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Chapter 6
The shaping of static characteristics of nonhomogeneous planar elements intended for effective heating systems

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Abstract: In the presented article planar heating structures on elastic base are proposed. These novel structures are intended for applications in, e.g., temperature control systems. An influence of electrically heated resistive layer geometry on the thermal properties of the system with a naturally cooled receiver is characterized. The impact of resistive layer geometry is determined, taking into account both thermal and electrical steady-state operating conditions. The results are discussed for a three-dimensional, non-linear numerical model of an exemplary system. The solution of coupled electrical and thermal phenomena is obtained using the finite element method (FEM). Factors which affect the temperature-voltage static characteristics of the exemplary heat receiver are identified.

Keywords: nonhomogeneous composite materials, heating mats, heating efficiency, numerical analysis.

Introduction

Modern composite materials and intelligent structures with adjustable mechanical [1], electrical [2] and thermal [3] properties constitute a group of intensively developed artificial hybrid structures. Their properties can be changed in a wide range to obtain desired parameters such as, for example, the resonant frequency [4] or the temperature in a steady and transient state [5], in an iterative process of adjusting and optimizing the geometry. Among these types of materials a specific group can be distinguished, where thermal properties are modified, and appears as a result of adjustments of proportions and a distribution of two different constituent substances [6, 7]. An example of this kind of hybrid structures are layered/laminar materials, in which at least one layer is heterogeneous and conductive, but the second layer is flexible – it serves both as a carrier and also provides electrical and thermal insulation.

The use of laminar materials in electro-thermal engineering and temperature control systems includes, among others, creating dedicated solutions for heating devices, radiators, or infrared heaters [8, 9, 10]. The dispersed heat generation that occurs as a result of the flow of an electric current through the resistive layer, is characterized by nearly 100% efficiency and possibility of regulating the power, by a simple control system changing the supply voltage. These systems can be used in domestic applications as well as, for example, at particular stage of a production process, in which large-scale (e.g. full-size heating mats) and small-scale components (e.g. local temperature control) are required. The most frequently used resistive materials are metal alloys, such as manganin, constantan, evanohm [10, 11, 12] with a very low resistance temperature coefficient ensuring the stability of heating system parameters, even at relatively high temperatures. This property, however, is associated with non-linear dependence of the dissipated thermal power – and thus a controlled temperature of a heated object – with a change of supply voltage.

The article discusses an influence of the structure of periodic elements, forming the heterogeneous resistive layer of a planar heating structure, on the temperature-voltage characteristics of an exemplary heating system, dedicated to a convectively cooled heat receiver. Two models of the system were considered: linear with constant material coefficients, and non-linear with a dependence of material resistivity of heating elements on receiver temperature. Elements with low and high heating power were investigated. The coupled steady-state analysis of electrical and thermal phenomena was performed using a three-dimensional numerical model, which was solved by the finite element method (FEM).

Model and methods

The electrical phenomena in a considered system are limited to a stationary electric field, which is described by the Laplace equation:

$$\nabla \cdot \left(\rho^{-1} \nabla V \right) = 0, \tag{6.1}$$

where: Ñ – differential Del operator, $V = V(x,y,z)$ – electric potential at a point $P(x,y,z)$ [V], ρ – electrical resistivity [Ωm]. A three-dimensional numerical model was characterized by a nonhomogeneous distribution of resistivity $\rho = \rho(x,y,z)$, generally speaking, different at any point P due to the appearance of a conductor as well as an insulating layer and a dielectric receiver at the top of the system. Moreover, it can be assumed that the value of resistivity has a linear dependence of the resistive layer temperature, hence:

$$\rho = \rho_0\left[1+\alpha\left(T - T_0\right)\right], \tag{6.2}$$

where: α – resistance temperature coefficient [1/°C], $T_0 = T_0(x,y,z)$ – reference temperature [°C], $T = T(x,y,z)$ – temperature at point P [°C], $\rho_0 = \rho_0(x,y,z)$ – resistivity at the reference temperature [Ωm].

