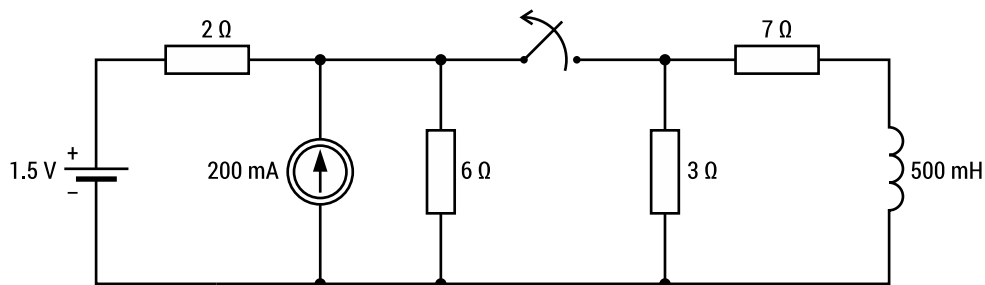e t = 5 ms, it instantaneously drops back to 5 V. Express the current flowing down through the 30 Ω resistor (as a function of time).
a) What is the 30 Ω resistor current at t = 0^+?
b) What would the final 30 Ω resistor current be if V_S stayed at 10 V?
c) What is the 30 Ω resistor current at t = 5 ms?
d) What is the final 30 Ω resistor current?
e) Write the full expression(s) for the 30 Ω resistor current as a function of time.
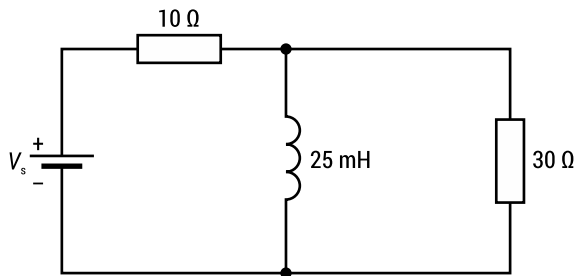
FIG. 2.15. The circuit for problem 2.15

Problem 2.16

A voltage source (from the previous task) of V_S = 18 V will be switched on at time t = 0. The maximum power dissipation of a 30 Ω resistor is 0.25 W. Check if it can be burned even if you know that current will flow through it for less than 20 ms. What should be the appropriate resistor to use in this circuit?

Problem 2.17

The switch in the circuit shown in the Fig. 2.17 closes at t = 0 and then opens at time t = 5 ms. Solve for the voltage over the 3k resistor (as a function of time).
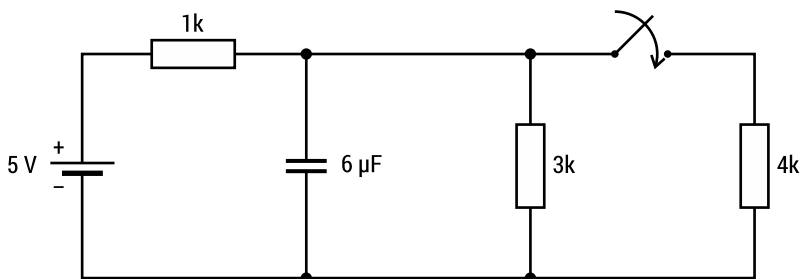
FIG. 2.17. The circuit for problem 2.17

Problem 2.18

You and your teammate wire the circuit (Fig. 2.18) on a protoboard. However, the 2k resistor is inserted in the board poorly and when your partner touches the resistor it becomes disconnected momentarily from the circuit at time 0 and then reconnected after 100 ms.

a) Describe the current flowing right through the 2k resistor by providing the mathematical formula(s) for t > 0 (split into multiple intervals if needed, but clearly label the time range of each interval).

b) Sketch a graph of this current for t > 0.
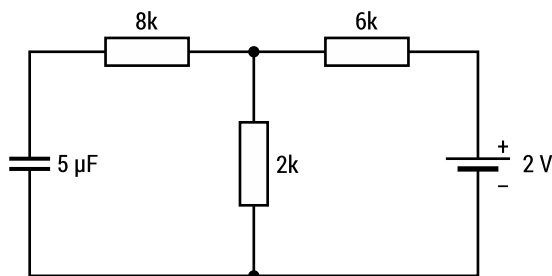
FIG. 2.18. The model of circuit connected at protoboard

Problem 2.19

A 12V DC power supply is connected to a 1 kΩ resistor and a 3 mF capacitor which are in series. The power supply has an internal source resistance of approximately 50 Ω.

a) Draw a diagram to model this circuit.

b) How long will it take the circuit to reach steady-state from the time the power supply is switched on?

c) Sketch the current through the 1 kΩ resistor as a function of time.

Problem 2.20

You and your teammate wire the circuit (Fig. 2.18) on a protoboard. However, the voltage source is poorly connected to the board and when your partner jiggles the cable,

it becomes disconnected momentarily from the circuit at time 0 and then reconnected after 100 ms.

a) Use transient analysis to describe the current flowing right through the 8 kΩ resistor by providing the mathematical formula(s) for t>0 (split into multiple intervals if needed, but clearly label the time range of each interval).
b) Sketch a graph of this current for t>0.
c) What is the value of current at time 50 ms and at time 150 ms?

Problem 2.21

In a lab you try to charge up a large capacitor (50 mF) using the circuit shown below (Fig. 2.21). However, when you turn on the supply, the 10 Ω resistor (0.5W, ±0.1%, 3ppm/°C, price 20 €) makes a loud pop, turns black, and smoke rises! The bad smell permeates the room.

* What was wrong? Quantify.
* What resistor should be used to avoid this accident?
* How long will it take the capacitor to charge up using the new resistor?
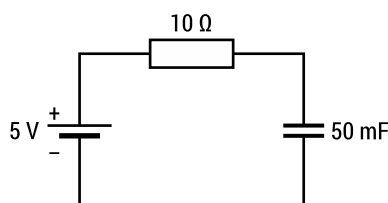
FIG. 2.21. Circuit for charging the capacitor

Problem 2.22

In the circuit below (Fig. 2.22), resistor 1k (on the right side) is suddenly removed, leaving a gap (an open circuit). Sketch the current through resistor 2k from this moment on.

* Label the graph scales to show the quantified maximum and quantified minimum current that flows.
* At what time is the current in resistor 2k slightly more than halfway between the maximum and minimum?
* At what time is the current in this resistor equal to exactly 20.0 mA?
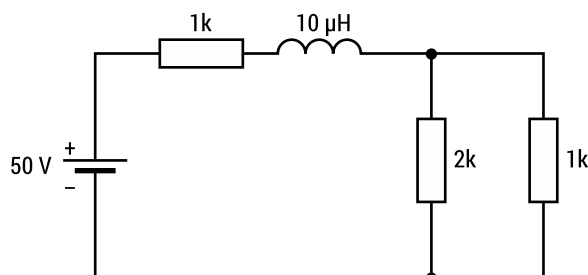
FIG. 2.22. Circuit connected at protoboard

Problem 2.23

The circuit shown below (Fig. 2.23) was in a steady state for a long time. At time t = 0, the 4 kΩ resistor was connected to the 1 nF capacitor for a moment and just after (10 μs) it is removed. Compute the voltage across the capacitor for all time. Give your answer as one or more formulas (clearly indicating the time range for each formula) and a sketch of the capacitor's voltage versus time.
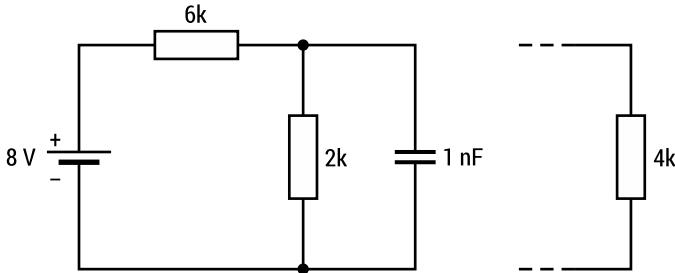
FIG. 2.23. **The circuit with removed resistor**

Problem 2.24

The voltage supply in the circuit shown below (Fig. 2.24) is switched on at t = 0. Compute the voltage over the 3k resistor for all time.
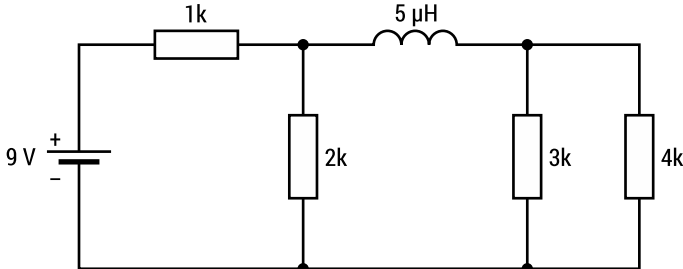
FIG. 2.24. **Circuit for calculating the load voltage (at 3k resistor) in transient state**

 SOLUTIONS

Inductor and diode » *keywords: Inductors*

Problem 2.25

Inductors are a bit mysterious, but by using a diode we might be able to *see* how they work. We have a circuit with a diode and an inductor, as shown in the figure below. We model the diode D_1 as an ideal resistor (R_L) of resistance 200 Ω. D_1 will be lit as long as it has a current of at least 2.5 mA running through it. The switch (Fig. 2.25) has been closed for a long time. When we open the switch, for how long after the switch

is open will the diode keep glowing? Try to connect the circuit in the real world, and see with your own eyes how the inductor is able to *store energy*.
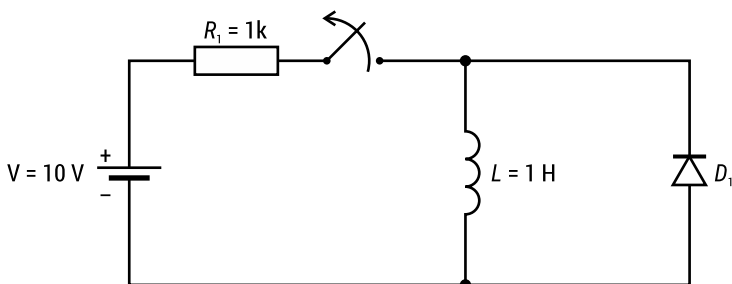
FIG. 2.25. Circuit with diode and inductor

 SOLUTION

Inductor and current source » *keywords: Inductors, Energy*

Problem 2.26
All the resistors in the circuit shown below (Fig. 2.26) have the same resistance R.
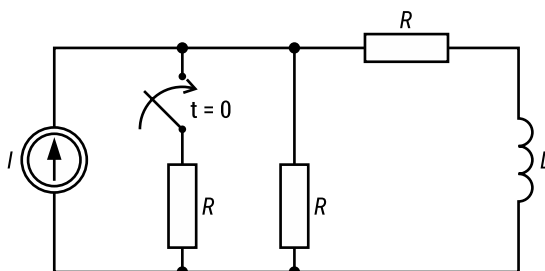
FIG. 2.26. An electric circuits with the current source in transient state

The switch has been open for a long time, at the time $t = 0$ the switch is closed. How big is the time constant τ for $t > 0$?

a) $\tau = \dfrac{L}{R}$

b) $\tau = \dfrac{2L}{3R}$

c) $\tau = \dfrac{3L}{2R}$

Problem 2.27

Look at the same circuit as in problem 2.26. Before the switch is closed the inductor has a stored energy W_0. What is the stored energy W_1 in this inductor when the switch has been closed for a long time?

a) $W_1 = \dfrac{W_0}{3}$

b) $W_1 = \dfrac{W_0}{9}$

c) $W_1 = \dfrac{4W_0}{9}$

SOLUTIONS

Make your own clock signal » *keywords: Logic gates, Capacitors*

In a digital circuit a clock signal is used to keep everything synchronized. This is often called CLK and it is a signal that switches between 1 and 0 with a constant frequency. This signal makes sure that everything in a computer happens in the right order. This clock signal can be generated in several different ways, one of which is shown below (Fig. 2.28).
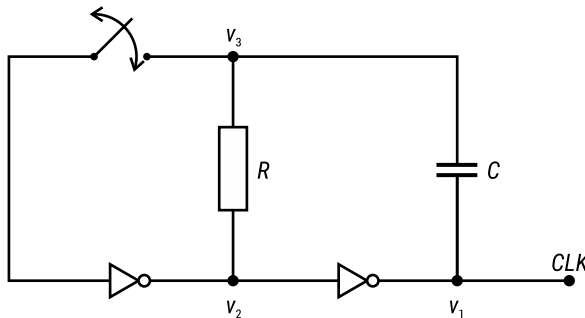
FIG. 2.28. A possible circuit for generating the clock signal

Problem 2.28

In the circuit we see three voltages v_1, v_2 and v_3, but where is the reference voltage ("ground")? **Hint:** Where is the source V_{DD}?

Assume that the inverters are ideal, meaning that all voltages below $\dfrac{V_{DD}}{2}$ are interpreted as logical 0 and that all voltages over $\dfrac{V_{DD}}{2}$ are logical 1. We can also assume

that the currents going into the inverters are zero. Start by assuming that the switch has been open for a long time and that it is then closed (this means that to start with $v_1 = 0$ and $v_2 = V_{DD}$).

Problem 2.29 Don't get stuck! (Use a few minutes then go to the next exercise)
Try to understand how the circuit works. **Hint:** How big is the voltage over the capacitor right after the switch is closed, and what happens next?

Problem 2.30
Study this simulation of the circuit: *https://tinyurl.com/2qgjx8o5* (here is another way of simulating the same circuit: *https://tinyurl.com/2mcmdzsf*). You can vary the simulation speed if needed.

Look at the voltages v_1, v_2 and v_3 and describe how the circuit works.

Problem 2.31
The period of v_1 is given by $T = 2\ln(3)\tau$. Design a clock generator with a frequency $f_c = 1$ kHz. Test if your circuit works as desired by simulating it: *https://www.falstad.com/circuit/circuitjs.html*

Problem 2.32 Extra Challenge
Derive the formula for the period that is given above. **Hint:** Look at the capacitor's voltage for half a period.

Problem 2.33 Extra Challenge
Design a clock generator with a period of 1 second and implement it in the real world. Connect a diode to the circuit to check if the light flashes with approximately the right frequency. NB: remember to connect a resistor in series with the diode so that it doesn't burn out.

Hint: To get the high period you will probably need a capacitor with high capacitance. These are often polarized. To avoid that it explodes when current runs through it in the wrong direction you can connect it as shown.
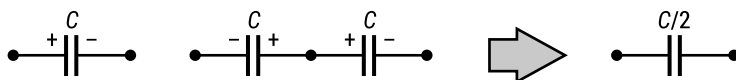
FIG. 2.33. **The equivalent capacitance for two capacitors**

Problem 2.34 Extra Challenge
Expand the design with another diode so that one lights when CLK is high, and one lights when CLK is low.

SOLUTIONS

CHAPTER 3. Basics of Electronics Elements

If you need some theoretical support for this part then use these links below or write in Internet search engine *Razavi Electronics 1, YouTube* or *Razavi Electronics 2, YouTube.*

- *https://ocw.mit.edu/courses/6-002-circuits-and-electronics-spring-2007/*
- *https://www.coursera.org/search?query=electronics&query%20=%20electronics*

Problem 3.1

The transistor, in the circuit shown in Fig. 3.1, will operate as a switch that turns on (or off) a 12V/60W automotive fog lamp. The transistor should be controlled by a digital signal: low level – 0 V, high level – 5 V. Two transistors are given: an NPN bipolar transistor 2SC6082 [3.1.1] and an N-channel MOSFET FDP8447L [3.1.2].

Which transistor will you choose? Make the appropriate calculations and justify your choice.
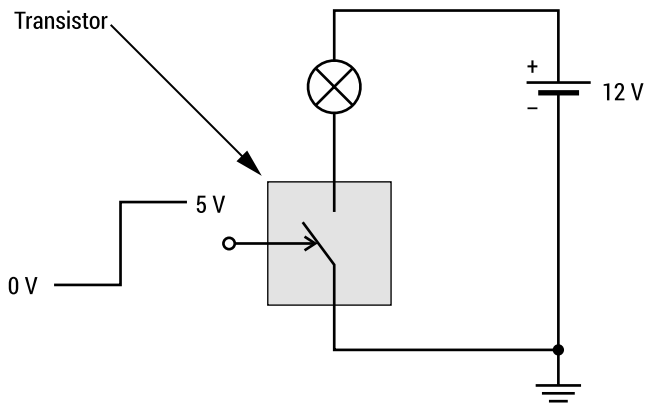
FIG. 3.1. Transistor as a switch

- [3.1.1] datasheet of 2SC6082, i.e. *2SC6082 (https://www.onsemi.com/download/data-sheet/pdf/ena0279-d.pdf)*
- [3.1.2] datasheet of FDP8447L, i.e. *FDP8447L N-Channel PowerTrench® MOSFET (onsemi.com)*
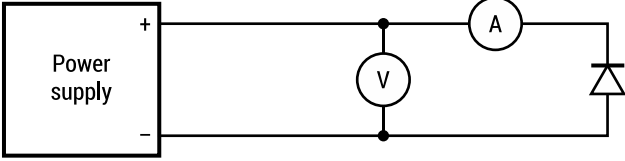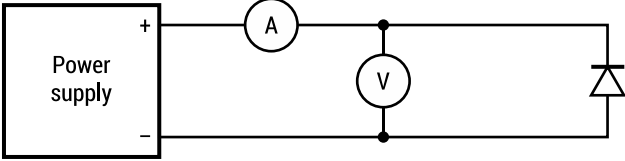
SOLUTION

Problem 3.2

During a laboratory exercise, two students (A and B) measured the reverse current of a 1N5408 rectifying diode at 30 V. They both used the same diode and the same digital multimeters [3.2.1], but they connected different measurement systems and got different measurement results (see Table 3.2). Justify which student got the correct results. The maximum DC Reverse Current of 1N5408 diode is 5 μA [3.2.2].

TABLE 3.2. Measurement systems built by students and obtained results

Student	Scheme of the measuring system.	Measurement results visible on displays of voltmeter and ammeter
Student A	Power supply (+/−), V across, A in series with diode	30.00 V 0.0000 mA
Student B	Power supply (+/−), A in series, V across diode	30.00 V 0.0030 mA

- [3.2.1] MY-65 digital multimeter – User manual;
- [3.2.2] *https://www.vishay.com/docs/88516/1n5400.pdf*

SOLUTION

Problem 3.3

During steady-state operation of a half wave rectifier with capacitor filter (Fig. 3.3a), determining the maximum value of the current flowing through the diode is relatively easy.

Can you estimate the maximum value of the current that will flow through the diode immediately after the rectifier is switched on (when the capacitor is fully discharged)?

Use the circuit shown in Figure 3.3b (transformer: 230V/12V, R_L = 200 Ω, C = 1000 μF). For simplicity, assume that the transformer is ideal (the secondary winding behaves as an ideal sinusoidal voltage source with zero internal resistance),

and R_s (e.g. with a value of 0.1 Ω – 1 Ω) represents the equivalent series resistance of the transformer output circuit.

Does the moment the rectifier is switched on (instantaneous value of the mains voltage) affect the maximum value of the diode current?

Can the current pulses occurring when the rectifier is switched on damage the diode? Which parameter in the diode data sheet will help answer this question?
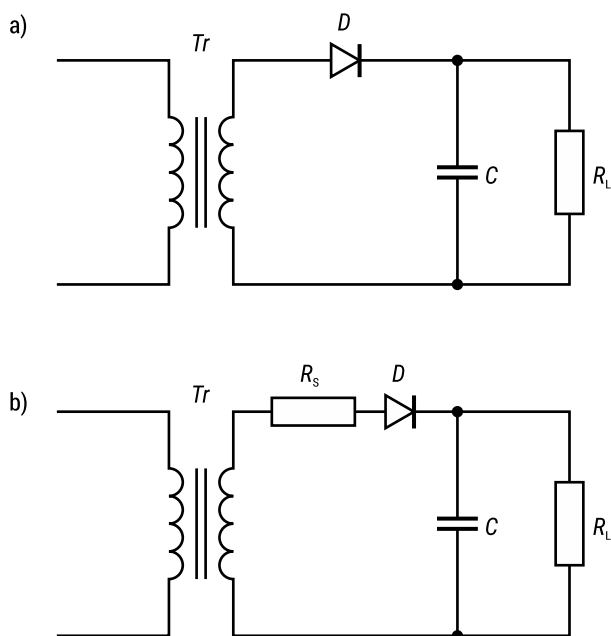
FIG. 3.3. Circuit diagram of a half wave rectifier with capacitor filter (a) and circuit diagram of a rectifier including equivalent series resistance of the transformer output circuit (b)

SOLUTION

Problem 3.4

Usually, a digital multimeter has a diode test function. With this function you can easily check if a diode is working (undamaged) and – in case of a working diode – determine its leads: anode and cathode. In other words – you can find the p-type region and the n-type region of a p-n junction. If you do not know how to do this, you can refer to the multimeter user manual [3.4.1] or watch a short video on YouTube [3.4.2].

Suppose you do not have access to the data sheet for a bipolar transistor or the name of the transistor is illegible.

Explain how, using a multimeter with a diode test function, you can:
a) determine whether it is a npn or pnp transistor;
b) identify the leads of the transistor (collector, base, emitter).

- [3.4.1] Fluke 80 series V – User manual, *untitled (fluke.com)*
- [3.4.2] *https://www.youtube.com/watch?v = mMXDa5hVzXA*

SOLUTION

Problem 3.5

In industrial environments, it is often necessary to measure the average temperature (e.g. on a factory floor or in an industrial process). Specialised integrated circuits (for example LM35, TMP36) are used as temperature sensors. Their output voltage is a linear function of temperature.

The following figures (Fig. 3.5a and 3.5b) show schematics of electronic circuits for determining the average value of three input voltages: V1, V2 and V3.