Steady-state thermal phenomena, including electrical heat sources, heat transfer (mostly thermal conduction) and convective heat transfer to an environment, are described by the equation:

$$\nabla \cdot \left[-\lambda \nabla T + h\left(T - T_a\right)\right] = Q_V, \tag{6.3}$$

where temperature T is a unknown variable dependent on:

- local distribution of thermal conductivity $\lambda = \lambda(x,y,z)$,
- heat transfer coefficient $h = h(x,y,z)$ of the environment with a constant temperature T_a,

as well as volumetric density of the dissipated power:

$$Q_V = \left|\mathbf{J}\right|^2 \rho, \tag{6.4}$$

which directly depends on an absolute value of a current density vector $\mathbf{J}(x,y,z)$.

The occurrence of non-zero value of the power $Q_V = Q_V(x,y,z)$ in the heating layer is the result of heat dissipation due to the electric current of density **J**, forced by regulated voltage U. To determine the distribution of current density **J**, it is required to solve Equation (1). Taking into account the influence of variable resistivity (2), the problem becomes complex and non-linear. Hence, a combination of electrical and thermal phenomena in the conducted analysis is necessary. This is possible through the simultaneous calculation of the local potential V and temperature T distribution, based on equations (1) and (3), including (2), using the iterative procedure in the finite element method.

The design of the heating system is based on the periodic arrangement of resistive elements Ω_e (Fig. 6.1) with adjustable internal geometry. In a structure contained in a cuboid with width and length of $\Delta l_x = \Delta l_y = 5$ mm and height of $\Delta l_z = 0.035$ mm, it is possible to modify a contact length (d_c), path (d_p) and diagonal (d_d) of the element. The presented geometric parameters for the two exemplary elements (Table 6.1) can be selected in order to achieve desired equivalent resistance $R_e = \mathrm{f}(d_c,d_p,d_d)$ determining, for the given parameters of a power source, the entire power dissipated in the system.

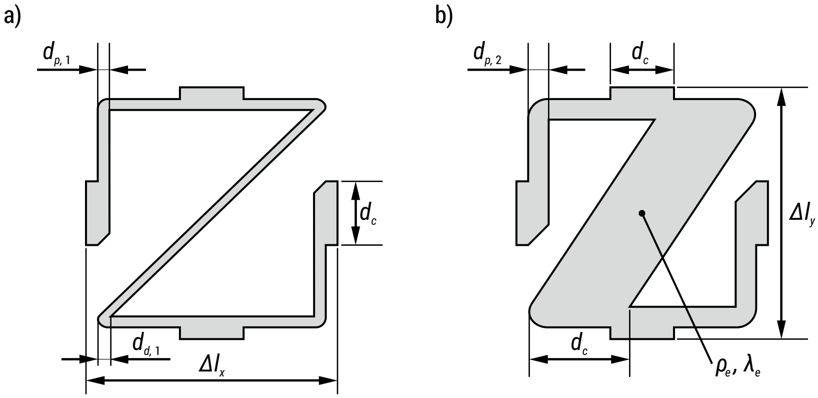


FIGURE 6.1. Heating elements Ω_e: a) lower power variant, b) higher power variant

The object temperature (T_O) at a measuring point is an output parameter of the system, subjected to the adjustment by the controller with a set temperature (T_{set}). This task can be successfully performed by well-known controllers (e.g. PI, PID) that are able to regulate the source voltage. The object temperature results directly from the power dissipated in the resistive layer as well as from voltage U which, in practical implementations, is limited to a certain maximum value U_{max}. Hence, if the source is able to provide sufficient current efficiency (e.g. it can be a typical DC power supply), it is possible to increase the dissipated power (and thus the receiver temperature) by modifying the geometry of the resistive element, so that it will have lower equivalent resistance.