FIG 3.5A. Circuit A
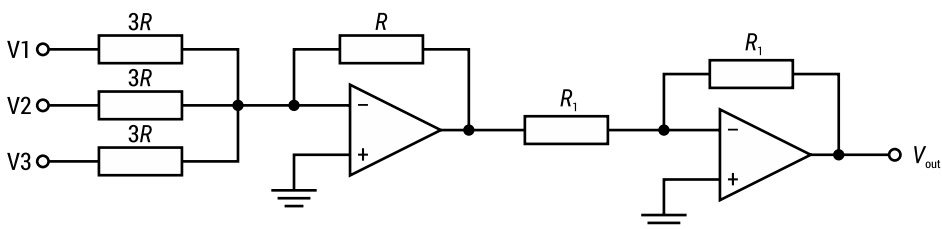
FIG. 3.5B. Circuit B

Prove that:

$$V_{out} = \frac{V1 + V2 + V3}{3}$$

List the advantages and disadvantages of both circuits.

 SOLUTION

Problem 3.6

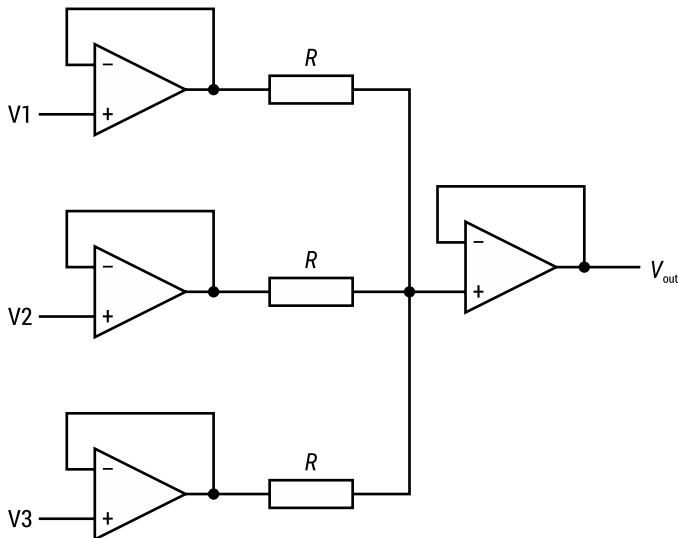Current sources are commonly used in many applications such as LED drivers, active loads, battery chargers, 4 – 20 mA current loops (industrial process control), sensor biasing and others.

The LM317 integrated circuit is an adjustable three-terminal positive-voltage regulator. Using the data sheet for this device [3.6.1], propose a 5 mA current source circuit. Calculate the maximum voltage drop across the load or the maximum value of the load resistance.

• [3.6.1] *https://www.ti.com/product/LM317?qgpn = lm317*

 SOLUTION

Problem 3.7

Your company bought a full reel of npn 2SC1815-O transistors – 3.000 pcs. You need to test all the transistors and select those which DC current gain $\beta = 100 \pm 10\%$ (under measurement conditions: $I_C = 5$ mA, $V_{CE} \approx 7$ V).

Propose a measurement circuit that will facilitate the selection and indicate the correct transistor by switching on the green LED.

 SOLUTION

Problem 3.8

Fig. 3.8a shows the circuit diagram of the water level detector. We can use this detector in closed opaque tanks to monitor the water level. When electrode B is above the liquid level the LED is off, but when both electrodes (A, B) are in the water the LED turns on. Instead of the LED, an optocoupler can be used to switch the water pump on/off.

Explain the principle of operation of this detector. Calculate the values of resistances $R_1 – R_3$. For calculations, assume that the resistance of the water (RW) between the electrodes is 250 kΩ. The current-voltage characteristic of the diode is shown in Fig. 3.8b.
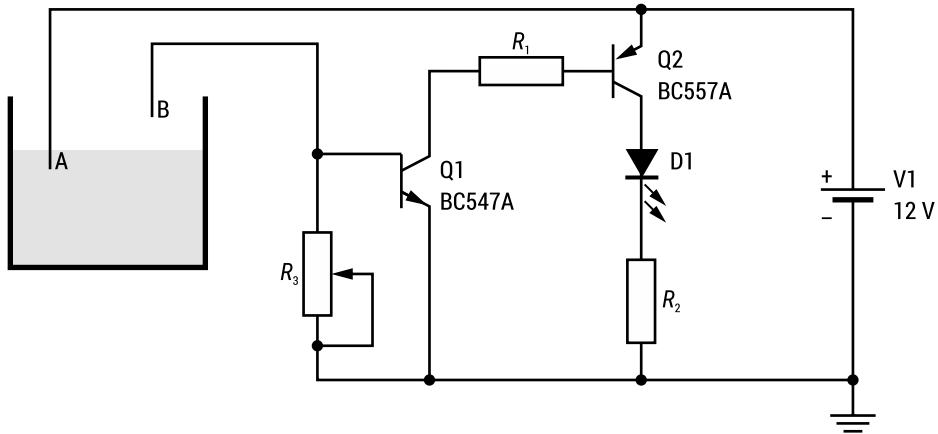
FIG. 3.8A. Circuit diagram of the water level detector
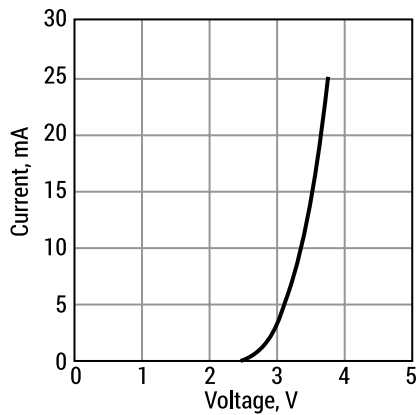
FIG. 3.8B. Current-voltage characteristic of the LED

SOLUTION

Problem 3.9

Some medicines or vaccines need to be refrigerated at the temperature specified by the manufacturer (for example, between 2°C and 8°C).

The TMP235 is a linear analogue temperature sensor. The output voltage of this sensor is linearly proportional to temperature on the Celsius scale and is described by the function:

$$T \text{ [in}°C] = (V_{out}\text{[in mV]} - 500 \text{ mV})/(10 \text{ mV})$$

Design an electronic circuit (using the TMP235 sensor) that measures the temperature in a refrigerator. The output voltage of this circuit in volts should strictly correspond to the temperature in Celsius degrees (1 V "equals" 1°C). An analogue

voltmeter can be used to display "temperature". In addition, exceeding a temperature of 6°C should activate an alarm, for example a red LED.

SOLUTION

Problem 3.10

The circuit shown in Fig. 3.10 automatically switches on the house number lighting (12V/21W bulb or LED panel 12V/3W) at dusk and switches it off at dawn.

The PGM5516-MP [3.10.1] photoresistor (PR) is used in this circuit. The operational amplifier U1 works as a voltage comparator.
a) Explain the principle of operation of the circuit.
b) Calculate the values of resistors $R_1 - R_3$. Find the parameters of the photoresistor in its data sheet.
c) How can the on/off threshold be adjusted?
d) Can the photoresistor be placed elsewhere in the circuit?
e) Suggest an improvement to the circuit so that momentary fluctuations in light intensity do not switch the bulb on and off?

- [3.10.1] *PGM5516-MP Token – Photoresistor | 90m W; 5÷10 kΩ; 540 nm; THT; 100 VDC; ØLED: 5.5 mm | TME – Electronic components*
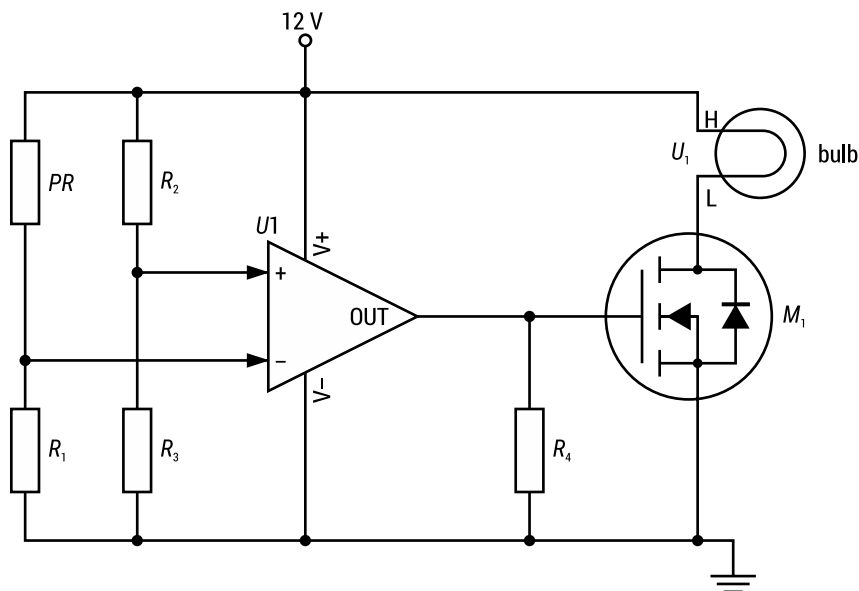
FIG. 3.10. Circuit diagram for automatic light switching on and off

SOLUTION

Make a mix table » *keywords: Operational amplifiers*

Problem 3.11
Show that the circuit (Fig. 3.11) satisfies:

$$v_0 = -\left(\frac{R_2}{R_{11}} v_1 + \frac{R_2}{R_{12}} v_2 \right).$$
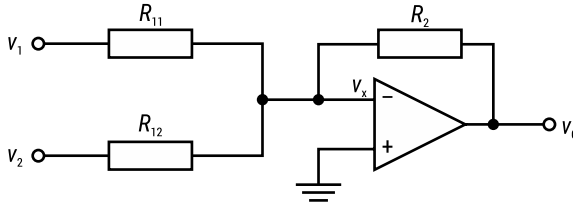
FIG. 3.11. A circuit that realizes the summing and amplification of signals

Problem 3.12
Now imagine that the inputs v_1 and v_2 in figure above are connected to different sound sources. One is connected to a CD-player and the other is connected to a microphone. A DJ wants to use this as a turn table so that he can mix in his voice over the music. How can you modify the circuit to accomplish this effect? **Hint:** potentiometer.

 SOLUTIONS

Amplify your music » *keywords: Operational amplifiers*

Problem 3.13
Design a circuit that gives a 10 times amplification using an operational amplifier.

Problem 3.14
Use an operational amplifier that it has a supply voltage of ±5 V (e.g. LF353) and wires up the circuit in real life. You can also use a circuit simulator if you don't have the equipment, link: *https://www.falstad.com/circuit/circuitjs.html*.
   At what input voltage will the circuit stop working as it should?
   Solution: *https://tinyurl.com/2g4b6acj*

Problem 3.15
Make the feedback resistor variable by replacing it with a potentiometer. Use a Jack-connector to send music at a low volume as the input of the circuit. Put another

Jack-connector on the output of the system and connect your headphones to it. Now see if you can use the circuit to amplify the sound to a comfortable volume.

 SOLUTIONS

Electronic thermometer » *keywords: NTC thermistors*

Problem 3.16

The circuit below has two resistors, one of which has a sort of slash through it, this indicates that it is a NTC thermistor. This is a temperature dependent resistor with a negative temperature coefficient. This means that the resistance R_1 decreases with increasing temperature T. If we look at a very small temperature range, it's behaviour can be approximated with a linear function

$$R_1 = R_0 - aT$$
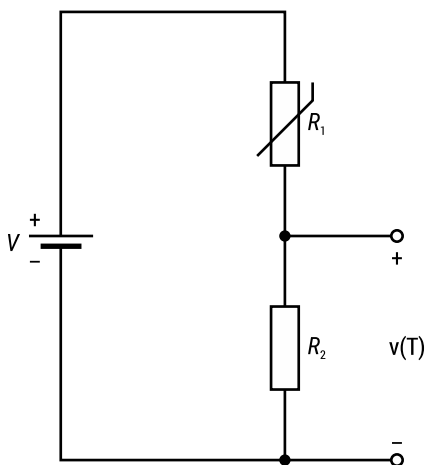
FIG. 3.16. The equivalent circuit for electronic thermometer

Here a is a positive constant and T has the unit °C.
a) Find an expression for the voltage v as a function of T.
b) Will the voltage v rise or fall as the temperature T increases? Justify your answer.
c) Now $V = 9$ V, $R_2 = 10$ kΩ, $R_0 = 20$ kΩ and $a = 400$ Ω/°C. The voltage source is a battery with an insignificant internal resistance and a battery capacity of 0.5 Ah.
d) How long will the battery last if the temperature in the room is $T = 25$°C?

 SOLUTION

CHAPTER 4. AC Circuits Analysis.

You can find the necessary theoretical information in *Basic Engineering Circuit Analysis* by J. David Irwin, R. Mark Nelms on Chapter 8, 9 and 10 (pp. 369-515).

The theory necessary for frequency analysis is in Chapter 12 of this book.

Introduction: Tasks for self-evaluation

1) Describe the construction of basic elements of a circuit: resistor, inductor, capacitor. Show the differences and common features and propose the method to recognize them when you see only at bunch of black boxes with two terminations on each one (you can use multimeters, power sources DC and AC and oscilloscope).

2) Try to specify what happens with energy at each type of element when it is connected to a power source. Find as many as possible expressions/formulas as possible to calculate the amount of energy at a given resistor, inductor or capacitor.

3) Describe in detail (in a form of mathematical function) and show graphically the dependences of ideal elements of circuit like: resistors (voltage in terms of current), inductors (magnetic stream in terms of current), capacitors (charge in terms of voltage).

4) What is the easiest way to recognize the voltage and current source (ideal and also the real case)?

5) For the parallel/series R_1C_1 /R_1L_1 circuit supplied by voltage AC source $v(t) = V_{amp}sin(\omega t)$ write the formula for impedance and admittance. Express them in algebraic and exponential forms to show the resistance, reactance, conductance, susceptance, their modules and arguments. Draw the vector diagram of currents and voltages. Use the parameters R_1, C_1, R_1, L_1, V_{amp}, ω to define the active and reactive powers in this circuit.

6) Explain the reasons and possible ways to increase the power factor of a load (compensation of reactive power). You may find some theory here below:
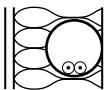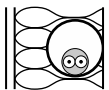
POWER FACTOR

7) With the use of resonance conditions find the resonant frequency value of the circuit presented by your teacher. Make the vector diagram of currents and voltages in resonant state of it.

8) Make the frequency analysis of the RC/RL circuits presented by your teacher. Determine the cut off frequency and draw the amplitude characteristic of these filters. Define the pass and stop bands of them.

Problem 4.1

The loads in the electrical circuit are 30 light sources – each of 30 W with nominal voltage 230 V. The power factor is 0.85. Select a proper power wire for the loads and choose an overcurrent protection device. Check if the voltage drop on the cable does not exceed 2%. For calculations assume the conductivity of copper $56\dfrac{m}{\Omega \cdot mm^2}$, cable length l = 18 m and that the cable is installed under plaster – A2 method (Table 4.1 from technical standard: *PN-HD 60364-5-52:2011/A11:2018-12*).

TABLE 4.1. The permissible current values depend on the cross-section of the wires and the way they are laid

Nominal cross-sectional area of conductor mm²	Installation methods			
	A1	A2	B1	B2
1	2	3	4	5
Copper				
1.5	14.5	14	17.5	16.5
2.5	19.5	18.5	24	23
4	26	25	32	30
6	34	32	41	38
10	46	43	57	52
16	61	57	76	69
25	80	75	101	90
35	99	92	125	111
50	119	110	151	133
70	151	139	192	168
95	182	167	232	201
120	210	192	269	232
150	240	219	300	258
185	273	248	341	294
240	321	291	400	344
300	367	334	458	394

Use the web side to select the appropriate circuit breaker.

- *https://www.se.com/ww/en/product-subcategory/1605-acti9-miniature-circuit-breakers-mcbs/?filter = business-5-residential-and-small-business*

Answer: Minimum cross-section of 1.5 mm^2. Proposed solution I_n = 10 A. Drop voltage 0.86% – acceptable.

Problem 4.2

Ironing is one of the daily activities we do in our homes. Living in a community of 5 people, assume that each of you irons a shirt in an identical time of 2 minutes.

Assuming that the iron heats up for one minute, calculate the cost of ironing the shirts:
a) One by one.
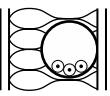b) Each with separate heating of the iron.
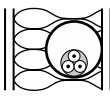
Assume that the power of the iron is 3500 W, and the cost of energy is 0.14 € per 1 kWh. Calculate the annual cost of electricity consumption for both options.

Answer: The annual cost a) 32.79 €; b) 44.71 €.

Problem 4.3

In a newly designed house with a garage, one of the loads is a three-phase welding machine. Using the technical data provided on the website, *https://www.toolnation. com/telwin-830827004-mastermig-400-migmag-welding-machine.html* select the cross-section of the cable which should be supplied to the milling machine. Assume in your calculations that the cable is led into an installation pipe on the wall – B2. In calculations assume the welding machine drive power factor 0.88 (lagg).

TABLE 4.3. The permissible current values depend on the cross-section of the wires and the way they are laid

Nominal cross-sectional area of conductor mm²	Installation methods of Table B.52.1			
	A1	A2	B1	B2
1	2	3	4	5
Copper				
1.5	13.5	13	15.5	15
2.5	18	17.5	21	20
4	24	23	28	27
6	31	29	36	34
10	42	39	50	46
16	56	52	68	62
25	73	68	89	80
35	89	83	110	99
50	108	99	134	118
70	136	125	171	149
95	164	150	207	179
120	188	172	239	206
150	216	196	262	225
185	245	223	296	255
240	286	261	346	297
300	328	298	394	339

(from technical standard: *PN-HD 60364-5-52:2011/A11:2018-12*)

Use the page to select the appropriate circuit breaker.

- *https://www.se.com/ww/en/product-range/65251-acti9-ic40/?parent-subcategory-id = 1605&filter = business-5-residential-and-small-business*

Answer: Minimum cross-section of 10 mm². Proposed solution $I_n = 40$ A.

Problem 4.4

Suppose you have an electric kettle with a power of 1730 W. Assume that one litre of water in the kettle boils for 2 min 10 s, and the electricity price of 1 kWh is 0.14 €.

a) Calculate the cost of one glass of tea if you boil 1 l of water in a kettle and make 4 glasses of tea (0.25 l).
b) Calculate the cost of heating one glass of tea if you drink the tea yourself, boiling 1 l of water in a kettle.
c) Assuming that 0.5 l of water in the kettle boils in 1 min 12 s, calculate the electricity consumption required to boil 0.5 l of water. Determine whether it is cheaper to boil 1 liter of water at once or to boil 0.5 liters of water twice.
d) Determine the annual (365 days) cost of boiling water for a family of four, assuming that everyone drinks tea in the morning and in the evening.
e) Calculate the current flowing through the kettle as it heats the water.

Answer: a) 0.0022 €; b) 0.0087 €; c) E = 0.035 kWh; d) Cost per year 6.42 €; e) I = 7.52 A.

Problem 4.5

A buck converter has been used to regulate the voltage in a circuit. On the output of the buck converter it is possible to obtain voltages of 5 V and 10 V. The converter manipulates the voltage so that on terminals 2-3 (feedback) there is always voltage of 0.8 V. Assuming that the receiver in this circuit draws 6 A of current, and that max. 5% of the current of the receiver can flow through the voltage divider, choose appropriate resistor values. You have resistors from series E24. Using the available resistor powers, select appropriate resistors.

E 24	10, 11, 12, 13, 15, 16, 18, 20, 22, 24, 27, 30, 33, 36, 39, 43, 47, 51, 56, 62, 68, 75, 82, 91

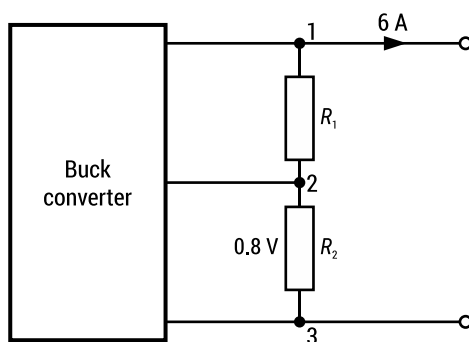| P, W	1/8; 1/4; 1/2; 1; 2; 5; 10

FIG. 4.5. An electric circuit with a buck converter for voltage regulation

Answer: 5 V – Resistor R_1 – resistance 15 Ω, power 2 W; Resistor R_2 – resistance 2.7 Ω, power 0.25 W; 10 V – Resistor R_1 – resistance 33 Ω, power 5 W; Resistor R_2 – resistance 2.7 Ω, power 0.25 W.

Problem 4.6

The current development of technology allows us to use many options that can reduce the electricity consumption of our TV sets. In addition to the standard TV switch-off, most of them have a standby mode in which it has a power of 0.2 W. Assuming that the television has a power output of 150 W and the energy cost is 0.28 € /kWh, calculate the following:

a) How much does it cost to watch TV for 3 hours a day for a year? Each time the TV is turned on and off manually.

b) What is the annual cost of using the standby function if the TV is turned on for 3 hours a day and the remaining time is left on standby?

Answer: a) E = 164.25 kWh, cost = 45.99 €; b) E = 1.53 kWh, cost = 0.43 €;

Problem 4.7

The photovoltaic system in the holiday home works off-grid during the summer time and the daily electricity consumption is 600 Wh. Since the holiday home cannot be connected to the power grid, energy storage is required so that you can use electricity when the panels are not working (i.e. evening, night, cloudy). Assuming a system voltage of 24 V, select the most suitable batteries so that electricity can be stored and used for 48 hours.

Leoch Battery 12V 24AH VRLA AGM	
Voltage	12 V
Nominal Capacity	24 Ah
Price	£ 55.44

https://www.sunstore.co.uk/product/leoch-battery-12v-24ah-vrla-agm/

75Ah 12v Leisure Battery	
Voltage	12 V
Nominal Capacity	75 Ah
Price	£ 83.95

https://www.sunstore.co.uk/product/75ah-12v-leisure-battery/

Varta Sealed Battery,12V, 140Ah (c20)	
Voltage	12 V
Nominal Capacity	140 Ah
Price	£ 160.82

https://www.tayna.co.uk/truck-batteries/varta/k8/

Ultramax LI100-24 24V 100Ah Battery	
Voltage	24 V
Nominal Capacity	100 Ah
Price	£ 862.18

https://batterymasters.co.uk/ultra-max-lifepo4-battery-24v-100ah.html

Answer: Varta Sealed Battery, 12V, 140Ah, cost = 321,64 £.

Problem 4.8

Select the component values in the air coil equivalent circuit diagram (Fig. 4.8a) such that the displacement between current I and voltage U is 60°. Calculate the power (P, Q, S) of such a system. Draw the appropriate phase diagrams. Select the R, L, C elements from the E24 series (10-11-12-13-15-16-18-20-22-24-27-30-33-36-39-43-47-51-56-62-68-75-82-91) – link: *https://www.bourns.com/support/technical-articles/standard-values-used-in-capacitors-inductors-and-resistors*
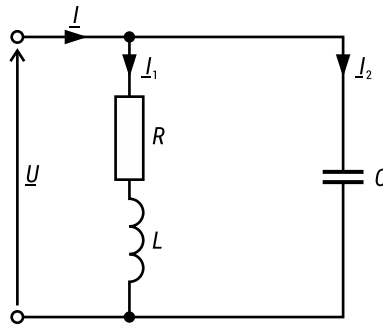

FIG. 4.8A. The equivalent diagram for air coil

 SOLUTION

Problem 4.9

Determine the voltage drop in the internal circuit (between the main switchboard and the boiler) if the circuit is installed with a 3-core cable with a cross-section of 2.5 mm^2 copper each (γ = 56 m/Ω mm^2) and the circuit length l = 15 m. What reactive power will be released during the operation of this device. A single-phase capacity boiler was used (water heater Warmtec ZF15-LJ50-6C 50 l; 1.5 kW; 230 V; 6.8 A; 0.8 MPa).

 SOLUTION

Problem 4.10

Determine the voltage drop in the inner circuit of the apartment building (between the main switchboard and the five-light chandelier). This part of the electrical installation was made with a 3-core cable with a copper cross-section of 1.5 mm^2 (γ = 56 m/Ω mm^2) with a circuit length of l = 22 m. Applied light sources – Philips; 13 W; 1521 lm; 220-240 V; cosφ = 0.5. Additionally, determine the reactive power consumed from the network by this device. Would it be economy to replace light sources of this kind given the cost of the modern light sources? The cost of one of each is about 4 €.

Problem 4.11

What difference will we notice in terms of the power consumed (P, Q) and the energy consumed when using four classic 40 W bulbs (equivalent to a 4-5 W light source, luminous flux 415 lm, efficiency 10.4 lm/W, power factor PF 1) each and after their replacement with modern light sources: LED Osram Value Classic (5.5 W; 470 lm; 85.5 lm / W; PF 0.5; 48 mA); LED Philips Master LED bulb DT (6 W; 470 lm; 78.0 lm / W; PF 0.7; 37 mA). The duration of use is 5 hours a day within a month (30 days). What is the tangent of the shift phase angle φ which determines the costs of energy consumption due to reactive power consumption? How long should the above modern light sources be used for the supplier to charge a penalty for excessive reactive power consumption.

Problem 4.12

We have a receiver (toroidal coil) available with the following parameters: power 300 VA; dimensions (outer diameter x inner diameter x height) 0.115 x 0.06 x 0.03 m; 500 turns; winding wire diameter 0.53 mm; $R = 5.3\ \Omega$; $I = 2.35$ A. How can we compensate for its reactive power.

Formula for receiver inductance: $L = \dfrac{\mu_r \mu_0 S n^2}{2\pi r}$

where:

L – self-inductance, H,

μ_r – relative magnetic permeability of the medium (soft ferrites – 6000),

μ_0 – magnetic permeability of vacuum ($4\pi \cdot 10^{-7}\ \dfrac{H}{m}$),

S – cross-sectional area of the torus,

n – number of turns,

r – mean radius of the torus.

Problem 4.13

Below are the invoices for electricity distribution services received by the public utility facility in November 2021 (Table 4.13 part A) and in January 2022 (Table 4.13 part B). What changes have been made in terms of energy consumption when we take into account its types? What devices should be connected to compensate the used reactive power? What amount (and kind) of reactive energy must a given facility use in order

for the energy company to be able to charge a penalty for the contractual consumption of reactive power? The average price in the given billing period for 1 kWh was respectively: November 2021– 0.4727 PLN; January 2022 – 0.6116 PLN.

TABLE 4.13. Data from public utility facility invoice: A) November 2021; B) January 2022.

A)	Readings			
	previous indication	current indication	multiplicant	payment in PLN
active power consumed, kWh	485.418	529.611	80	5535
reactive power consumed, kvarh	12.296	13.624	80	106
reactive power delivered, kvarh	45.037	49.648	80	369
B)	Readings			
	previous indication	current indication	multiplicant	payment in PLN
active power consumed, kWh	583.521	633.888	80	6176
reactive power consumed, kvarh	14.815	16.189	80	110
reactive power delivered, kvarh	63.589	78.772	80	1215

SOLUTION

Problem 4.14

The generator has an output impedance ($C_1 = C_2 = 10\ \mu F$, $R_1 = 200\ \Omega$, $R_2 = 100\ \Omega$) as shown in Fig. 4.14. What impedance should be connected to it in order to achieve matching due to active power? Provide the necessary calculations.
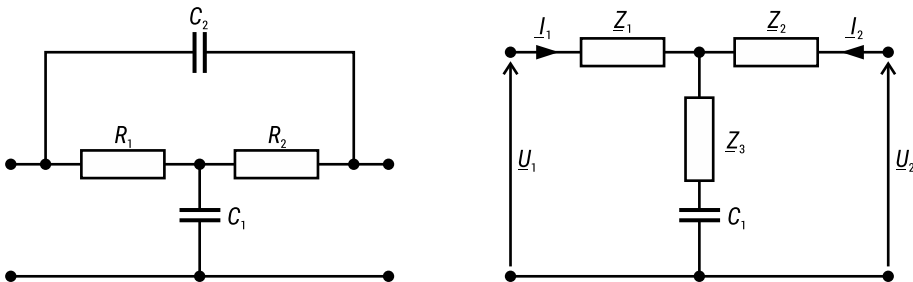

FIG. 4.14. Schematic impedance arrangement in audio generator

SOLUTION

Problem 4.15

The capacitive voltage divider (Fig. 4.15) used in medium voltage lines, was powered from an alternating voltage source with a voltage of 10 V and a variable frequency within the range of 50-500 Hz. How will the capacitive voltage divider ($C_1 = 10$ µF, $C_2 = 22$ µF) affect the parameters of the voltage network?
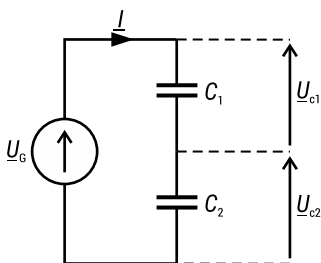

FIG. 4.15. Simplified schematic of the capacitive voltage divider

SOLUTION

Problem 4.16

The amount of electricity that will flow in the apartment's power supply circuit when the following devices are turned on: TV set, the ceiling lamp, the fridge, the electric kettle, microwave oven and the washing machine:
1) TV set – Samsung 65" QLED 4K, 116W, 230 V, 50 Hz, 102 kWh, PF 0.86;
2) ceiling lamp – 3 x light source OSRAM DULUXSTAR 21W, PF 0.7;
3) fridge – Whirlpool CB 281W, 281W, PF 0.59;
4) electric kettle – Tefal KI700830, 1850W, PF 0.7;
5) microwave oven – LG, 1000W, PF 0.92;
6) washing machine – Samsung (WF70F5EOW2W), 2400W; 2.4 kWh, PF 0.7.

SOLUTION

Problem 4.17

Fig. 4.17 presents the resistive-capacitive voltage divider used in medium voltage lines. Derive the dependence on the voltage that will be measured by the voltmeter connected to parallel elements C_2 and R_2. Calculate the value of the voltmeter when the total voltage is 15 kV, the frequency 50 Hz and $C_1 = 15$ nF, $C_2 = 5$ nF. Use the condition to obtain the frequency independent voltage ratio: $R_1 \cdot C_1 = R_2 \cdot C_2$.
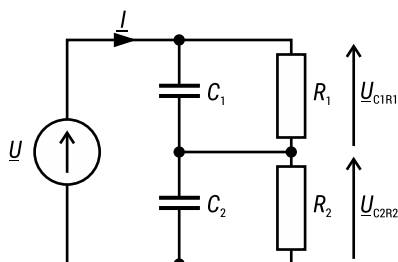
FIG. 4.17. Simplified schematic of the resistive-capacitive voltage divider

 SOLUTION

Problem 4.18

Design a low-pass filter, passive (5 kHz), thanks to which (in the case of a signal that changes in time) very quick changes of state (contact vibrations) occurring when a button is pressed, will be eliminated. Use the available elements (Fig. 4.18):

1) capacitors – 100 nF, 22 nF, 22 μF
2) adjustable potentiometer – 1 kΩ, 10 kΩ.

   Make the necessary calculations and present the amplitude and phase characteristics.

- *https://www.youtube.com/watch?v = GqCn94Ksdbo*
- *https://www.youtube.com/watch?v = Oy9Nsql4iIw*
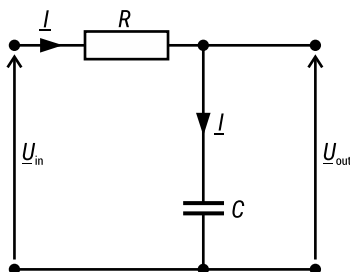- *https://www.youtube.com/watch?v = P-Grp0HIljI*

FIG. 4.18. Schematic of the low-pass filter

 SOLUTION

Problem 4.19

Design a 250 Hz high-pass filter (passive) for acoustic system correction (octave equalizer). Use the available elements (Fig. 4.19):

1) capacitors – 220 nF, 330 nF, 470 nF
2) adjustable potentiometer – 2 kΩ, 5 kΩ, 10 kΩ.

Make the necessary calculations and present the amplitude and phase characteristics.

- *https://www.youtube.com/watch?v = GqCn94Ksdbo*
- *https://www.youtube.com/watch?v = Oy9Nsql4iIw*
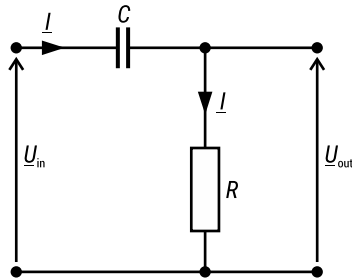- *https://www.youtube.com/watch?v = P-Grp0HIljI*

FIG. 4.19. Schematic of the high-pass filter

SOLUTION

Problem 4.20

Design the bass filter, passive for 500 Hz cut-off frequency to improve the bass and cut off the highs. Select the values of the elements (inductance – L, capacitance – C) from the E24 series.

Useful links:

- *https://www.bourns.com/support/technical-articles/standard-values-used-in-capacitors-inductors-and-resistors*
- *https://electronicsreference.com/analog/lc_filter/*
- *https://www.coilcraft.com/en-us/edu/series/what-is-an-lc-filter/*
- *https://resources.pcb.cadence.com/blog/lc-filter-for-power-supply-design-tips*
- *https://www.tdk.com/en/tech-mag/noise/06*
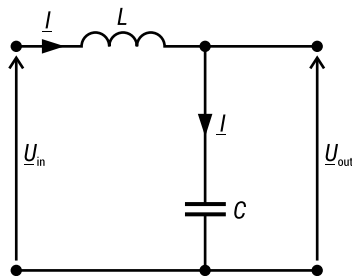- *https://www.industrial-electronics.com/lc_3.html*

FIG. 4.20. Schematic of the bass filter

SOLUTION

Problem 4.21

Design the band-pass filter, passive for radio band (88 MHz -108 MHz) having at your disposal the elements $R_1 = R_2 = 20 \ \Omega$. Select the values of the capacitors C_1 and C_2 from the E24 series.

- *https://www.youtube.com/watch?v = GqCn94Ksdbo*
- *https://www.youtube.com/watch?v = Oy9Nsql4iIw*
- *https://www.youtube.com/watch?v = P-Grp0HIljI*
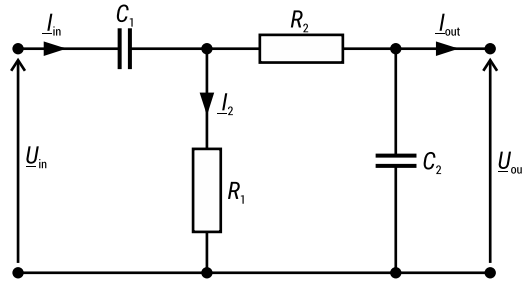
FIG. 4.21. **Schematic of the band-pass filter**

SOLUTION

Problem 4.22

Loudspeakers have been installed in the house. At what distance from the source and with what cable (parameters R, L, C) should the speakers be connected so that the sound does not lose its quality? A loss in quality is a signal change by 1 dB in the entire range of the acoustic band (20 Hz ÷ 20 kHz).

FIG. 4.22. **Equivalent diagram for connecting amplifier (Amp) − loudspeaker (LS) with a loud-speaker cable**

SOLUTION

Problem 4.23

The factory of an impedance $\underline{Z}_f$ (Fig. 4.23) is connected to a medium voltage grid (U_g, R_g) by an overhead line (R, L, C). This factory has ordered an active power of at least 2.8 MW and a reactive power of 0.8 Mvar. Will these powers be ensured if the grid voltage drops to U_g = 8.3 kV?

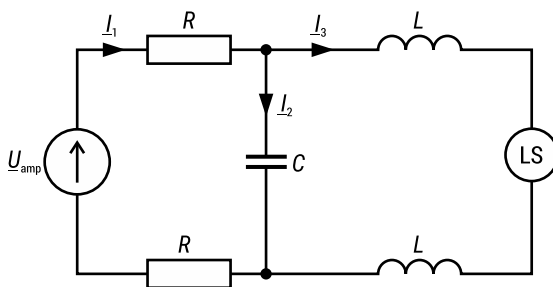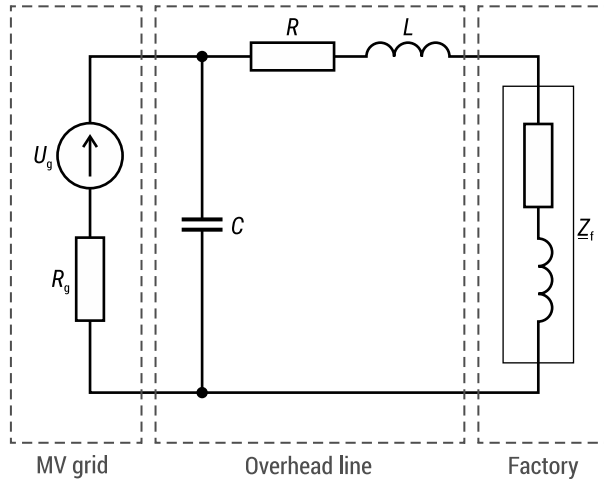FIG. 4.23. The electrical circuit representing a medium-voltage (MV) grid connected to a factory by an overhead line

Parameters:
- Grid voltage U_g = 8.3 kV = 8300 V;
- Grid resistance R_g = 1.7 Ω;
- Grid frequency f = 50 Hz;
- Line resistance R = 0.9 Ω;
- Line inductance L = 6.37 mH;
- Line capacitance C = 10 μF;
- Factory impedance $\underline{Z}_f$ = (18 + j7.2) Ω.

Answer: For U_g = 8.3 kV the active power of factory is 2.44 MW and reactive is 0.975 Mvar. While the reactive power will be ensured (0.975 Mvar > 0.8 Mvar), the active power will not be satisfied (2.44 MW < 2.8 MW).

SOLUTION

Problem 4.24

During a failure three loads were supplied by the emergency power devices, i.e., accumulators with inverters ($C + E_1$, $C + E_2$). The capacitance C of accumulators introduces capacitive reactive power into the system (Fig. 4.24); hence a choke L_c was

used to compensate this reactive power. Calculate the total active and reactive powers of sources and – on this basis – answer: if a power factor meets the requirement $\cos\varphi > 0.9$?

Note: calculate a power factor as $\cos\varphi = \dfrac{P_{E1} + P_{E2}}{S_{E1} + S_{E2}}$.
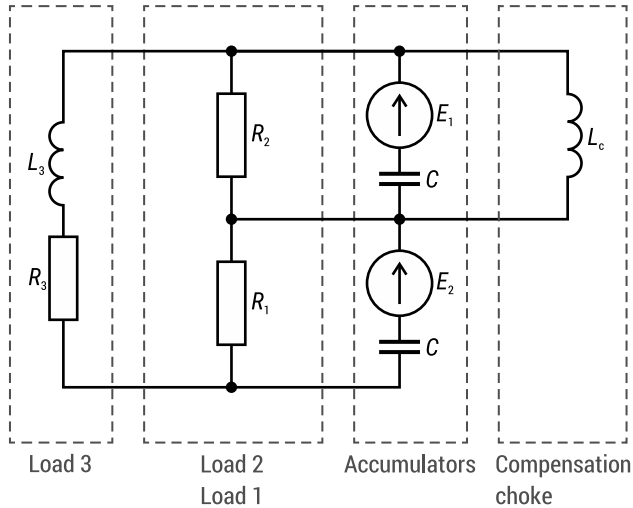
FIG. 4.24. Loads supplied by a series connection of two packs of accumulators

Parameters:
- Source 1 voltage E_1 = 115 V (rms), phase = 0°;
- Source 2 voltage E_2 = 105 V (rms), phase = 0°;
- Source frequency f = 50 Hz;
- Accumulators' capacitance C = 0.8 mF;
- Compensating inductance L_c = 240 mH;
- Load 1 resistance R_1 = 120 Ω;
- Load 2 resistance R_2 = 40 Ω;
- Load 3 resistance R_3 = 180 Ω;
- Load 3 inductance L_3 = 114.6 mH.

Answer: The power factor is $\cos\varphi$ = 0.974, so it meets the imposed requirement.

 SOLUTION

Problem 4.25

Two electromagnets (for lifting and dropping metal debris from conveyor belts) were connected in parallel to a current source. Are both electromagnets supplied by identical currents, despite using different coils? During comparison, take into account amplitude and phase of these currents.

FIG. 4.25. The considered device: a) two pairs of electromagnets connected in parallel to a current source; b) equivalent electrical circuit
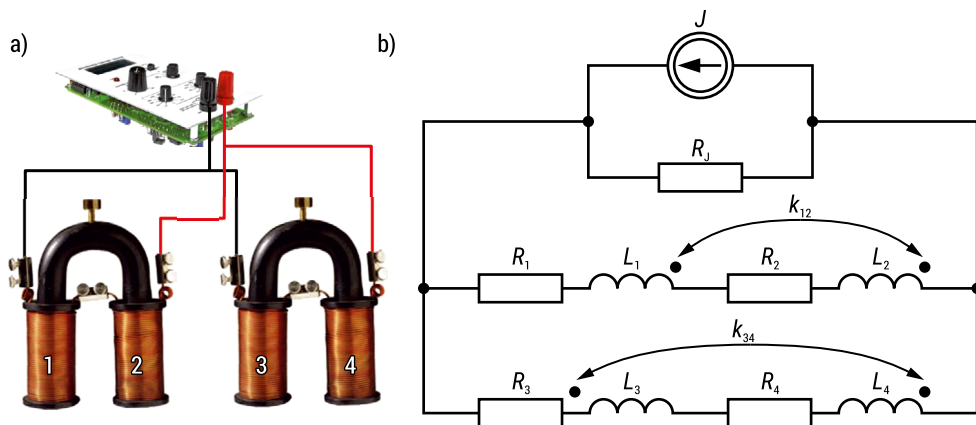
- *www.dorlingkindersley-uk.co.uk*
- *www.elektormagazine.com/magazine/elektor-201404/26220*

Parameters:
- Source current $J = 4$ A;
- Source current frequency $f = 0.5$ Hz;
- Current source resistance $R_J = 680$ Ω;
- Coil 1 resistance $R_1 = 6$ Ω;
- Coil 1 inductance $L_1 = 1.4$ H;
- Coil 2 resistance $R_2 = 1.8$ Ω;
- Coil 2 inductance $L_2 = 0.4$ H;
- Coil 3 resistance $R_3 = 1.5$ Ω;
- Coil 3 inductance $L_3 = 1.65$ H;
- Coil 4 resistance $R_4 = 7$ Ω;
- Coil 4 inductance $L_4 = 2.42$ H;
- Coupling coefficient $k_{12} = 0.6$;
- Coupling coefficient $k_{34} = 0.4$.

SOLUTION

Answer: The RMS values of currents are identical (1.99 A), yet their phases are slightly different (-2.81° and 2.13°). Hence, despite clearly diverse parameters of coils currents are nearly the same.