The considered temperature control system (Fig. 6.2) consists of 24 elements located on a surface of non-conductive polymer substrate Ω_B, with thermal conductivity λ_B = 0.3 W/m/°C and dimensions l_x = 30 mm, l_y = 20 mm, l_z = 1 mm. On the resistive layer a rectangular (and also electrically non-conductive) heat receiver with λ_O = 10 W/m/°C and height l_O = 5 mm is placed. The compact, thin-layer heating structure consisting of any number of Ω_e elements, allows for adapting the shape of the periodic heater to almost any application. The electrical and thermal properties can be changed, primarily based on the modification of a single periodic element geometry and placing it (or embedding it) in a thin (e.g. l_z = 1 mm) supporting layer Ω_B. Furthermore, the mechanical strength to bending or cracking and the multiplication of connections between elements (each with 4 contact surfaces with the adjacent ones), increase the reliability of the entire structure, even in the case of possible mechanical stress or damage.

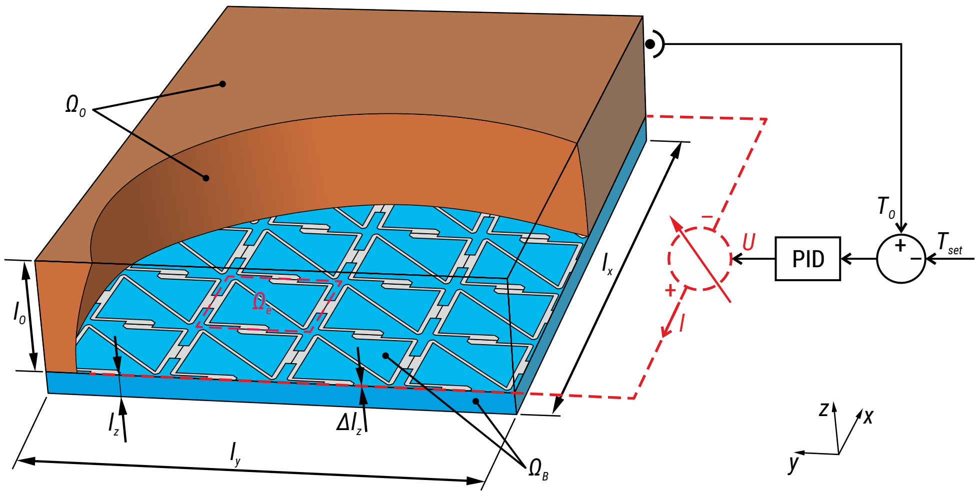


FIGURE 6.2. Heating system with power regulator and a receiver Ω_0 placed on the periodic heating structures, embedded on an elastic insulator Ω_B

TABLE 1. ELectric [11], thermal [12] and geometric parameters of heating elements Ω_e

Variant	ρ_e,0 [Ωm]	α [1/°C]	λ_e [W/m/°C]	T_{max} [°C]	d_c [mm]	d_p [mm]	d_d [mm]
v(1,1)	$2{\cdot}10^{-7}$	0.005	90	250	1.25	0.2	0.2
v(2,1)	$2{\cdot}10^{-7}$	0.005	90	250	1.25	0.4	2.0
v(1,2)	$2{\cdot}10^{-7}$	0	90	400	1.25	0.2	0.2
v(2,2)	$2{\cdot}10^{-7}$	0	90	400	1.25	0.4	2.0

In the numerical model of the system there was assigned, on all external surfaces, the Hankel-Robin boundary condition with a locally determined heat transfer coefficient h, which corresponds to convective heat transfer to an environment with a constant temperature $T_a = T_0 = 25°C$. The system was supplied using an ideal DC voltage source with an adjustable value of U (maximum voltage $U_{max} = 2V$), connected to outer edges of the heating structure (Fig. 6.2 – dashed red lines). Simulations were performed by changing the value of source voltage in a range $0 \div U_{max}$. Four basic variants of the heating system were considered: the first variant $v(1,1)$ refers to an element Ω_e with a lower, and variant $v(2,1)$ to a higher dissipated power with temperature-dependent resistivity. On the other hand, variants $v(1,2)$ and $v(2,2)$ correspond respectively to identical elements, but with a constant resistivity, independent of the system temperature ($\alpha = 0$).

Results discussion

For the discussed variants numerical calculations of the heater model, with an exemplary receiver, were performed. By increasing a supply voltage from 0 to 2 V, the volume distributions of electric potential V (Fig. 6.3) and temperature T (Fig. 6.5) were obtained. Moreover, the dependence of an average absolute (T) and relative (T_w) temperatures of the receiver (Fig. 6.4a), and the absolute current flowing through the heating structure (I) and the relative one (I_w) were found (Fig. 6.4b). Based on the second characteristic, it is possible to calculate the total power dissipated in the system, or equivalent resistance of a resistive path, from the point of view of power source terminals. Thus, on this basis, it becomes possible to analytically estimate electric power consumption or optimal settings of the control system.

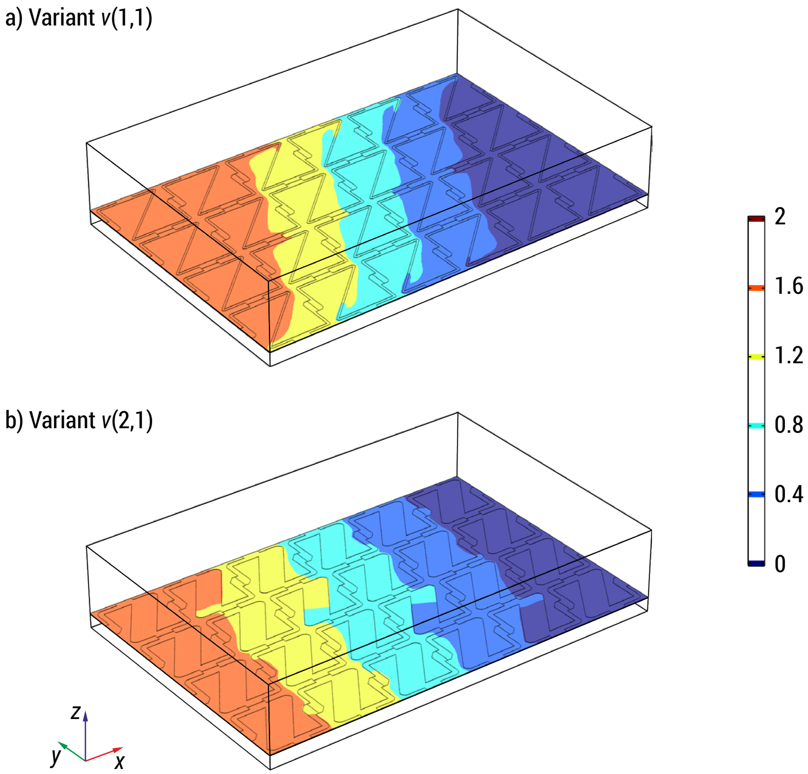


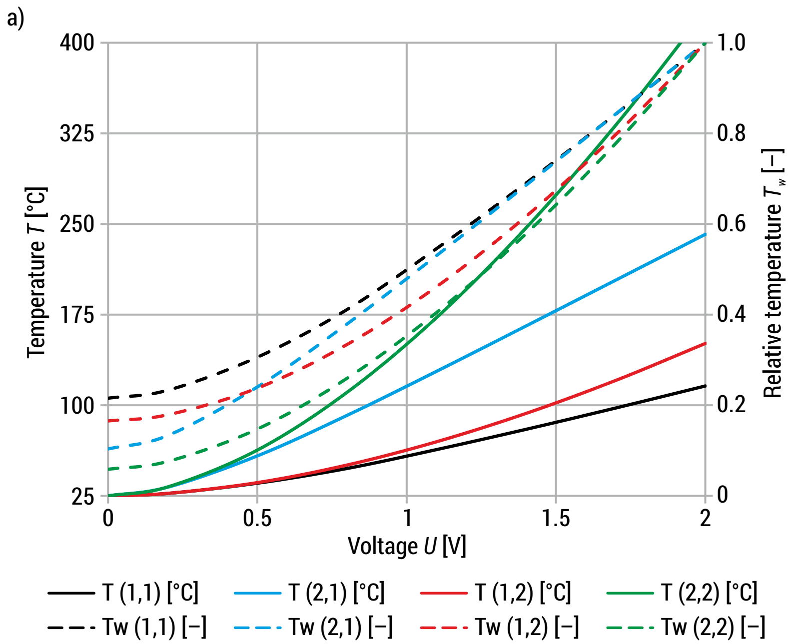
FIGURE 6.3. Electric potential distribution V in [V] on the upper plane of a resistive layer at $U = Umax$ and $\alpha = 0{,}005$ 1/°C: a) system with lower power elements, $v(1,1)$; b) system with higher power elements, $v(2,1)$