Problem 4.26
The part of electronic circuit is supplied by both current and voltage source. A voltage drop on the capacitor C cannot be higher than 10 V. Calculate the RMS voltage drop on capacitor and check whether the permissible value was exceeded.
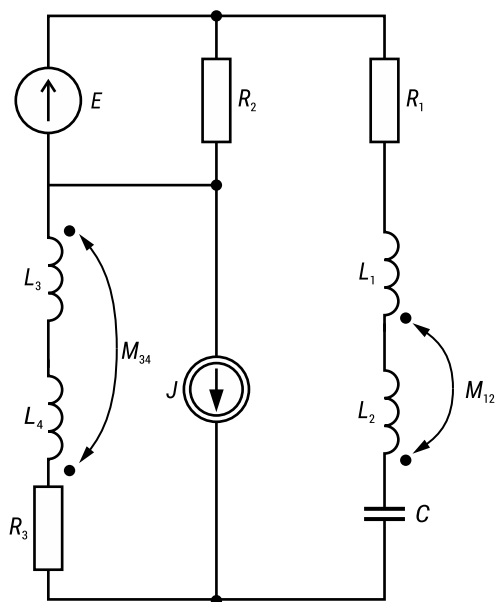
FIG. 4.26. The considered electronic circuit

Parameters:
- Source current J = 100 mA (rms), phase = 0°;
- Source voltage E = 9 V (rms), phase = 0°;
- Source frequency f = 250 kHz;
- Capacitance C = 3.6 nF;
- Resistance 1 R_1 = 150 Ω;
- Resistance 2 R_2 = 1600 Ω;
- Resistance 3 R_3 = 300 Ω;
- Inductance 1 L_1 = 35 µH;
- Inductance 2 L_2 = 20 µH;
- Inductance 3 L_3 = 47 µH;
- Inductance 4 L_4 = 120 µH;
- Mutual inductance M_{12} = 25 µH;
- Mutual inductance M_{34} = 41 µH.

SOLUTION

Answer: The voltage drop was 9.44 V hence the maximum permissible value was not exceeded.

Problem 4.27

The pressure sensor was made using two magnetically coupled planar coils (Fig. 4.27). Calculate a circuit with coupled elements and investigate how the RMS voltage U_{m2} at the terminals of meter R_m will change after applying a force and pushing coil 1. When no force was applied this voltage was $U_{m1} = 4.2$ V.
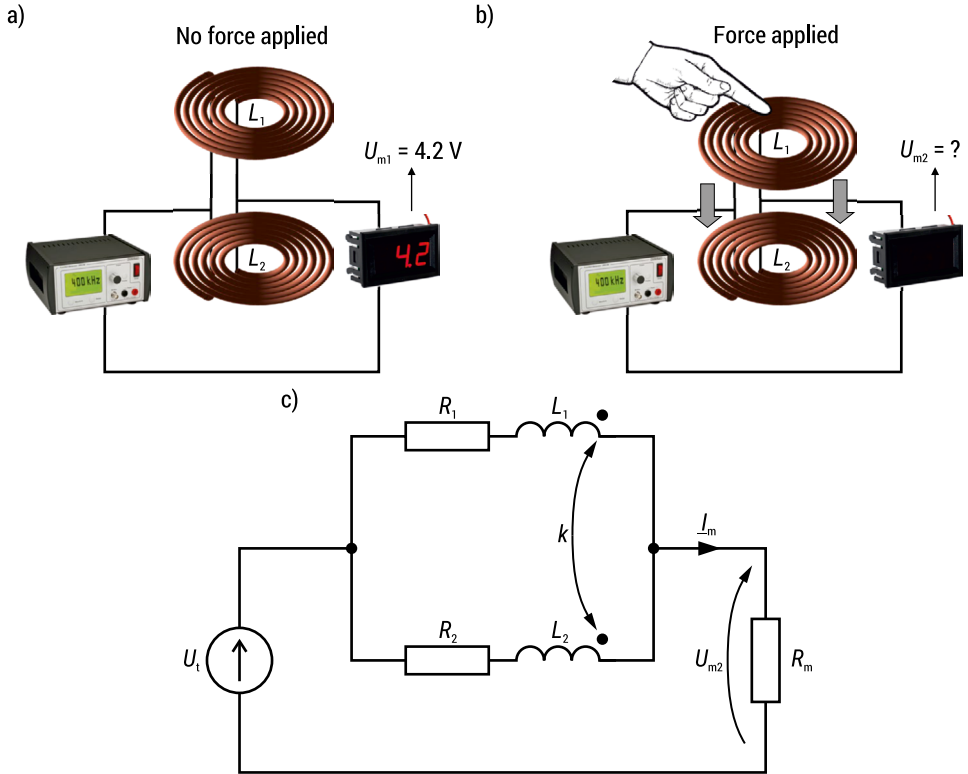
FIG. 4.27. The visualization of an inductive sensor operation where a) no force is applied at any coil, and b) coil 1 was pressed by external force, as well as c) corresponding electrical circuit of a sensing device

- *www.biolab.fr/boutique-biolab/sciences-physiques-chimiques/alimentation/g%C3%A9n%C3%A9rateur-bf-0,001hz-%C3%A0-100-khz-detail*
- *www.meesho.com/digionn-digital-voltmeter-dc-5v-30v-2-wire-056-panel-voltage-meter-red-led-battery-monitor/p/22stmu*

Parameters:
- Test voltage $U_t = 5$ V;
- Test voltage frequency $f = 400$ kHz;
- Meter resistance $R_m = 100$ Ω;
- Coil 1 resistance $R_1 = 15$ Ω;
- Coil 1 inductance $L_1 = 22.5$ µH;

- Coil 2 resistance $R_2 = 25\ \Omega$;
- Coil 2 inductance $L_2 = 50\ \mu H$;
- Coupling coefficient $k = 0.8$.

SOLUTION

Answer: After putting a force on a sensor the measured voltage will decrease by 0.44 V. Hence, it is possible to detect this force by measuring voltage on the output of the proposed measuring system.

Comment: *In engineering practice, an arrangement of magnetically coupled coils can act as a method of measuring displacement or pressure. The measurement is based on a change of electrical quantity at the circuit's output. So, if any coil is moved by a force, then coupling coefficient k will change. As a result, an equivalent impedance of circuit will change also. Hence, by measuring impedance or any electrical quantity depending on it (e.g. voltage, current) it is possible to detect the movement or pressure.*

Problem 4.28

Using identical parameters calculate a circuit, but this time with negatively coupled coils (Fig. 4.28). Investigate how the RMS voltage U_{m2} at the terminals of meter R_m will change (when no force was applied it was $U_{m1} = 4.2$ V). Finally, compare voltage change with the one from Problem 4.27 and, on this basis, decide which configuration of coils is better – with positive or negative magnetic coupling?

Parameters:
- Test voltage $U_t = 5$ V;
- Angular frequency $\omega = 8\pi \cdot 10^5$ 1/s;
- Voltage meter resistance $R_m = 100\ \Omega$;
- Coil 1 resistance $R_1 = 15\ \Omega$;
- Coil 1 inductance $L_1 = 22.5\ \mu H$;
- Coil 2 resistance $R_2 = 25\ \Omega$;
- Coil 2 inductance $L_2 = 50\ \mu H$;
- Mutual inductance $M = 26.8\ \mu H$.

SOLUTION

Answer: After putting a force on the sensor with negative magnetic coupling, the measured voltage will increase by 0.36 V. This change in voltage drop is less significant than in a system with positive magnetic coupling (Problem 4.27). Hence, the positive coupling would ensure higher sensitivity.
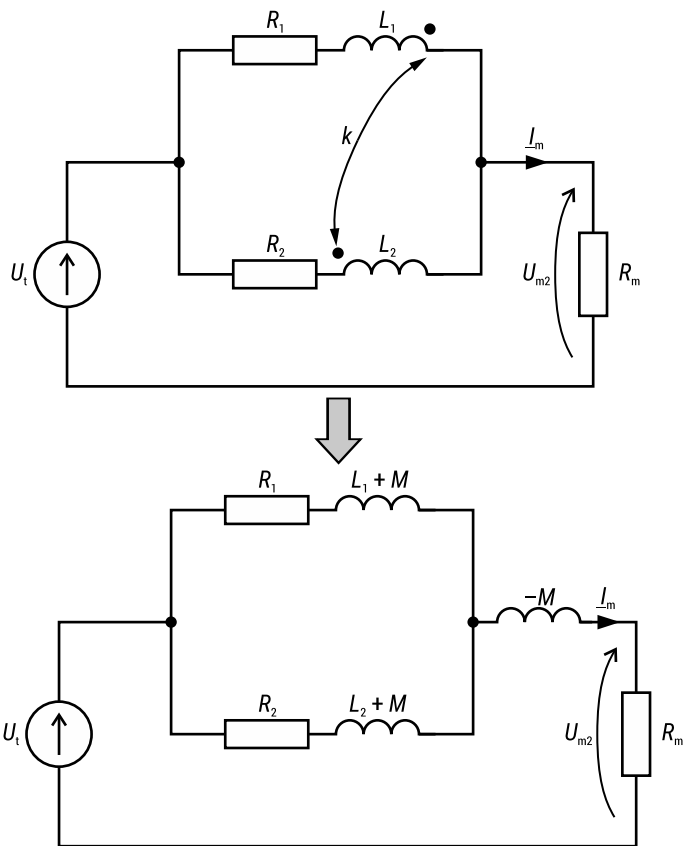
FIG. 4.28. Equivalent electrical circuits of a sensing device with negative coupling between coils

Problem 4.29

During laboratory classes a capacitor for reactive power compensation will be tested (Fig. 4.29). For this purpose it will be connected to autotransformer supplied by mains voltage. The highest allowable RMS current of the autotransformer is $I_{a,max}$ = 4 A, while for the capacitor it is $I_{c,max}$ = 1.5 A. Check if the currents be exceed these values during the testing.

Parameters:
- Mains voltage U_m = 230 V;
- Mains voltage frequency f = 50 Hz;
- Mains internal resistance R_m = 10 Ω;
- Capacitor resistance R_c = 6.8 Ω;
- Capacitor capacitance C = 70 μF;
- Winding 1 inductance L_1 = 300 mH;
- Winding 2 inductance L_2 = 55 mH;
- Coupling coefficient k = 0.69.
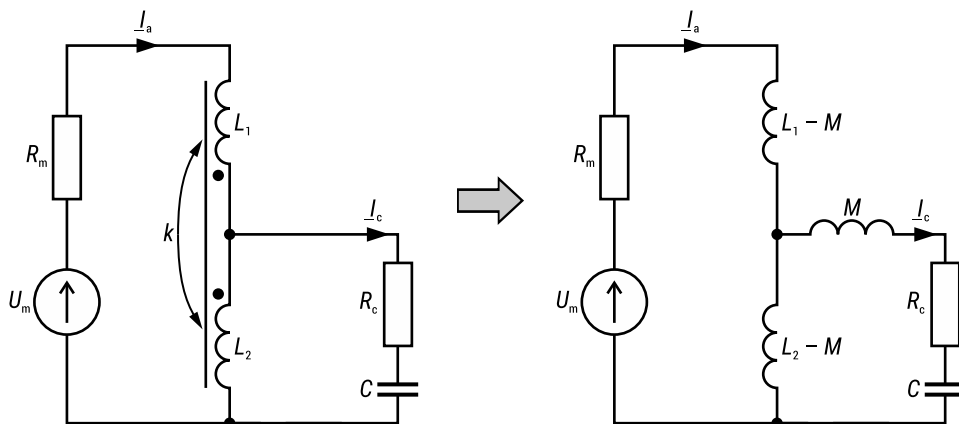
FIG. 4.29. Equivalent electrical circuit of a test stand

SOLUTION

Answer: The RMS current of the autotransformer will be I_a = 3.8 A and for the capacitor it is I_c = 1.38 A, hence the maximum allowable values will not be exceeded.

Problem 4.30

The system for inductive power transfer (Fig. 4.30), consisting of two magnetically coupled coils, was used to load a car wirelessly. Calculate currents flowing through a source and load. Then, on this basis, find the efficiency of this system.

Efficiency is defined as $\eta = P_L / P_s$, where P_L is the load's active power and P_s is the source's active power.

Parameters:
- Inverter voltage U_s = 440 V;
- Inverter internal resistance R_s = 0.5 Ω;
- Inverter frequency f = 90 kHz;
- Load equivalent resistance R_L = 3.9 Ω;
- Transmitting coil resistance R_T = 2.6 Ω;
- Transmitting coil inductance L_T = 70 µH;
- Receiving coil resistance R_R = 0.58 Ω;
- Receiving coil inductance L_R = 15 µH;
- Coupling coefficient k = 0.84.

SOLUTION

FIG. 4.30. **Wireless power transfer system utilizing two insulated but magnetically coupled coils: a) simplified diagram of inductive power transfer (IPT) device, and b) corresponding electrical circuit of the IPT**

Answer: The efficiency of the presented wireless charging system is 68.7%.

Problem 4.31

Another system for inductive power transfer (Fig. 4.31) was made using resonators (coils with capacitors) and a moving robot was supplied wirelessly. Calculate current flowing through a load and a voltage drop on current source. On this basis find the efficiency of this system.

Efficiency is defined as $\eta = P_L / P_s$, where P_L is the load's active power and P_s is the source's active power.

Parameters:
- Source current $J_s = 0.8$ A;
- Source frequency $f = 55$ kHz;
- Load impedance $\underline{Z}_L = (10.5 + j3.15)$ Ω;
- Transmitting coil resistance $R_T = 1.1$ Ω;
- Transmitting coil inductance $L_T = 125$ µH;
- Transmitting coil capacitance $C_T = 67$ nF;
- Receiving coil resistance $R_R = 0.97$ Ω;

- Receiving coil inductance L_R = 110 µH;
- Receiving coil capacitance C_R = 76 nF;
- Coupling coefficient k = 0.7.

SOLUTION

a)

b)
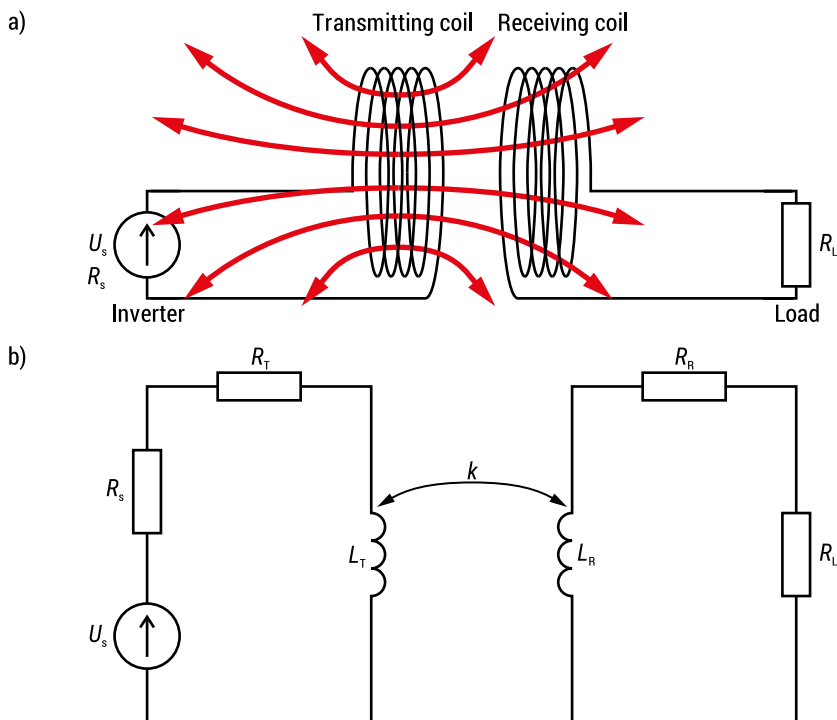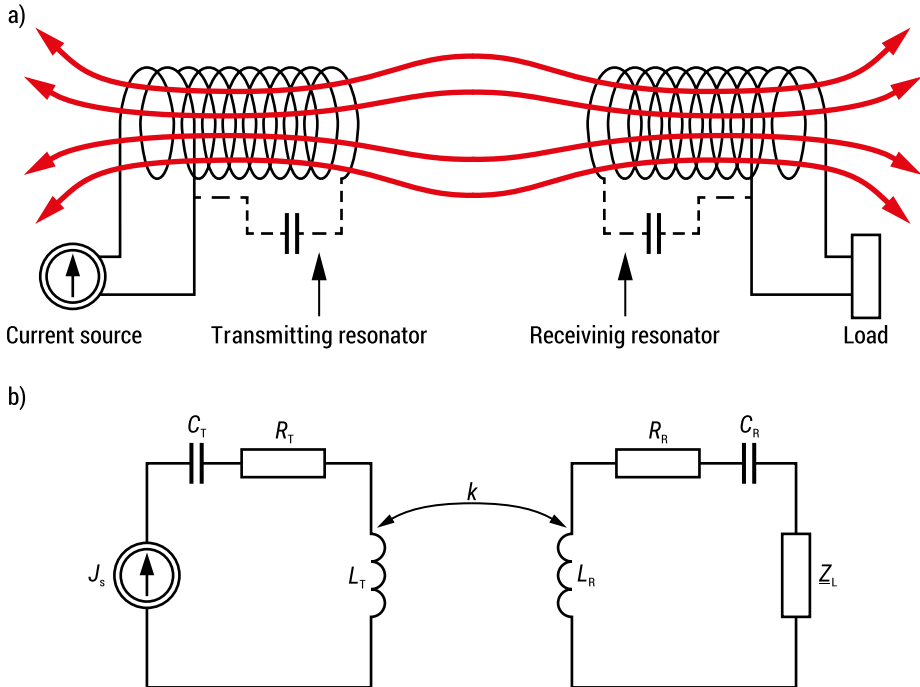
FIG. 4.31. Wireless power transfer system utilizing two magnetically coupled resonators: a) simplified diagram of inductive power transfer (IPT) device, and b) corresponding electrical circuit of this system

Answer: The efficiency of this inductive power transfer system is 90%.

Problem 4.32

The Separated Extra-Low Voltage (SELV) system was used to supply fluorescent lamps with magnetically coupled chokes (in US electricity grid). The system includes an isolation transformer where the output RMS voltage E_2 cannot exceed 50 V due to SELV requirements and nominal voltage of lamps. By calculating the circuit below check if this voltage was appropriate.

Parameters:
- Primary voltage E_1 = 110 V;
- Voltage frequency f = 60 Hz;
- Resistance of lamps R_L = 15 Ω;

- Inductance of chokes $L_C = 27.9$ mH;
- Mutual inductance of chokes $M_C = 25.1$ mH;
- Transformer's windings resistances $R_1 = R_2 = 0.3$ Ω;
- Transformer's windings inductances $L_1 = L_2 = 900$ μH;
- Transformer's mutual inductance $M_{12} = 765$ μH.

SOLUTION
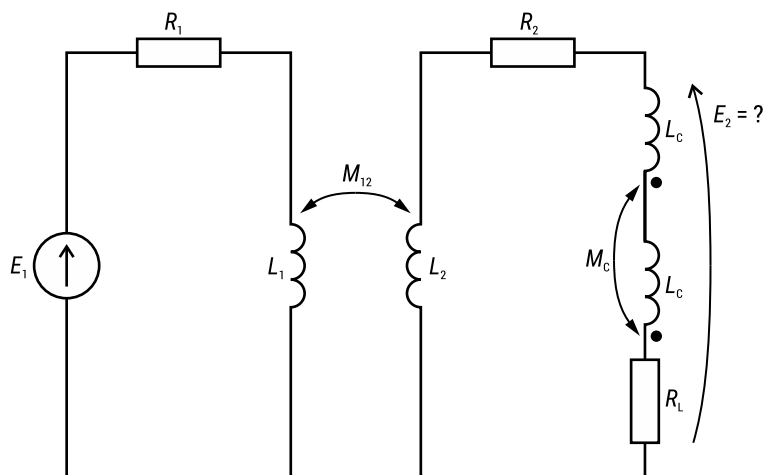
FIG. 4.32. Diagram of an isolation transformer supplying fluorescent lamps (R_L) witch chokes (L_C)

Answer: The secondary side RMS voltage was 69.5 V hence it has exceeded the maximum value of 50 V.

Problem 4.33

The heating system (R) will be supplied by a core transformer and a compensation of reactive power will be needed. Calculate the total reactive power and answer how much of it must be compensated? Assume that $\tan\varphi_r = 0.4$ is a required power factor by the electricity supplier.

FIG. 4.33. The schematic of a heating system connected to an ideal voltage source by core transformer

Parameters:
- Source voltage U_s = 400 V;
- Source frequency f = 50 Hz;
- Heating system resistance R = 0.75 Ω;
- Primary winding resistance R_1 = 4 Ω;
- Primary winding inductance L_1 = 0.35 H;
- Secondary winding resistance R_2 = 0.16 Ω;
- Secondary winding inductance L_2 = 0.014 H;
- Magnetizing resistance R_u = 500 Ω;
- Coupling coefficient k = 0.98.

SOLUTION

Answer: For a presented circuit, a compensation of ΔQ = 505 var of the reactive power will be needed.

Problem 4.34

Consider the identical situation as in Problem 4.33, but with more non-ideal elements. Take into account the source's internal resistance (R_s = 1 Ω), lower coupling coefficient (k = 0.95) and the load's inductance (L = 240 µH). Now, calculate the total power of a source and reactive power to compensate.

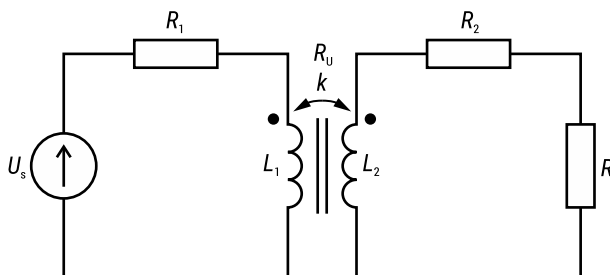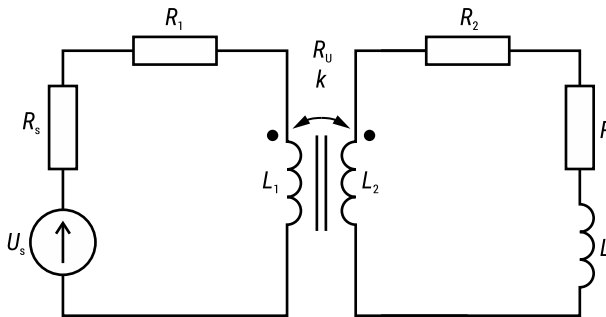FIG. 4.34. The schematic of a heating system connected to a real voltage source by core transformer

Parameters:
- Source voltage U_s = 400 V;
- Source frequency f = 50 Hz;
- Source resistance R_s = 1 Ω;
- Heating system resistance R = 0.75 Ω;
- Heating system inductance L = 0.24 mH;
- Primary winding resistance R_1 = 4 Ω;
- Primary winding inductance L_1 = 350 mH;
- Secondary winding resistance R_2 = 0.16 Ω;

- Secondary winding inductance L_2 = 14 mH;
- Magnetizing resistance R_u = 500 Ω;
- Coupling coefficient k = 0.95.

 SOLUTION

Answer: For this system, a compensation of ΔQ = 2837 var of the reactive power will be needed.

Problem 4.35

To receive DCF77 signal a pair of coupled coils (R, L) and capacitor (C) were connected to a radio (R_{out}). The receiver works properly if it operates in resonant state at frequency f = 77.5 kHz. Find the input current and, on this basis, answer if device resonates at frequency f. Make an assumption that resonance appears when an imaginary part of the input current is less than 0.01 A.

a)
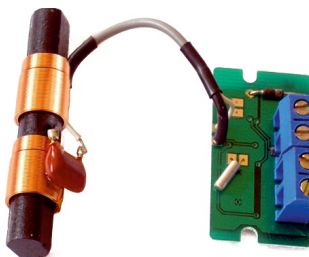
b)
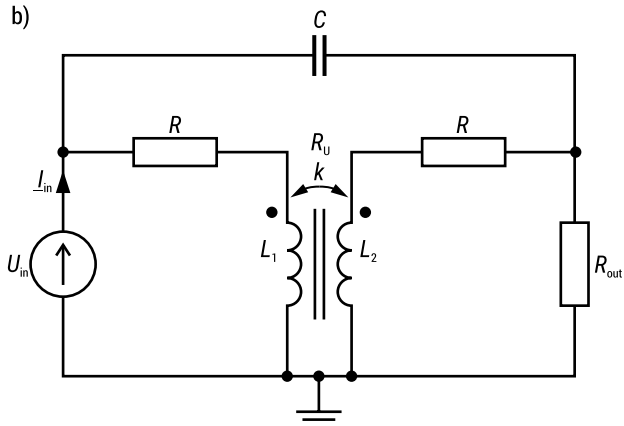
FIG. 4.35. The designed DCF77 signal receiver: a) coupled chokes with parallel capacitor, and b) equivalent electrical circuit of the device

Parameters:
- Input voltage U_{in} = 9 V;
- Input frequency f = 77.5 kHz;
- Output resistance R_{out} = 50 Ω;
- Coil's resistance R = 2.5 Ω;
- Coil's inductance L = 140 μH;
- Magnetizing resistance R_u = 140 Ω;
- Magnetizing inductance L_u = 110 μH;
- Capacitance C = 95.7 nF.

 SOLUTION

Answer: Input current is $\underline{I}_{in}$ = (0.273 + j0.002) A. Hence, the device will resonate at frequency 77.5 kHz, since Im[$\underline{I}_{in}$] = 0.002 < 0.01 A.

From sound to voltage & Intro to Thevenin » *keywords: Thevenin*

Find a website on your phone or computer that can generate a sine wave. Use a mini jack connector to connect the sound source to a breadboard. Put the source on max volume and measure the V_{rms} with a multimeter.

Your sound source probably has stereo output, but for this we only need mono. Therefore, you just need to measure between ground and one of the channels. See the figure:
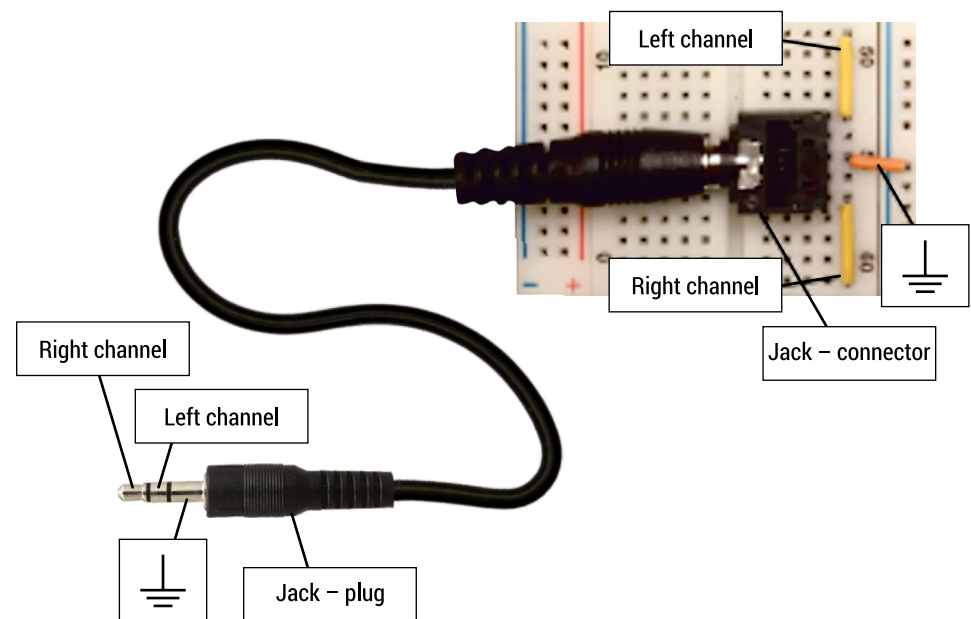
FIG. 4.36. **The view of jack connections**

Problem 4.36

Connect a load resistance of varying value between one of the output channels and ground, and then measure the voltage (V_{rms}) and current (I_{rms}). Try with different values and fill in this table (don't go below 33 Ω to make sure too high current isn't flowing).

Why do you think the voltage over the resistor gets lower as the resistance gets lower? What would have happened if we had an ideal voltage supply connected directly to a resistance?

R, Ω	V_{rms}, V	I_{rms}, mA
∞		

Problem 4.37

After filling out the table make a plot of v with respect to i. Try to find a straight line $v = ai + b$ that fits the points as good as possible (this is probably easiest to perform with pen and paper, or using e.g. Geogebra, Excel or MatLab). What values do you get for a and b, and what units do they have?

Problem 4.38

Now draw the Thevenin equivalent for your laptop. What value must the internal resistance (Thevenin resistance) have, to give the behaviour that you measured?

SOLUTIONS

CHAPTER 5. Three-Phase Circuits

To acquire or recall the necessary basic knowledge see the videos
* *https://box.biaman.pl/f/7e13c70010f64031b0fd/?dl = 1*
* *https://box.biaman.pl/f/1e9e93b186934971b130/?dl = 1*
and
* *https://youtu.be/RToXdTYQteo* (with the consent of Jim Pytel)

Problem 5.1
In a 3-phase balanced Y-Y system, the source voltage is V_{ph} = 230 V rms. The impedance per phase is $\underline{Z}_Y$ = (8+j6) Ω. If the line impedance per phase is (0.7+j0.5) Ω, check if the circuit breaker of the rated current 10 A is sufficient to protect this circuit. Also calculate the losses of active power across the line and answer if the load voltage decrease does not cross 10% (regarding the nominal value).

Notice: The circuit breaker will be mentioned as OCPD (Over Current Protection Device).

Problem 5.2
In the 3-phase balanced Y-Y system, the load voltage is V_0 = 430∠-5° V rms, the line impedance is (1.0+j1.0) Ω and the source voltage is V_{ph} = 440 V rms. Find the load impedance, its power and the value of supplying current.

Problem 5.3
In a balanced Y-Δ system, the source voltage is V_{ph} = 230°V rms. The line and load impedances are (5+j0.5) Ω and (22+j11) Ω respectively. What OCPD (give the rated current) should be installed to protect the supplying lines? What is the value of total load power?

Problem 5.4
A balanced Δ-connected motor having an impedance of (110+j60) Ω per phase is fed from a line having an impedance of (4+j5) Ω per phase. The line-to-line voltage at the sending end of the line is 1200 V rms. What should be the OCPD (give the rated current) at this line? And what is the cost of active energy used by this motor during one week (8h per day, 5 days, 0.6 €/kWh).

Problem 5.5
A balanced 3-phase distribution line is used to supply two balanced Y-loads that are connected in parallel:
- Load 1: 42 kVA at 0.80*pf*, lagging.
- Load 2: 18 kVA at 0.65*pf*, lagging.

The line voltage at the load is 700 V rms at 50 Hz. Check if the OCPD of rated current 32 A will be adequate to protect the supplying line and if the combined power factor (*pf*) of the load does not require additional compensation system (tan φ_{lim} = 0.4).

Problem 5.6
A balanced 3-phase distribution line is used to supply four balanced Y-loads that are connected in parallel:
- Load 1: 1.5 kVA at 0.80*pf*, lagging
- Load 2: 2.5 kVA at 0.92*pf*, leading.
- Load 3: 1.0 kW and 0.4 kvar
- Load 4: 2.0 kW at unity pf.

The line voltage at the load is 440 V rms and the line is protected by the OCPD of rated current 16 A. Find the line current in the distribution line. What is the total active energy used by an equivalent Δ-load during one (8h) day?

Problem 5.7
The wire impedance per phase in each transmission line of a 3-phase system is (5+j3) Ω. The lines feed three balanced loads that are connected in parallel.
- Load 1: 37 kVA at 0.72 *pf*, lagging
- Load 2: 64 kVA at 0.83 *pf*, leading;
- Load 3: 55 kW and 29 kvar.

The line voltage at the load end is 3300 V rms. What should be the rated current of line OCPD and the line voltage at the source end.

Problem 5.8
In a balanced Y – Δ three phase system, the source line voltage is 330 V rms. The impedance per phase of delta load is (30+j27) Ω. The line impedance is (2+j3) Ω. Find the annual cost of energy losses in this line (300 days, 10 hours/day, 0.6 €/kWh) .

Problem 5.9
In a 3-phase balanced system (f = 50 Hz) the Y-source line voltage is 400 V rms and the load consumes 16 kW of power at pf of 0.72 lagging. Three capacitors, each with the capacitance of 200 μF, are connected in parallel with the original load in a Y configuration. By how much will the relative reactive energy bill decrease after connection of the capacitors?

Problem 5.10

A balanced 3-phase distribution line is used to supply three loads. L_1 is Y-load with 1.5 kVA at 0.80pf, lagging, L_2 is a welding machine with impedance (10+j100) Ω connected between R-T phases, L_3 is a one-phase resistance heater at power of 2 kW, connected with S-phase. The line voltage is 400 V rms at 50 Hz. Determine if the 3-phase OCPD of rated current 16 A assures the continuous work of these loads.

SOLUTIONS

Problem 5.11

A 3-phase electric heater with nominal power P_n = 6000 W and nominal voltage U_n = 230 V is used to heat the water in a tank (Fig. 5.11). What should be the nominal current of the overcurrent circuit breaker protecting the installation when circuit breakers with the following currents I_n = 6 A; 10 A; 16 A; 20 A; 25 A are available?

FIG. 5.11. The electric circuit for 3-phase heater

Answer: I_n = 10 A

SOLUTION

Problem 5.12

A three-phase electric heater with a nominal power of P_n = 9000 W consists of three identical heating coils. The heater is supplied from a power network with a line voltage of U_n = 400 V. The heater coils are Δ-connected (Fig. 5.12). What should be the nominal current of the installation overcurrent circuit breaker protecting the heater when circuit breakers with the following currents I_n = 6 A; 10 A; 16 A; 20 A; 25 A are available?
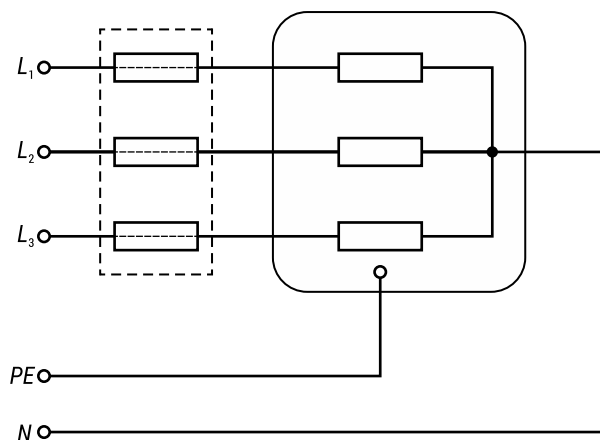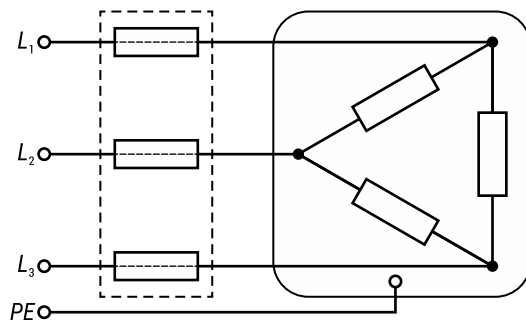
FIG. 5.12. The electric circuit for 3-phase Δ-connected heater

Answer: $I_n = 16$ A

SOLUTION

Problem 5.13

A 3-phase electric heater with nominal power $P_n = 15$ kW is used to heat the warehouse. The heater is supplied from a power network with a phase voltage of $U_n = 230$ V. The heating elements are Δ-connected. The heater operates at full power for 12 hours a day. A photovoltaic installation has been installed in close proximity to the warehouse, which has caused the phase voltage increase from $U_n = 230$ V to $U_n' = 242$ V. Calculate the percentage increase in warehouse heating costs in one day due to the increase in phase voltage.

SOLUTION

Problem 5.14

The three-phase electric heater consists of three heating coils Y-connected (Fig. 5.14a).

FIG. 5.14. The electric circuit for 3-phase Y-connected heater before (a) and after repair (b)

The nominal power of the heater is P_n = 3x1 kW with the nominal phase voltage U_n = 230 V. The heater has been damaged. After its repair the length of the first coil decreased by 5% and the length of the second coil by 10% (Fig. 5.14b). Calculate the power of the repaired heater and the current that will flow in the neutral line.

 SOLUTION

Answer: P' = 3164 W, I_N = 0.42 A

Problem 5.15
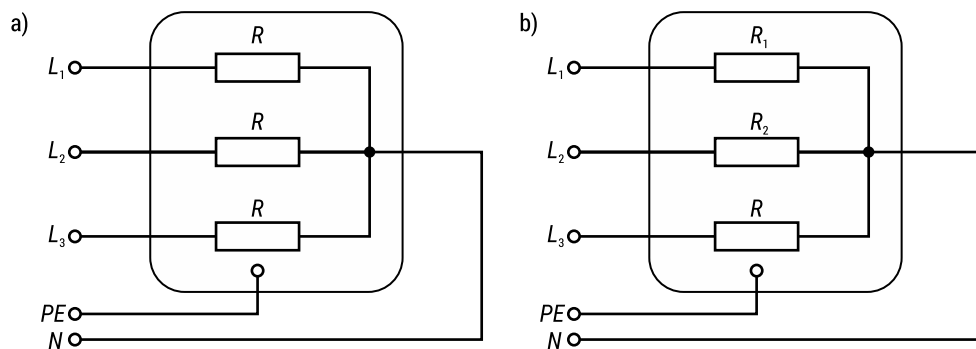
A three-phase electric heater for the boiler consists of three resistance elements. Each element has two terminals located on the heater casing. The nominal power of the heater is P_n = 3×2000 W, and the phase voltage U_n = 230 V. The heating elements should be Δ-connected (Fig. 5.15a). An inexperienced electrician has connected the heating elements in a Y-system (Fig. 5.15b). Calculate the power of the heater working in such a system.
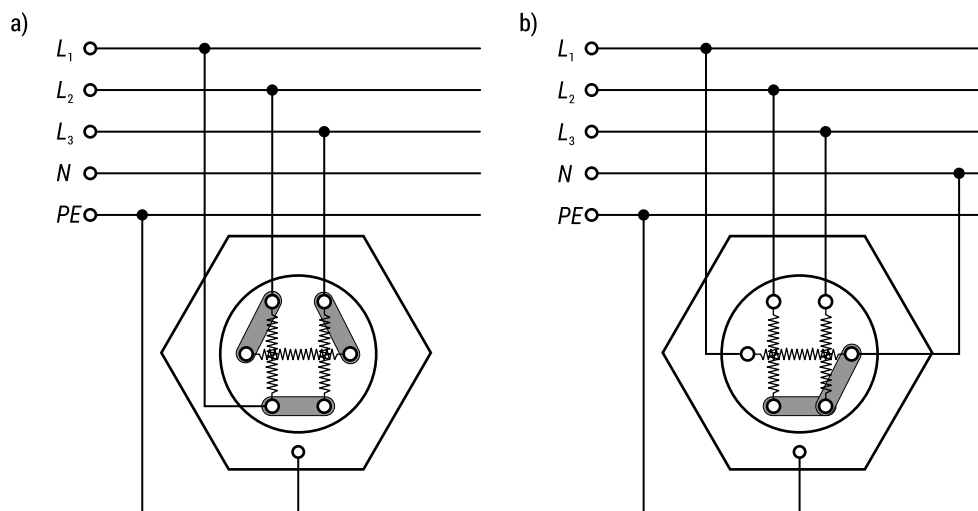
FIG. 5.15. **The electric circuit for 3-phase heater connected in Δ (a) and in Y (b)**

 SOLUTION

Answer: P = 2000 W

Problem 5.16

An aluminum plate of the vulcanisation press is heated using 24 electric tubular heaters. The way the heaters are connected to the three-phase power network with the phase voltage U_n = 230 V is shown below (Fig. 5.16). The resistance of one heating element is R_1 = 13 Ω. During the operation of the press, some of elements were damaged.

It turned out that the cost of purchasing the same elements is very high and economically unjustified. However, access was gained to other, much cheaper elements. It was therefore decided to replace all the heating elements. The new elements can operate at 230 V or 400 V and the resistance of one element is $R_2 = 58 \ \Omega$. Determine how many new elements should be used and how they should be connected to the power network so that the power of the system before and after the replacement of the elements is similar.
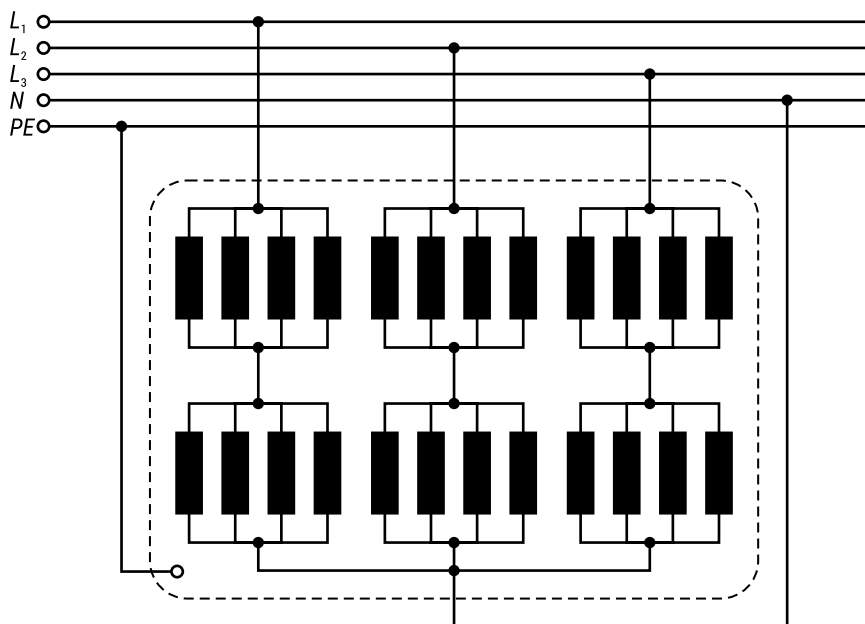
FIG. 5.16. The electric circuit for connecting heaters at vulcanization press

SOLUTION

Problem 5.17

In a room there is a three-phase electric motor supplied from a power network with the line voltage of $U_l = 400$ V. The motor windings are Δ-connected. The motor parameters are: nominal power $P_{n1} = 15$ kW, motor efficiency $\eta_{n1} = 0.895$, power factor $\cos \varphi_{n1} = 0.9$ (lagging). It was decided to install a second three-phase motor in the same room with the parameters: $P_{n2} = 7.5$ kW, $\eta_{n2} = 0.88$, $\cos j_{n2} = 0.88$. The room is supplied with power by a copper wire 5×6 mm^2. Check whether, after installing the second motor, the cable cross-section will be sufficient due to its long-term current carrying capacity. In the case of 5-core copper conductors laid in the room, the long-term current carrying capacities are: 1.5 mm^2 – 17 A, 2.5 mm^2 – 24 A, 4 mm^2 – 31 A, 6 mm^2 – 40 A, 10 mm^2 – 55 A.

Answer: The cross section of the cable supplying the motors will be too small.

Problem 5.18

A pump supplying the system with water for fire extinguishing is driven by a three-phase electric motor. The motor is supplied from the power network with a phase voltage of U_n = 230 V. Reducing the supply voltage by more than 5% may cause a problem with starting the motor. Calculate what the minimum nominal cross-sectional area of the copper wires supplying the motor should be, so that the voltage drop across them does not cause the 5% limit to be exceeded. The cable supplying the motor has a length l = 50 m, the nominal current of the motor is I_n = 14.8 A, and the inrush current I_r = 5.5·I_n. The nominal cross-sections of the cables: S_n = 1.5; 2.5; 4; 6; 10; 16; 25; 35; 50; 70 mm^2. During a fire there is a danger that the cable may heat up to a temperature of 100°C. Check that at this temperature the voltage drop on the supply line will not be too high.

Answer: The minimum cable cross-section should be S' = 10 mm^2.