The distribution of an electric field in a resistive layer depends on the geometry of an element. For a structure with higher equivalent resistance (Fig. 6.3a), the isolines of an electric potential do not line up parallel to the *OY* axis (i.e. the edges to which a voltage source is connected). The observed "curvature" is a result of an electric current flowing through also curved resistive paths of the elements. However, for the geometry with higher dissipated power (Fig. 6.3b) the isolines of electric potential are nearly parallel to the *OY* axis, which means that the current flows in a different way than in a previous case. Hence, the distribution of local heat sources on the heating surface will be different. When comparing the potential distributions for elements with $\alpha > 0$ or $\alpha = 0$, no change in distribution or the value of V was observed, but only a change in the value of the supply current I has appeared.

The dependence of average receiver temperature T on the output voltage U is highly nonlinear (Fig. 6.4a). The variants with lower (or correspondingly higher) dissipated power, with the two considered values of the coefficient α, have the same geometry but differ in the value of dissipated heat power. This results from different values of currents I flowing through the resistive path (Fig. 6.4b), since for $\alpha > 0$ the electrical resistance of the conductor increases with increasing temperature. For example, the Ω_O receiver is heated at $\alpha = 0.005$ 1/°C to a maximum temperature $T(1,1)_{max} = 115$°C, and for $\alpha = 0$ to $T(1,2)_{max} = 150$°C.

The self-occurring power limitation at non-zero α simultaneously leads to the linearization of the $T = \mathrm{f}(U)$ characteristic, when the voltage exceeds a certain threshold value (in this case it is $U \approx 0.7$ V). Then, a relative temperature T_w increase has identical an shape and values, both for the variant of element with lower and higher power (Fig. 6.4a – curves $T_w(1,1)$ and T_w (2,1)). Moreover, in the variant $v(2,2)$, where $\alpha = 0$ is assumed, a permissible system temperature may be exceeded above the specified voltage (e.g. $T_{allowed} = 250$°C), while in the variant $v(2,1)$ elements with the same geometry may be safely supplied in the entire voltage range, since they do not show a sudden increase in power, and thus in the temperature of the object (receiver). Still, according to Ohm's law, the temperature-dependent resistance will affect the shape of a current-voltage characteristic (Fig. 6.4b), where the relation $I = \mathrm{f}(U)$ becomes non-linear.

The temperature distribution on the surface of the resistive layer differs under identical supply conditions for individual geometry variants (Fig. 6.5a, 6.5c). The modification of the current flow path, resulting from the structure of the element, changes both an absolute temperature value (e.g. $T_{max} \approx 123$°C for $v(1,1)$ and $T_{max} \approx 250$°C for $v(2,1)$) and also the distribution of local heat sources. Elements with a higher dissipated power have provided an area of the highest temperatures in the central part of the system (Fig. 6.5c), while lower power elements at opposite vertices (Fig. 6.5a). The distribution of heat sources directly affects temperature distribution within the receiver. However, generally speaking, in both geometrical variants of heating elements, the wide area with the highest temperatures is located in the center of the receiver (Fig. 6.5b, 6.5d) and extends to opposite edges. Nevertheless, the uniformity of temperature distribution at $U = U_{max}$ for the entire receiver volume, defined as $T_{min}/T_{average}$, is high for the variant $v(1,1)$ and equal to 0.94, and for $v(2,1)$ it is equal to 0.96.



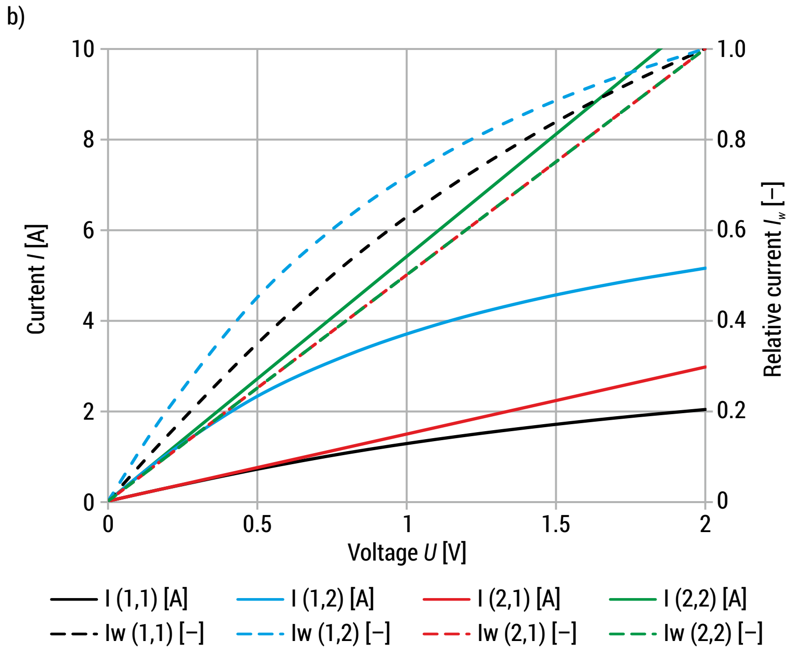


FIGURE 6.4. Absolute and relative characteristics for four variants: a) temperature vs. source voltage; b) current vs. source voltage