Problem 5.19

A three-phase power network with a phase voltage of U_p = 230 V supplies three single-phase loads. A cordless kettle with a nominal power P_1 = 1900 W and $\cos \varphi_1$ = 1 is connected to the first phase, a microwave oven with P_2 = 1300 W and $\cos \varphi_2$ = 0.92 is connected to the second phase, and a vacuum cleaner (operating with minimum power P_3 = 380 W) with $\cos \varphi_3$ = 0.57 is connected to the third phase. Calculate the currents flowing in each phase. What fuses should protect each phase?

Answer: First phase: I_1 = 8.26 A, fuse 10 A; second phase: I_2 = 6.14 A, fuse 10 A; third phase: I_3 = 2.90 A, fuse 6 A (one triple OCPD is usually used for the network and if it is divided into separate sections, each section is protected by a single OCPD)

Problem 5.20

There are four buildings in the industrial plant powered by a three-phase power network. The powers consumed by buildings are as follows:
- building no. 1: S_1 = 54 kVA, $\cos \varphi_1$ = 0.92 (lagging)
- building no. 2: S_2 = 32 kVA, $\cos \varphi_2$ = 0.87 (leading)

- building no. 3: $P_3 = 18$ kW, $\cos\varphi_3 = 1$
- building no. 4: $P_4 = 15$ kW, $Q_4 = 10$ kvar

There is an additional charge for the reactive power consumed by the plant depending on the amount of power and its character (inductive or capacitive). Calculate the reactive power consumed by the whole plant and its character.

SOLUTION

Answer: $Q_T = 15.4$ kvar, inductive character.

CHAPTER 6. Electromagnetic Fields

To acquire or recall the necessary basic knowledge about the electromotive force read this file

ELECTROMOTIVE FORCE

In order to better understand the phenomenon presented in problems 6.1-6.4, it is recommended to watch the following videos:
* *https://www.youtube.com/watch?v = d73e3QiMdSU*
* *https://www.youtube.com/watch?v = uW-M8eBHq9U*
* *https://www.birmingham.ac.uk/teachers/study-resources/stem/physics/electromotive-force.aspx*
* *https://www.youtube.com/watch?v = shJAV59NS6k*
* *https://www.youtube.com/watch?v = gwSbQBo55QI*
* *https://www.youtube.com/watch?v = rqq11Id_x5E*
* *https://www.youtube.com/watch?v = tC6E9J925pY*
* *https://www.youtube.com/watch?v = uESCEU5Thqs*

Problem 6.1
Calculate the greatest electromotive force induced between the wing tips of a flying airplane. The plane is flying at a speed of v = 900 km/h. The wingspan of the aircraft is 30 m. The vertical component of the Earth's magnetic field induction is B = **30** μT.

The video will help you understand the phenomenon: *https://www.youtube.com/watch?v = PqfxMWBe4PI*

SOLUTION

Answer: e = 0.225V

Problem 6.2
A rod of l = 10 cm long is placed in a constant magnetic field, the induction of which is $\boldsymbol{B}$ = 2 T. The rod is perpendicular to the magnetic field line and rotates around an axis paralel to the field line and passing through one of the rod ends. The rod makes one revolution per second. Calculate the electromotive force (EMF).
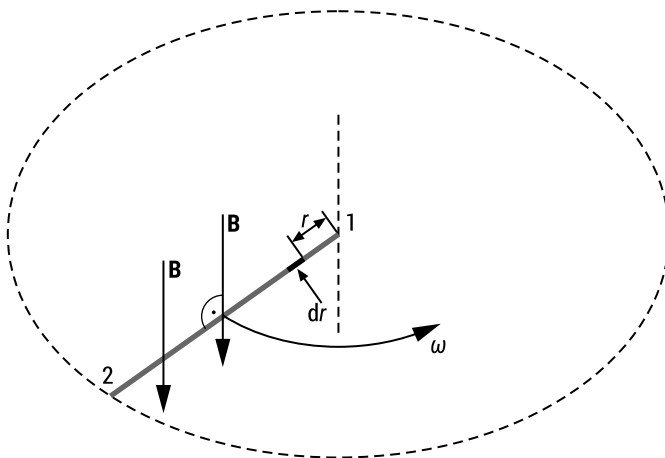
FIG. 6.2. A rod in a magnetic field rotating around one end (1)

The video will help you understand the phenomenon: *https://www.youtube.com/watch?v = p4LDwOJTjBc*

 SOLUTION

Answer: $e = 63$ mV

Problem 6.3

A rectangular frame with dimensions $a \times b$ rotates around the Z axis, which is the axis of symmetry and is perpendicular to the field line. The frame rotates at angular velocity ω in a uniform magnetic field of induction B. The frame is composed of n turns. Calculate the electromotive force induced in the frame taking into account the movement of the conductors in the magnetic field.

FIG. 6.3. A frame rotating in a magnetic field

These videos can be helpful in understanding the phenomenon:

- *https://www.youtube.com/watch?v = milGuJZBFlU*
- *https://www.doubtnut.com/question-answer-physics/a-rectangular-frame-of-wire-abcd-has-dimensions-1mxx-05m-and-a-total-resistance-of-30omega-it-is-pul-14156541*

SOLUTION

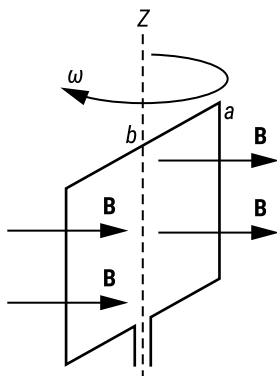Necessary to reading
- *electrical properties*

- *Maxwell's equations*

Problem 6.4

Calculate the density quotient between the two currents (k_J): the displacement current and the conduction current in a cable made of copper with electrical parameters: $\varepsilon_r = 1$, $\sigma = 5.6 \cdot 10^7$ S · m^{-1}. The electric field in copper changes according to the formula

$$E = E_0\left(x, y, z\right)\sin\omega t$$

a) Calculate k_J at frequency f = **50 Hz.**
b) Calculate k_J at frequency f = **1 MHz.**
c) Also, calculate the frequency at which both currents will be equal.
d) Also, calculate the frequency at which both currents will be equal, assuming that the environment is ground with electrical parameters ($\varepsilon_r = 2$, $\sigma = 1 \cdot 10^{-2}$ S · m^{-1}).

SOLUTION

Problem 6.5

What condition must so that in a dielectric environment with constant values of electric permittivity and magnetic permeability (ε and μ), the magnetic field intensity is determined by the following relation

$$H = H_0\cos\left(\omega t + \beta z\right) \cdot 1_y$$

The goal is to determine the electric field that is along with this magnetic field.

Also the Poynting vector should be determined for the electromagnetic field, which was described by the relations from the task. Calculate its mean value for the period and interpret the result.

SOLUTION

Necessary to read
- *Poynting's vector*

Continuation of the task

The Poynting vector for the electromagnetic field is equal to

$$P = E \cdot H$$

Due to the fact that in the task we have already defined the vector of the electric field:

$$E = \frac{\beta}{\omega\varepsilon} H_0 \cos(\omega t + \beta z) \cdot 1_x$$

and the vector of the magnetic field:

$$H = H_0 \cos(\omega t + \beta z) \cdot 1_y$$

then we can substitute both equations to the above dependence

$$P = E \cdot H = \frac{\beta}{\omega\varepsilon} \cdot H_0 \cos(\omega t + \beta z) \cdot 1_x \cdot H_0 \cos(\omega t + \beta z) \cdot 1_y =$$

$$= \frac{\beta}{\omega\varepsilon} \cdot H_0 \cos(\omega t + \beta z) \cdot H_0 \cos(\omega t + \beta z) \cdot 1_x \cdot 1_y \, P =$$

$$= \frac{\beta}{\omega\varepsilon} \cdot H_0^2 \cos^2(\omega t + \beta z) \cdot 1_z =$$

$$= \frac{\beta \cdot H_0^2}{\omega\varepsilon} \cdot \frac{1 + \cos(2\omega t + 2\beta z)\beta \cdot H_0^2}{2} \cdot 1_z$$

The average value for the period is

$$T = \frac{1}{f}$$

$$\bar{P} = \frac{1}{T} \int_0^T P dt = \frac{1}{T} \cdot \frac{\beta \cdot H_0^2}{\omega\varepsilon} \cdot 1_z \int_0^T \frac{1 + \cos(2\omega t + 2\beta z)}{2} \, dt = \frac{\beta \cdot H_0^2}{2\omega\varepsilon} \cdot 1_z$$

The obtained equation is independent of z and taking into account the Assumption obtained earlier:

$$\beta = \pm\sqrt{\omega^2 \varepsilon\mu}$$

Based on the above considerations, it can be seen that the analysed electromagnetic wave carries power in the positive direction of the *Oz* axis.

Problem 6.6

Jacob's Ladder

How to Make a Jacob's Ladder Travelling Electric Arc

Jacob's Ladder is a very interesting and phenomenal experiment. It is an electrical device made of two metal rods arranged in the shape of the letter "V". The rods form electrodes that do not touch each other. There is a large potential difference between them, created by a transformer of a dozen or several dozen kilovolts. The difference is so big that the air breaks through and an electric arc is created. Under the influence of the current flow, the air in the arc heats up and, thanks to convection, rises upwards. As a result, the entire arc rises creating an effective phenomenon. As the arc travels up, it will increase in length and eventually break. A new arc appears at the bottom of the ladder.

FIG. 6.6. Jacob's Ladder – rising arc (*www.youtube.com/watch?v = 6hhaaSSCU5k*)

This phenomenon is used, i.a., in on the roof of the tram (two V-shaped wires). This is to "get rid" of excess energy, so that important components of the railway traction and the trams themselves will not be damaged.

Explanation of the phenomenon

It is known from theory that the electromagnetic field is a physical field in which forces of an electromagnetic nature act on a physical object carrying an electric charge. The electromagnetic field is a system of two fields: an electric field and a magnetic field. These fields are related to each other through the relations described by Maxwell's equations:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

An electric arc is a continuous electrical discharge, usually in a gas at or above atmospheric pressure. A characteristic feature is a very low internal electrical resistance. The discharge takes place between two electrodes made of electrically conductive material or other elements fulfilling their role. The electrons "jump" from one rod to another.

Problem 6.7

Microwave oven and experiment
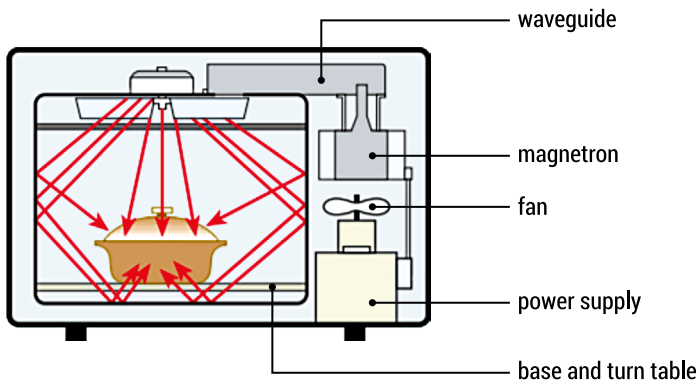1) The principle of operation of a microwave oven (Fig. 6.7a)

FIG. 6.7A. The basic scheme of the construction of a microwave oven (*https://electrical-engineering-pics.blogspot.com/2014/11/schematic-diagram-of-typical-microwave.html*)

Magnetron
The magnetron is a self-excited resonance-based oscillating device that converts DC input energy into high-frequency electricity. The energy conversion takes place in a specially shaped anode chamber placed in a strong magnetic field. The electrons emitted from the cathode travel towards the anode but are deflected by the magnetic field on a spiral trajectory. The interaction between the electron beam and the anode produces an electromagnetic wave. The electronic flow in the generating magnetron has the shape of a "spoke" and rotates in the interaction space. The energy output is coaxial and takes place via a loop contained in one of the resonators. Emission and secondary emission processes take place in the magnetron interaction space, electronic

clots are formed, and energy is transferred to the high-frequency field. In a microwave oven, the most important thing is to generate a frequency wave 2.45 GHz through the magnetron.

- Link to the video: *https://www.youtube.com/watch?v = bUsS5KUMLvw*

The microwave oven does not heat the food directly inside it. It is the polar particles (mainly water, but also sugar and fat) contained in food that are set to vibrate in rotation, they begin to vibrate quickly, absorbing the waves. The vibrations are so intense that they pass to the particles of the heated food through friction. As a result, the heated food becomes warmer with each passing second.

This phenomenon uses electromagnetic waves (more specifically high-frequency radio waves) generated by a magnetron. Generally speaking, this device draws energy from the socket, converts it into high-power waves and releases it through a wave conductor into the chamber containing the heated food.

Waves reflect on the walls and microwave door, but not on food, as they must penetrate inside the microwave oven. The higher the power, the more energy the magnetron takes, resulting in faster vibrations of molecules and faster heating of food.

This is how a microwave oven converts electricity into heat in the form of electromagnetic radiation. The generated heat concerns only heated food, because the air inside the chamber practically does not change its temperature. For this reason, the food will not have a brown or crispy crust, which is possible with an oven.

Standing waves
Sometimes the waves don't travel. Fig. 6.7b shows an experiment that can be performed at home.

FIG. 6.7B. **Standing waves formed on the surface of milk contained in a glass bowl** (*https://cnx.org/contents/TqqPA4io@5.93:_jcAMgg6@5/16-6-Fale-stoj%C4%85ce-i-rezonans*)

Put a bowl with milk on the housing of an ordinary fan. The vibrations that the fan creates will create spherical standing waves in the milk. Such waves result from the superposition of two or more traveling waves that are identical but move in opposite directions. These waves overlap and disturbance is added. If they have

the same amplitudes and lengths, we can observe alternating effects of interference amplification and suppression. The resultant wave looks as if it was standing still, hence the name standing wave.

Standing waves in microwave oven

The three-dimensional wave equation with boundary conditions gives the standing waves in the three dimensions x, y, and z which are consistent with the equation

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2} + \frac{1}{\lambda_z^2}$$

The wavelengths λ_x, λ_y and λ_z are determined by the linear relationships L_x, L_y and L_z.

$$L_x = l \cdot \frac{\lambda_x}{2}, \qquad L_y = m \cdot \frac{\lambda_y}{2}, \qquad L_z = n \cdot \frac{\lambda_z}{2}$$

where *l*, *m*, *n* are natural numbers.

The solutions for the data (*l*, *m*, *n*) are marked as resonator modes. In commercial microwave ovens, the dimensions L_x, L_y and L_z disperse considerably. The equations given above allow a number of resonator frequencies, which are close to the emitted magnetron frequency.

The quality factor Q is a measure of the energy loss and gives the ratio of the energy stored in the resonator to the energy losses per cycle. Energy losses in microwave ovens are caused by four factors. Firstly, microwaves can escape through the oven housing, although manufacturers assure that this factor is negligible. Secondly, losses arise due to the absorption of waves in the walls. Thirdly, because the food is absorbing the waves, which of course is the desired mechanism. There is also a chance that the microwaves are coupled back to the magnetron especially when the microwave is used empty. This should be avoided to ensure a long service life for the magnetron.

Typically, in a volume of about $15 \cdot 10^3$ cm^3 and a penetration depth of $\delta \approx 1$ μm, Q values of 10^4 can be found, which for a frequency of 2.45 GHz leads to a frequency band of only 0.25 MHz – that is, resonance. They are very narrow compared to the frequency band of the magnetron.

Why does the heated thing need to be on a rotating plate?

A standing wave is characterized by constantly taking the value of zero (knots) in some places and the maximum value in other places. If the knots are on the food being heated, the food will not be heated in these places. In order to ensure that food is evenly heated inside the microwave oven, it is necessary to insert a rotating plate.

Absorption of a radiation beam in matter (Lambert's law)

All matter more or less absorbs radiation. In a microwave oven, the material must absorb energy well. Quantitatively, such absorption is described by Lambert's law

$$I(x) = I_0 \cdot e^{-\alpha x}$$

The absorption coefficient α often depends on the wavelength. Exactly for the relaxation frequency of water, the coefficient will be large and effective absorption will be only in the outer layer of the food (about 0.5 cm). A high value of the imaginary part of electric permeability indicates that the substance will absorb energy from the electromagnetic wave well in this frequency range.

2) The course of the task 1.

Check in a microwave oven how much and at what time both liquids will be heated: filtered water and oil. Each of them with a capacity of 150 ml was placed in glass vessels. The microwave power was set to 800 W.

It is worth adding that the water used was previously filtered in a special jug, so it did not have many impurities.

	Time: 60 s	Time: 120 s	Time: 180 s
Filtered water	60°C	85°C	100°C
Oil	30°C	42°C	50°C

Conclusions: It can be observed that water ($\varepsilon_r' = 80$) heats up faster than oil ($\varepsilon_r' = 2.7$). Water overheating (*) was observed. The superheated water is above its boiling point, but it does not boil. Water cannot change its state of aggregation, the energy contained in it accumulates until it reaches a critical moment. It boils immediately and explodes on any contact with dirt. The water has no way to boil, because the boiling has to start somewhere. It can be dangerous, because it is enough to put one grain of sand or a small crystal of salt into the vessel and the boiling process will start so rapidly that the water will practically explode!

(*) Superheating is a physical reaction that is sometimes referred to as boiling retardation or boiling delay. The phenomenon occurs when water is heated to a temperature above the boiling point without actually boiling. The boiling process, however, requires appropriate conditions – incl. the possibility of volatilization of the gas formed from the liquid, moreover, it does not occur simultaneously in the entire volume of the liquid and requires initiation and time (*https://microwavemealprep.com/ can-water-be-superheated-in-microwave/)*

An overheated liquid can be obtained through:
- very slow heating of a pure liquid in a vessel with smooth walls, with no vibrations and mechanical shocks (e.g. water up to 120°C under normal pressure);

- very fast heating (so fast that the substance "does not have time" to evaporate) – in this way you can obtain an overheated liquid, even when it is heavily contaminated;
- heating the liquid to high pressure, where its boiling point is higher, and then gradually, carefully reducing the pressure;
- heating the liquid with microwaves, which does not boil it – however, adding a sugar cube to such heated tea initiates its immediate boiling.

An overheated liquid is in a state of delicate thermodynamic equilibrium, the so-called metastable state. Such a balance is easily damaged by a sufficiently large disturbance. In order to bring a liquid to a gaseous state, it is enough to initiate the boiling process in any way. This could be, for example:
- throwing the so-called boiling seeds into the liquid, which can be any contamination – other liquid, gas, solid and even elementary particles;
- shaking a vessel with a liquid;
- effect of sound on the liquid;
- treating the liquid with an electrical impulse.

As a result of such initiation, the liquid begins to boil in its entire volume, causing the liquid content to be ejected outside the tank due to a much larger volume of vapours released.

3) The course of the task 2.

Checking the heating of the chocolate bar without using the turntable in a microwave oven.

As a result, points where the chocolate had melted were observed after the chocolate bar was removed. The distance between these points was measured, which made it possible to determine the distance between the peaks of the standing wave, which is half the wavelength:

$$\frac{\lambda}{2} = 6 \text{ cm}$$

To verify the correctness of the result, the data should be substituted for the formula for the speed of light, i.e. the speed of propagation of an electromagnetic wave in a vacuum or material medium

$$\lambda = c \cdot T = c \cdot \frac{1}{f}$$

After conversion, you can designate *c:*

$$c = \lambda \cdot f = 12 \cdot 10^{-2} m \cdot 2.45 \cdot 10^{9} s^{-1} \approx 3 \cdot 10^{8} m \cdot s^{-1}$$

Problem 6.8

A n-turn solenoid of length l and a circular section of radius a is given. The length of the solenoid $l \gg a$, and the magnetic permeability of the area inside the solenoid is μ. Let current $i(t)$ flow through the solenoid.
1) Find the magnetic field intensity inside the solenoid.
2) Find the intensity of the electric field inside the solenoid.
3) Determine the magnetic field energy inside the solenoid and its time derivative.
4) What about the energy of the induced electric field (after all, it occurs inside the solenoid)?

Problem 6.9

Prove that the displacement current flowing in the capacitor, e.g. during its charging, is equal to the conduction current flowing in the conductors connecting the capacitor with the rest of the circuit.

The purpose of this task is to use and validate the law of flow in this task. According to this law, the currents should be equal. The task is to prove this assumption and to check whether in such considerations it is possible to omit one of the currents.

CHAPTER 7. Digital Electronics

Each set of exercises is marked with some keywords with topics that are covered. To find the desired topics use "Ctrl +F" and search for one of the following keywords: Transistors, Energy, Superposition, Digital Design, Flip-flops, DAC/ADC, MUX, Counter.

Why Digital electronics? We are all surrounded by digital systems, our phones, laptops and headphones are just some examples. In the day to day we view these as black boxes, there is some magic that happens inside these devices, but that is way beyond us. Learning about the basics of digital electronics is learning about the pieces that make up this magic (a bit like the molecular biology of computers). When you finish all the exercises it will hopefully be more understandable how we can go from high and low voltages to a circuit that does some "thinking".

Why do the exercises? While you are diving into the world of digital electronics these exercises will give you the opportunity to use what you have learned and get an idea of how your new knowledge can be connected to real world applications.

iPhone with NMOS-logic » *keywords: Transistors, Energy*

The A7 processor, which was used in the iPhone 5S, was one of the world's first processors with one billion (10^9) transistors. This was designed using CMOS technology (as most other processors today).

Problem 7.1
Imagine that NMOS-technology was used instead, and that the circuit consisted of $N = 10^9$ inverters of the type shown in the figure below. Assume that at any given time, 50% of these inverters are in the state $A = 1$, $Q = 1$. The resistor has the value $R = 1$ kΩ, the supply has the value $V_{DD} = 3.5$ V. Estimate how much effect this circuit will consume (we assume the transistors work as perfect switches that are either on or off).

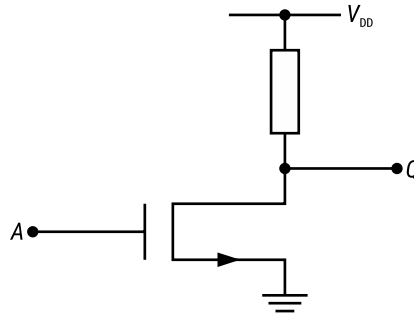FIG. 7.1. A single inverter in NMOS-technology

SOLUTION

Traffic Light » *keywords: Digital Design*

In this exercise you will design a digital circuit that can be used to control a traffic light. The control signals s_1 and s_2 control the logical system that outputs d_1 (for red), d_2 (for yellow), and d_3 (for green).
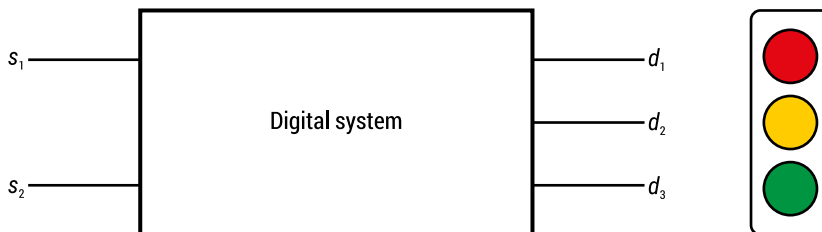

FIG. 7.1A. The „black" box for control the traffic light

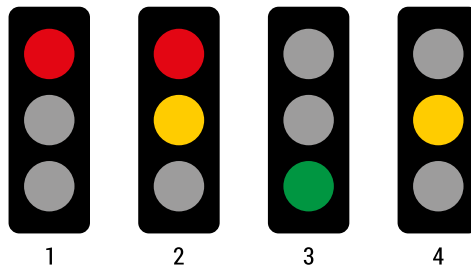We want the system to have the behaviour that is shown in the figure below.

FIG. 7.1B. Desired behaviour of the traffic light

Problem 7.2

Finish the truth table, write the logical expressions, and realize the *Digital system* that is hidden in the box, using logical ports. **Hint:** The situations 1, 2, 3, 4 are the inputs of the system.

s_1	s_2	d_1	d_2	d_3
0	0			
0	1			
1	0			
1	1			

SOLUTION

Comparator » *keywords: Digital Design*

Comparing values is an essential part of any computer, and one of the fundamental operations that the computer uses. You might also notice that you need this every time you write an "if statement" in any code language. We will now look at one possibile way to realize a simple comparator that has two one-bit inputs A and B, and three outputs f_1, f_2 and f_3 that indicate whether we have the case A = B, A > B, or A < B respectively.

Problem 7.3
Realize a circuit (with logical ports) that fulfils the function A = B.

Problem 7.4
Realize a circuit (with logical ports) that fulfils the function A > B.

Problem 7.5
Realize a circuit (with logical ports) that fulfils the function A < B.

Problem 7.6
Draw the total system (with logical ports), and simulate it to check if it works as intended (here is a possible circuit simulator that can be used: *http://www.falstad. com/circuit/circuitjs.html*).

SOLUTIONS

Let the computer do the math for you! (Subtraction)
» *keywords: Digital Design*

Computers are all about automating tasks, and some of the first computers were used to automate calculations (calculators). In fact, the word computer used to be the job title of a person that worked with doing a lot of calculations by hand! (If you are interested: *Computer (occupation) – Wikipedia*)

You are tired of subtracting numbers by hand and want to make a logical circuit that does it for you. (We will only consider problems that result in non-negative numbers in these exercieses).

Problem 7.7
Start by solving this equation:

	1	1	0
–	0	1	1
=			

Problem 7.8
Why do you think a half subtractor needs two outputs? Fill out the truth table for the system.

Hint: Perform the subtraction A-B for each of the cases and sett *Borrow* to 1 if you need to borrow.

A	B	Answer	Borrow
0	0		
0	1		
1	0		
1	1		

Problem 7.9
Write down and simplify the logical expressions for each of the outputs (using Boolean algebra).

Problem 7.10
Realize the half subtractor using logical ports.

As you might have noticed in problem 7.7, we sometimes need to consider the previous number that was subtracted when we subtract bigger numbers. That is why we now want to make a full-subtractor that takes in three numbers and spits out two, as shown in the figure below.
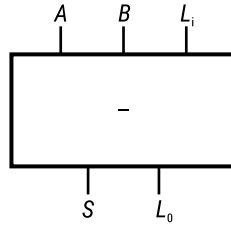
FIG. 7.10. A substractor with its inputs and outputs

Problem 7.11

Set up the truth table for this system. **Hint:** L_1 is what has been borrowd (removed) from you, L_0 is what you borrow (from somewhere else).

Problem 7.12

Design the interior of the figure show above using logical ports. Then simulate the circuit to check if it works as expected (here is a possible circuit simulator that can be used: *http://www.falstad.com/circuit/circuitjs.html*).

Problem 7.13 Challenge

Try to simplify the circuit so that it only uses two XOR-ports, two NOT-ports, one AND-port, and one OR-port. Then simulate the circuit to check if it works as expected.

Problem 7.14

Utilize the box shown in the figure above to design a system that can subtract two three-bit numbers A and B, such that *A − B = Answer*. (Remember that we do not consider negative results for now).

SOLUTIONS

Let the computer do the math for you! (Addition)
» *keywords: Digital Design*

The adder is an immensely important component in most all digital electronics, and you will get to see some implementations of it in some of the later exercises. By utilizing components you know of you will now get the chance to design a circuit that uses high and low voltages to calculate the sum of two numbers! This is the first obvious step towards making computers that can solve problems for us.

Adding of small numbers

We will now look into addition of small binary numbers; these are numbers that can be represented with one bit. We call them *a*, and *b*.

Problem 7.15
Fill out the table below with all possible combinations of *a* and *b*. Then perform the operation $a + b$ and write *s* as the sum of the operation, and *c* as the carry of the operation (do this row by row). Notice that + in this case means *addition* and not logical *OR*.

a	b	s	c
0	0	0	0

We will now try to make a system that performs the addition that is described in the table.

Problem 7.16
Write the Boolean equations for each of the outputs of the system (*s* and *c*).

Problem 7.17
Now use logical gates to draw the circuit that performs this operation. You can use all types of logical gates that you know of. You have now successfully designed a digital circuit that performs simple addition!

Problem 7.18 Extra Challenge
Design the same system using just NAND-gates.

Adding several numbers
We will now try to design a system that adds three one-bit numbers.

Problem 7.19
We call the three input numbers *a*, *b*, and c_i. Fill out the table below as before.

a	b	c_i	s	c_0

Now, look at this as a digital system that performs addition on three inputs a, b, and c_i, and gives out the result as two outputs s and c_0. Here s is the least significant bit and c_0 is the most significant in the two-bit result. The system is shown below.
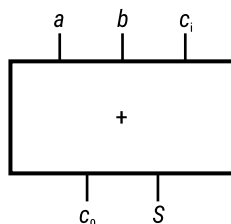
FIG. 7.19. A system that adds three one-bit numbers

Problem 7.20
Write the Boolean equation for c_0 and for s.

Problem 7.21
Design the system using logical ports. You can use all types of logical gates that you know of.

Adding even bigger numbers

Problem 7.22 Challenge
Use the box you just designed (that can add three numbers) as a building block to design a bigger system that can add two three-bit numbers.

   Hint 1: Clarify to yourself how you would add two three-bit numbers by hand.
   Hint 2: No seats left? *Carry* on!

SOLUTIONS

Digital Inverter » *keywords: Digital Design*

An important operation in all modern processors (e.g. the CPU in a PC) is to take a positive binary number x and transform it into $y = -x$. The figure below illustrates this in the case where x is a two-bit number.
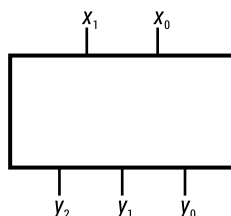
FIG. 7.23. An inverter of two-bit number

The operations can be realized with a system that has two inputs x_0 and x_1, that are the least and most significant digit in x respectively, and three outputs for the three digits in y. Since x is assumed to be positive we haven't included the sign bit, but we need the sign bit for the number y (that is negative); y is given in *two's compliment* from.

Problem 7.23
Set up the truth table for the system.

Problem 7.24
Set up the Boolean equations for the system.

Problem 7.25
Realize the system with logical ports.

SOLUTIONS

Digital to Analog Converter (DAC) » *keywords: Digital Design, superposition, Thevenin*

You have probably seen a digital radio before. This radio recieves a digital signal (1's and 0's), and then it gives you sound as its output. The sound comes from a changing voltage that is used to move air with the speaker (thus making sound). But how does the radio turn the digital signal into an analog signal?

You guessed it, it's the digital-to-analog-converter (DAC). In the next series of exercises you will get the opportunity to design a DAC yourself. This is not only used in radios but is an essential component in almost all modern digital electronic devices that interact with the world. Can you think of any examples?

- *Necessary Reading material*
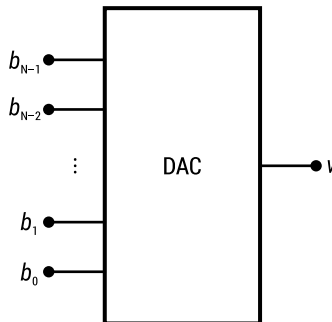
FIG. 7.26. A *N*-bit DAC system with *N* inputs

Both ADC and DAC circuits can be realized in many different ways. In this exercise we will look at one possible DAC. Before looking at a possibile implementation lets specify the functionality. A N-bit DAC is a system with N inputs b_0, b_1,..., b_{N-1} and an output v as shown in the figure below.

For $N = 16$ we have the possibility to represent $2^N = 65536$ different values on the input and the same amount of voltage values on the output. But what voltage should for example the bit value $m = 65536$ be equal to? One thing is certain: we don't want to have $v = 65535$ V on the output! What we do in practice is to ensure that we have an output value that is proportional to the input, thus:

$$v = V_0 \sum_{n=0}^{N-1} b_n 2^n$$

where V_0 is a constant voltage that takes an appropriate value.

Problem 7.26

Justify (show) why the biggest attainable value on the output is

$$v_{max} = \left(2^N - 1\right)V_0.$$

Hint: A finite geometric series can be useful here.

Problem 7.27

If we look at an example where $N = 16$ bit and v takes values between 0 and 5 V, what should V_0 then be?

As mentioned, the behaviour discussed can be realized in several different ways. A known metod is shown for $N = 3$ bit in the figure below.
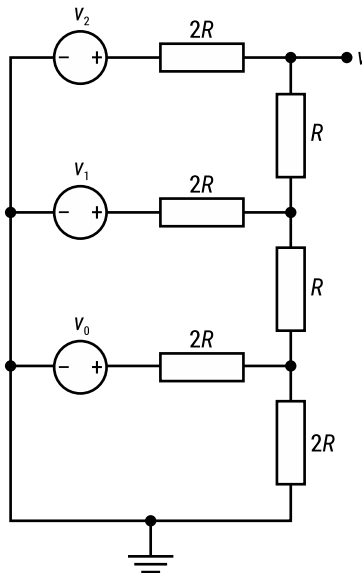
FIG. 7.27. **The electric circuit for realizing the 3-bit DAC**

What happened to the input b_n? We have simplified the figure by including these in the voltage sources v_0, v_1 and v_2. These have the value 0 when the corresponding bit is equal to zero and the constant value V when the bit is 1, thus

$$v_n = b_n V, \text{ for } n = 0, 1, 2.$$

Problem 7.28
Assume that you have made the circuit in the figure above. How could you test it? Discuss with a partner if possible.

A theoretical derivation
In the next problems we will show step by step that the circuit above (in the figure) works as we want ($v = V_0 \sum_{n=0}^{N-1} b_n 2^n$). We start with the superposition principle and define v_0 as the value v takes if both $v_1 = 0$ and $v_2 = 0$. We also define v_1 as the value v takes if both $v_0 = 0$ and $v_2 = 0$. And define v_2 as the value v takes if both $v_0 = 0$ and $v_1 = 0$.

The superposition principle then gives us the output of the DAC as

$$v = v_0' + v_1' + v_2'$$

To continue we need to find values for v_0', v_1' and v_2'. This can be done by drawing good figures and simplifying them.

Problem 7.29
Draw the circuit with v_1 and v_2 turned off (suppressed) and show that:

$$v_0' = b_0 \frac{V}{8}.$$

Problem 7.30
Draw the circuit with v_0 and v_2 suppressed and show that:

$$v_1' = b_1 \frac{V}{4}.$$

Problem 7.31
Draw the circuit with v_0 and v_1 suppressed and show that:

$$v_2' = b_2 \frac{V}{2}.$$

Problem 7.32
Show that:

$$v = \frac{V}{8} \sum_{n=0}^{2} b_n 2^n.$$

Problem 7.33
Realise and test the model based on the circuit above. Use resistors of several tens of kilo-ohms, so that the DA-converter doesn't draw too much current. To get the most accurate result use the same resistor for R and $2R$.

 To test the system you must realise the voltages $b_n V$. This can be accomplished in several different ways. The easiest might be to use cables that you either connect to +5 V or ground, based on the value you want (1 or 0).

SOLUTIONS

Digital Counter » *keywords: Digital Design, Flip-flops, MUX*

Counters have numerous essential applications in digital systems, for example retrieving the next address in memory, keeping the time and detecting problems (*watch dog timer*). You might also be able to come up with several creative ways to utilize the digital timer. As you might imagine, counting is all about doing things in the right order, you can reflect on what this entails in a digital system before starting the exercises.

* *Necessary Reading material:*

Problem 7.34 Don't get stuck! (Take a few minutes and then check the solution)
Why is the figure below not a good way of implementing the EN signal on a D-flip flop?
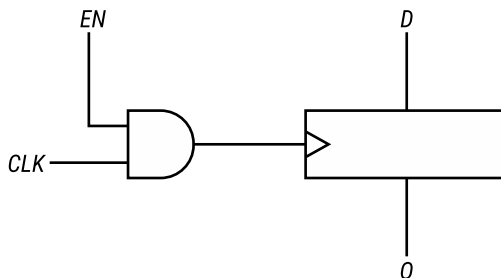
FIG. 7.34. **D-flip flop with enable signal that controls the clock**

Problem 7.35

Use the figure shown below as a starting point. Design the contents of the block above the flip flop so that the EN signal controls whether the flip flop keeps its value or gets the value of D on the next clock period.
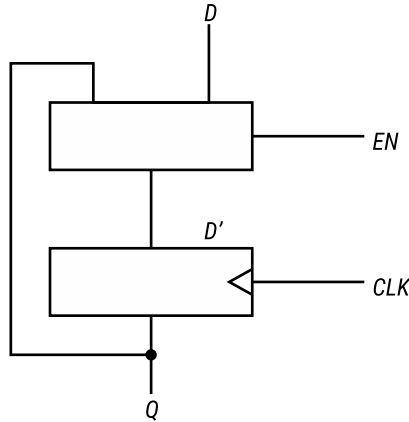
FIG. 7.35. **D-flip flop with enable signal that controls the input**

When we turn on the power of a digital system, we don't have any control over which state the system is in (Why?). We therefore want a to add a RESET-signal. The RESET should set the stored values to zero.

Problem 7.36

Modify the design you have on problem 7.35 so that you also have a RESET-signal.

Counter

A counter is a synchronous system with N outputs and a clock signal as input. The output forms a N-bit binary word x. If we denote the value at clock period n as $x[n]$, then the behaviour is given as:

$$x[n+1] = \begin{cases} x[n]+1, & \text{if } x[n] < 2^N - 1 \\ 0, & else \end{cases}$$

This means that we want x to go from 0 to $2^N - 1$ and then start at 0 again. Thus, a four-bit counter will be as shown in the figure below.
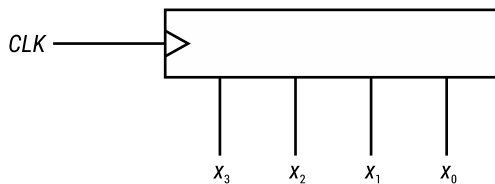
FIG. 7.36. **A possible realization of four-bit counter**

The four outputs form a four-bit binary number x that takes values between 0 and 15. This is shown in the table below:

x	x_3	x_2	x_1	x_0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

The clock signal changes values between 0 and 1 periodically. Every time CLK changes value from 0 to 1 the output x should change value, as shown in the figure below.
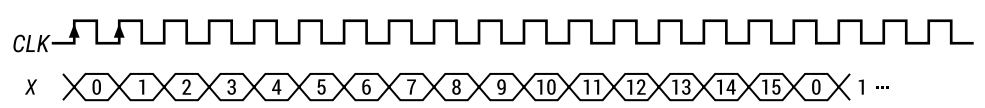
FIG. 7.36. Desired output x with respect to the clock

Problem 7.37
Design a four-bit counter that behaves as described by the equation above. Available components: all types of ports, registers, adders, multiplier, transistors (you don't need all of these, think about what you need). **Hint:** Keep it simple, try to think of which operations normal counting entails and use digital components to perform the operations.

Problem 7.38

Simulate the circuit with a circuit simulator and check if it behaves as expected (here is a possible circuit simulator: *http://www.falstad.com/circuit/circuitjs.html*). **Hint:** you can find useful components if you press on the *draw* button.

SOLUTIONS

Digital Dice

» *keywords: Digital electronics, Flip-flop, Energy, Counter, MUX*

A possible design of a digital dice is shown in the figure below. Here we have diodes that indicate the result of the "throw" of the dice. If we for example get 3, the diodes D_1, D_7 and D_6 will light up. The idea is to perform a "throw" by holding a button and then releasing it. Repeating this action several times should then give seemingly random results.
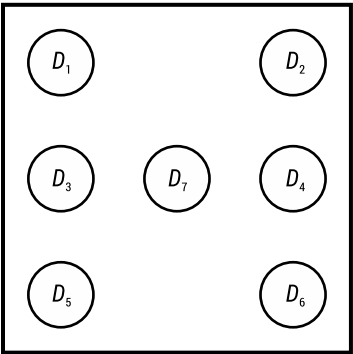
FIG. 7.39A. Digital dice

Problem 7.39

The table below shows which diodes should be on at what numbers. Finish the table (if you don't remember how the different numbers look on a dice this can be found on google).

Result	D_1	D_2	D_3	D_4	D_5	D_6	D_7
1							
2							
3	ON	OFF	OFF	OFF	OFF	ON	ON
4							
5							
6							

The whole system can be drawn as shown in the figure below.
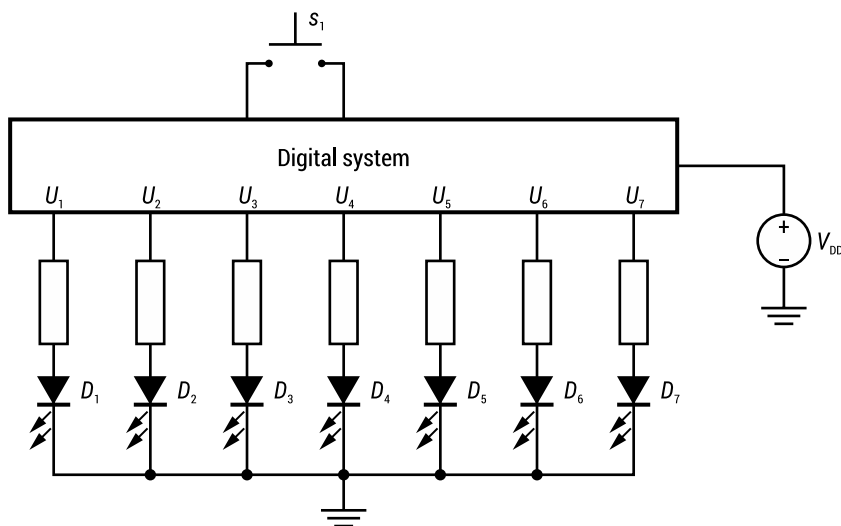
FIG. 7.39B. **High level figure of the dice circuit**

Here we have V_{DD} which can be any type of source (battery, USB-connection, or something else). We also have 7 diodes $D_1 - D_7$ with one resistor connected to each of them. The button S_1 is what we press to perform a "throw" of the dice.

The rest of the system is hidden inside a black box. This is what you will have to design the content of. The box has 7 outputs $U_1 - U_7$, each of these supply voltage to one diode. Thus if U_n has a high voltage ($\approx V_{DD}$) then the diode D_n will light up.

Problem 7.40
If a diode is on it will have a voltage $V_D = 2$ V and a current $I_D = 20$ mA. Assume that each output U_n can be modelled as an ideal voltage source of $V_{DD} = 5$ V (when D_n is on) and 0 V when D_n is off. How big does the series resistance need to be?

Problem 7.41
How much power is consumed by the diodes when the dice rolls 1, and how much is consumed when it rolls 6?

More details
Now we look closer at the digital system we previously looked at. The figure below shows a possible design for the inside of the box.
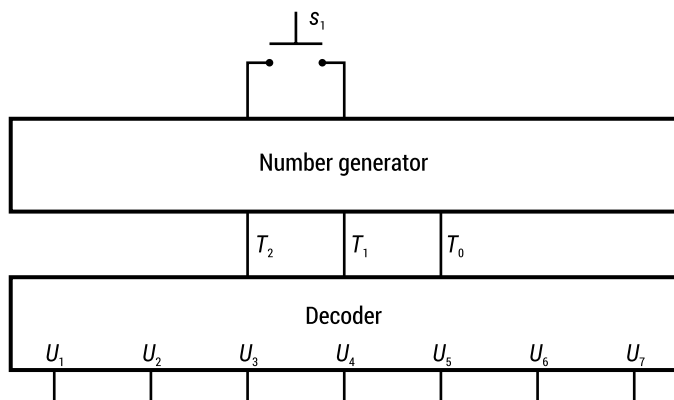
FIG. 7.41. **Possible design of the *"Digital System"* shown in fig. 7.39b**

The digital system is split into two parts: *Number generator* and *Decoder*. Every time the user of the dice makes a throw by pressing the button the number generator gives a (seemingly) random number between 1 and 6, represented by the three bits T_2, T_1, and T_0. These bits are the input to the decoder, which makes sure that the right diodes light up depending on the number that is given.

Problem 7.42
Finish the table for the *Decoder* that is show below.

Result of the throw	T_2	T_1	T_0	U_1	U_2	U_3	U_4	U_5	U_6	U_7
1										
2										
3	0	1	1	1	0	0	0	0	1	1
4										
5										
6										

Problem 7.43
Write the logical expressions for the outputs U_1, U_2, ..., U_7 based on the input signals T_2, T_1, and T_0. **NB:** Here there are several right answers. Some are more elegant while others more systematic and rigorous. Several methods and answers are all correct, as long as the expression matches with the truth table.

Problem 7.44
Realize the *Decoder* using logical ports.
   Hint: There are several correct answers here, just as in the previous exercise.

Random Numbers?

Our goal is to produce a number that seems to be random. The job of the *Number generator* is to give us a bit combination $T_2T_1T_0$ every time the button S_1 is pressed, that doesn't seem to have any relation to the last number that was given. One possible way to accomplish this is to make the value depend on how long the button is pressed. If our *Number generator* counts very fast from 1 to 6 and immediately starts at 1 again, and does this several thousand times per second, then the number given by how long the button is pressed, will seem random. The counting starts when the button is pressed and stops when it is released.

To realize the system, we will go step by step forward. To start with we want a counter that can count as shown:

$$1 - 2 - 3 - 4 - 5 - 6 - 1 - 2 - 3 - 4 - 5 - 6 - 1 - 2 \cdots$$

without stopping. This is called a cyclical counter. One possible design of such a system is shown in the figure below.
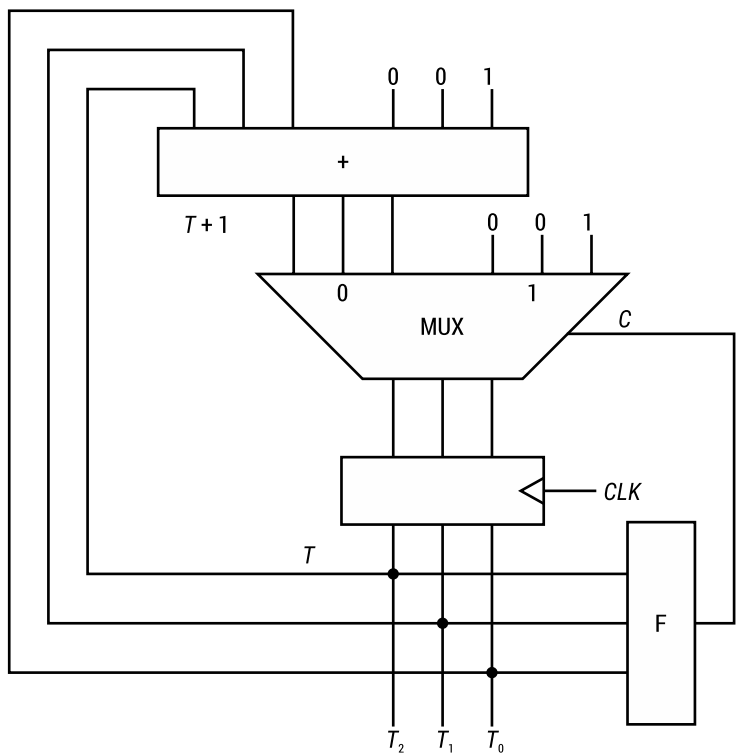
FIG. 7.45A. **Possible design of the** *"Number Generator"* **shown in fig. 7.41**

Here we have a three-bit register, that holds the bits $T_2T_1T_0$, that represent numbers between 1 and 6. We now define this three-bit number as T. In the figure, we have also drawn a three-bit adder. This always adds 1 to T, so we get $T + 1$ on the output.

But if the value of T already is 6 then we don't want 7 as the next output, but rather 1. The multiplexer (MUX) has the job of choosing between $T + 1$ and 1 depending on the current value of T. The MUX is controlled by the control signal C. When $C = 0$ the next value will be $T + 1$, and when $C = 1$ the next value is 1. The system also has a clock signal CLK that feeds into the system.

Problem 7.45

Design the contents of the box F, so that C has the right value.

Hint: The input signals are T_2, T_1, and T_0. Start by setting up the truth table for the system, then find the logical expression for C, then realize the system by drawing it with logical ports. You have then realized the function $C = F(T_2, T_1, T_0)$.

We now have a cyclical counter. We go further and expand our design, so we also have an activation button to perform throws. We want the counter to start when the button is pressed down and to stop when it is released. Then the last value that the counter had should be on the output of the system, and remain there until the next press of the button, so that the diodes can light as previously described. This is possible to realize in several different ways. One possibile concept is shown in the figure below.
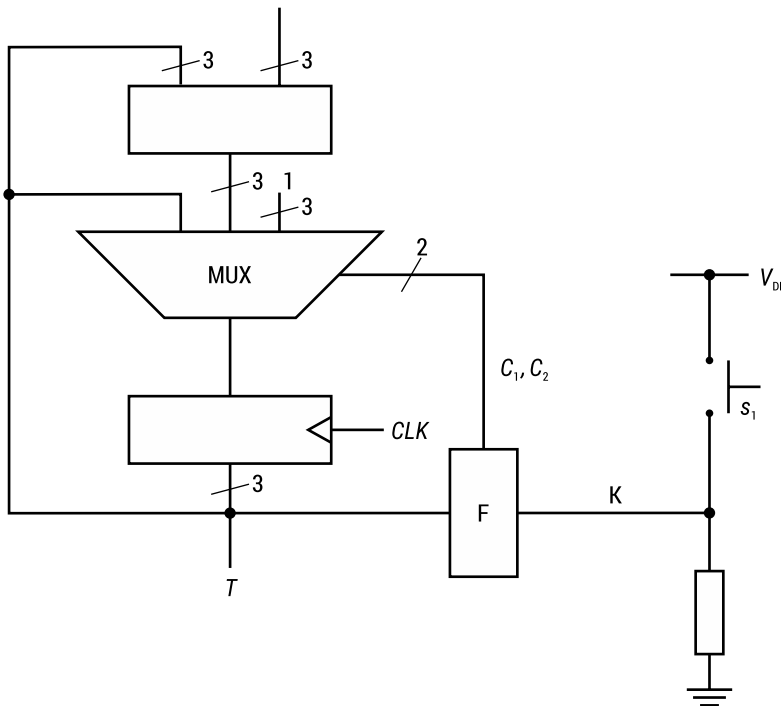
FIG. 7.45B. *"Number generator"* with activation button

(To avoid a too complicated figure with a lot of wires we use one wire with a slash through to indicate several wires. The number beside the slash line indicates how many wires there actually are.)

The main idea is to use a MUX with two control signals C_1 and C_2, so that it can choose between three alternatives. In addition to the alternatives $T + 1$ and 1 that we previously had, we now also have the current value T. When this value is chosen the contents of the register will be frozen. The behaviour of the MUX is shown in the table below.

C_1	C_2	Meaning	Next output value
0	0	Continue counting	$T + 1$
0	1	Restart at 1	1
1	0	Freeze the value	T
1	1	Not in use	-

The control signals are produced by the combinational function $F(T_2, T_1, T_0, K)$. In addition to the signals $T_2 T_1 T_0$ we now also have the signal K that indicates if there is a "throw" or not.

The default state of the system is frozen. When the button is pushed the counter starts, and keeps counting until the button is released. Then the system goes back to the frozen state but now with a new value on the output (that is the last number that was counted to).

A truth table for C_1 and C_2 is shown in the table below.

K	T_2	T_1	T_0	C_1	C_2
0	0	0	0	X	X
0	0	0	1		
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1	X	X
1	0	0	0	X	X
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1	X	X

Some combinations of the input value are not possible in practice. We have marked these with X to indicate this. This means that it makes no difference which value they have.

Problem 7.46
Make a design that realizes the control signals C_1 and C_2. Start by finishing the truth table, then find the logical expressions, and finish by realizing the system with ports.

Problem 7.47 Extra Challenge
In the figure above there is a resistor placed between the switch S_1 and ground. What is the purpose of this?

Problem 7.48
Design the full circuit in a circuit simulator (e.g. this one is a possible to use: *http:// www.falstad.com/circuit/circuitjs.html*). You can also try to realize the circuit in real life if you want to see with your own eyes that it is working!

Problem 7.49 Extra Challenge
We have not looked at the design of the clock generator yet. Design a clock generator with component values so that the counter counts from one to six 100 000 times per second.

SOLUTIONS

Sine wave Generator
» *keywords: Digital Electronics, Counter, DAC/ADC, Flip-flops*

In the following set of exercises, we will design a digital sine wave generator together. This will provide some insight into how bigger digital systems can be constructed, and will also give insight into the connection between the analog and the digital world. A complete digital system can often be divided into several parts:
- Functional block – the parts that bring the data in and out, processes it, and save it.
- Data buses that combine different parts of the system.
- A clock generator that is responsible for synchronization.
- A controller that sends control signals to different blocks so that they perform the right action at the right time.

A sine wave generator is an instrument that generates a voltage $x(t)$ with the following behaviour:

$$x(t) = V_0 \sin(2\pi f t)$$

Here V_0 is the amplitude, t is the time, and f is the frequency.

This can be realized with an analog circuit, but we will now look at how this can be done with a digital circuit.

What we first do is to generate a stair shaped voltage signal $\hat{v}(t)$ that approximates the ideal sine signal, as shown in the figure below:
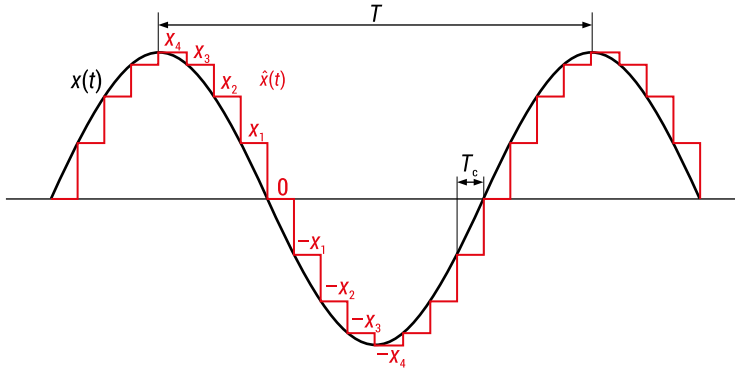
FIG. 7.50A. Desired output $\hat{x}(t)$

Notice that the time intervals between the "steps" T_c are the same (constant). This means that the DAC keeps the output value constant for the time T_c before it changes value. This can be accomplished by using a clock signal that has period T_c. In the figure you can see that we have chosen T_c so that we have $T = \dfrac{1}{f}$ with $T = MT_c$ where $M = 16$. So we have 16 steps during one period of the sine wave.

Problem 7.50

Assume that the DAC has perfect resolution in the voltage area $-V_0$ to V_0 (meaning it is not restricted in accuracy). Show that the step value x_1 in the figure has the value:

$$x_1 = V_0 \sin \frac{\pi}{8}$$

Then find the values for x_2, x_3 and x_4. **Hint:** find an expression for x_n.

A block diagram of the total system is shown in the figure below.
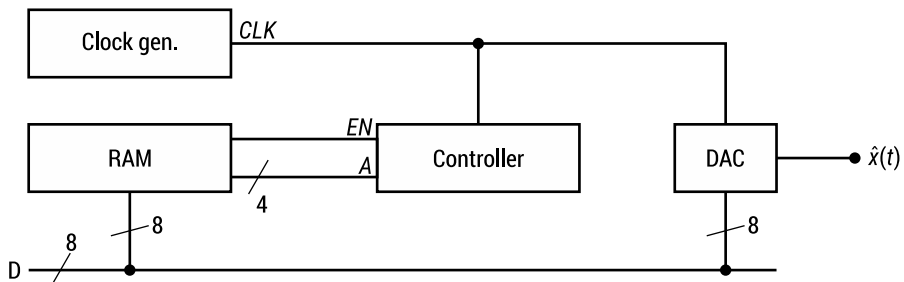
FIG. 7.50B. A block diagram of the total system

We can see that we get the signal $\hat{x}(t)$ from the DAC. The DAC gets a digital signal of 8 bits from the data bus D. The bus is also connected to the RAM block. The controller determines what address is retrieved from the RAM.

DAC

We have an 8-bit DAC as shown in the figure below
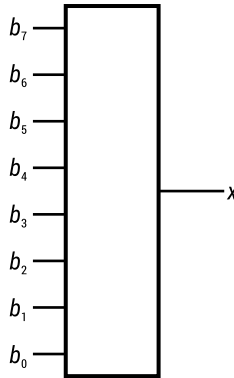
FIG. 7.50C. **Block diagram of 8-bit DAC**

This has 8 inputs denoted b_0, b_1,..., b_7 and one analog output x. The DAC interprets this as an 8-bit binary number B on two's compliment form, meaning that:

$$B = -b_7 2^7 + \sum_{i=0}^{6} b_i 2^i$$

Problem 7.51

Show that B can take values between –128 and +127. **Hint:** The equation for a finite geometric series can be helpful here, but there are several ways to get to the solution.

Problem 7.52

We want the DAC to give out the analog voltage value so that:

$$x = B\Delta V$$

How big should ΔV be so that the lowest possible value on the output is –5 V? What is the biggest possible value of x in this case?

Problem 7.53

Assuming we want to make a sine wave of amplitude $V_0 = 5$ V using the 8-bit DAC. How accurately can the DAC represent the value of x_1? How big is the error? **Hint:** Try to find the value of B first.

Problem 7.54

Show that the binary value for the step x_1 is $B_1 = 00110001$.

Problem 7.55

Use the same method as in the previous two exercises and show that the binary values for x_2 and x_3 are $B_2 = 01011011$ and $B_3 = 01110110$.

Problem 7.56

What about x_4?

The sine wave generator works by using the RAM-module as a look up table. It simply reads the right values from the RAM in the right order, and feeds them into the DAC. During one clock period 16 values come out of the DAC. The time diagram shows how the different signals change value during a clock period.
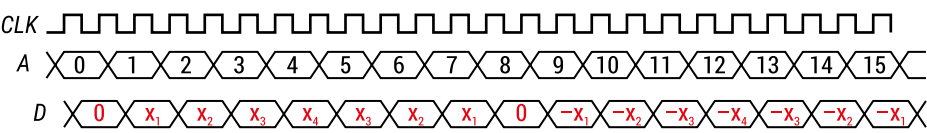
FIG. 7.56. **Time diagram of A and D**

We see that A starts at $A = 0$ and increases by one each clock period. When it is at $A = 15$, it starts over.

Problem 7.57

Design a controller that works as described by the figure above. Counting from 0 to 15 and over.

Problem 7.58

The table below shows the binary contents of the RAM module. The table shows the address values in decimal A_{10} and in binary A_2. It also shows the 8-bit value stored B, and the name of the variable that is to be saved at the address. Complete the table and fill out what is missing. **Hint:** Use two's complement.

A_{10}	A_2	B	Name
0	0000	00000000	0
1	0001	00110001	x_1
2			x_2
3			x_3
4			x_4
5			x_3
6			x_2
7			x_1
8			0

A_{10}	A_2	B	Name
9			$-x_1$
10			$-x_2$
11			$-x_3$
12			$-x_4$
13			$-x_3$
14			$-x_2$
15			$-x_1$

Problem 7.59

In most cases the amount of memory used is an important factor in the design of a system. Now we use a memory of 16 words, but we only have 9 different values that are saved. So it might be possible to design a system that only has these values saved. We need to redesign the controller in that case. A possible solution is shown in the Fig. 7.59.
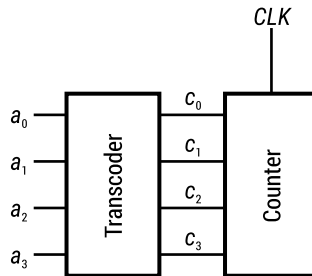
FIG. 7.59. Block diagram of controller

Here we assume that we only use the 9 first memory locations, thus the ones with address 0 to 8, as shown in the table below.

A_{10}	Saved values
0	0
1	x_1
2	x_2
3	x_3
4	x_4
5	$-x_1$
6	$-x_2$
7	$-x_3$
8	$-x_4$

The controller contains a counter that counts periodically from 0 to 16, as before. But instead of letting the output directly generate the addresses for the RAM, instead we use a *transcoder* that changes the number from the one the counter gives to the address that contains the value we want to connect to the DAC at that moment. The *transcoders* behaviour is described by the table below. Fill out correctly the table below.

C_{10}	C_2	A_{10}	A_2	Output value
0	0000	0	0000	0
1	0001	1	0001	x_1
2	0010	2	0010	x_2
3	0011	3	0011	x_3
4	0100	4	0100	x_4
5	0101	3	0011	x_3
6	0110	2	0010	x_2
7	0111	1	0001	x_1
8	1000	0	0000	0
9	1001	5	0101	$-x_1$
10	1010	6	0110	$-x_2$
11	1011	7	0111	$-x_3$
12	1100	8	1000	$-x_4$
13	1101	7	0111	$-x_3$
14	1110	6	0110	$-x_2$
15	1111	5	0101	$-x_1$

Problem 7.60
Find the logical expression for one or more of the bits $a_0 - a_3$ as a function of the counter input $c_0 - c_3$. Start with for example a_3.

Problem 7.61
Design a sine wave generator with adjustable frequency between 1 and 5 kHz. **Hint:** Assume that you have a 10 nF capacitor available and as many resistors and potentiometer as needed.

SOLUTIONS

References:

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Annex

This part contains five examples of problems that may be elaborated by students of 1st semester of bachelor level at electric engineering programme

Problem No. 1

You have for your disposal the 9 V battery. Design and build a circuit for supplying the load of nominal voltage $U_N < 9$ V and current that can change from 0 to $I_{Lmax} = 5$ mA. The supply voltage at the output can be in the range of $U_N \pm 10\%$. The voltage U_N (one for your choice) is given by the teacher:

a) $U_{out} = \dfrac{1}{2} U_{in}$

b) $U_{out} = \dfrac{1}{3} U_{in}$

c) $U_{out} = \dfrac{1}{4} U_{in}$

The problem report should include:
- course on which the problem was implemented (name, code, form),
- problem title,
- team member list (names and surnames, field of study, semester),
- problem completion date,
- the aim of the problem, the assumptions,
- description of the problem implementation (e.g. circuit diagram, results of calculations and measurements),
- description of the contribution of each team member,
- conclusions.

SOLUTION

Problem No. 2

Design and build a system that allows smooth voltage regulation and that performs the following functionality: If, while increasing the input voltage, it reaches the value of U_1, then the first LED should start to light. If the input voltage reaches U_2, the second

LED should light up. You have to use the 9 V battery as supplying source. As the values of voltages U_1 and U_2, assume (one for your choice):
a) $U_1 = 4$ V, $U_2 = 6$ V
b) $U_1 = 3$ V, $U_2 = 5$ V
c) $U_1 = 3$ V, $U_2 = 6$ V
d) $U_1 = 3.5$ V, $U_2 = 5.5$ V

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SOLUTION

Problem No. 3
Design and build a circuit in which the intensity of the light falling on the photoresistor controls the switching on or off of the LED. Use a 9 V battery to power the system. Consider two modes of operations of the circuit:
a) when the photoresistor is uncovered, the LED is on, and when the photoresistor is covered, the LED goes out,
b) when the photoresistor is uncovered, the LED is off, and when the photoresistor is covered, the LED starts to glow.

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Problem No. 4

Select the nominal current of the overcurrent circuit breaker protecting the circuit from which the selected loads in Table 1 are powered.

TABLE A1. Parameters of loads

No.	Load	Parameters
1.	Fan with heater	Nominal voltage: U_n = 230 V Nominal power: P_n = 1000 W Resistive load
2.	Fan with heater	Nominal voltage: U_n = 230 V Nominal power: P_n = 2000 W Resistive load
3.	Fan	Nominal voltage: U_n = 230 V Nominal power: P_n = 16.5 W Inductive load (cos φ = 0.73)
4.	Iron	Nominal voltage: U_n = 230 V Nominal power: P_n = 1000 W Resistive load
5.	Lighting − bulb (75 W)	Nominal voltage: U_n = 230 V Nominal power: P_n = 75 W Resistive load
6.	Lighting − bulb (100 W)	Nominal voltage: U_n = 230 V Nominal power: P_n = 100 W Resistive load
7.	Lighting − bulb (175 W)	Nominal voltage: U_n = 230 V Nominal power: P_n = 175 W Resistive load
8.	Cordless kettle	Nominal voltage: U_n = 230 V Nominal power: P_n = 2200 W Resistive load
9.	Resistive load	Resistance: R = 98 Ω
10.	Inductive load	Resistance: R = 282 Ω Inductance: L = 0.8 H
11.	Electromagnetic ballast FL-4000 for fluorescent lamp	Nominal voltage: U_n = 230 V Nominal power: P_n = 40 W Working in an inductive system (cos φ = 0.5)
12.	Electromagnetic ballast FL-4000 for fluorescent lamp	Nominal voltage: U_n = 230 V Nominal power: P_n = 40 W Working in a capacitive system (cos φ = 0.5)
13.	Capacitive load	Capacitance: C = 10 µF

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SOLUTION

Problem No. 5

Any sound file containing, for example, music or spoken word is given. Using any program, record additional sound (disturbance) of a specific frequency (e.g. squeak, noise) to this file. Design a circuit whose input is a given sound signal. The circuit should filter out the extra interfering sound.

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Two sound files should be attached to the report: with and without interference.

List of tables

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