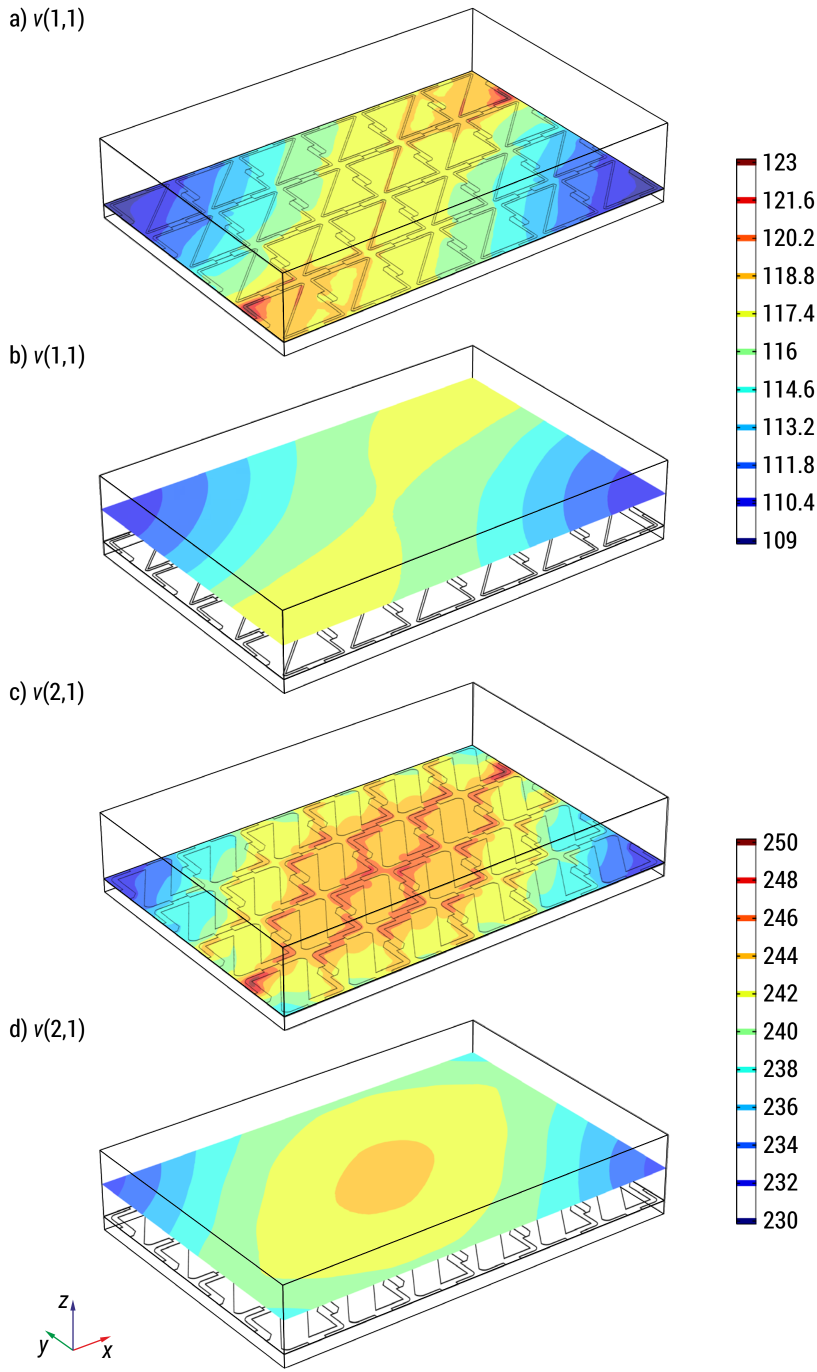


FIGURE 6.5. Temperature distribution T in [°C] at $U = U_{max}$ and $\alpha = 0.005$ 1/°C on the upper plane of the resistive layer for: a) lower power elements, c) higher power elements; as well as in the center of the receiver for: b) lower power elements, d) higher power elements

TABLE 6.2. Linear approximation coefficients of a function T = f(U) as well as relative errors of the linear approximation

Variant	a	b	$\|\Delta T_{\%}\|$
v(1,1)	0.4676	0.0483	8.05%
v(2,1)	0.5090	−0.0231	6.90%
v(1,2)	0.5102	−0.063	17.04%
v(2,2)	0.5708	−0.1854	22.48%

In the last step, an approximation of relative temperature function T_w = f(U), depending on the voltage in the range 0.4 ÷ U_{max} for 4 considered variants, was made. In the case of variants v(1,1) and v(2,1) linear approximation was achieved with the average relative errors $|\Delta T_{\%}|$ of 8.05% and 6.90%, respectively. These values are more than two times smaller than approximation errors of strongly nonlinear functions of variants v(1,2) and v(2,2), in which the assumed resistivity is temperature-independent. The presented results (Table 2) indicate that it is possible to approximate the dependence of object temperature on the source voltage using a linear function, with an average error below 10% in a system with variable resistivity. Hence, the conclusion can be made that with a higher resistance temperature coefficient α and operating temperature T_{set}, the relationship T = f(U) between temperature and voltage U of supply source becomes much more linearized.

Conclusions

In the article a heating system with temperature regulation, arranged of low-power elements, and an exemplary heat receiver was proposed and discussed. The obtained numerical results indicate the ability to adjust the range of dissipated power by a proper change of resistive elements geometry. Material selection of the element, characterized by specific resistivity and the resistance temperature coefficient, has an impact not only on the amount of dissipated heat, but mainly on the shape of static characteristics T = f(U) of the heating system. It is also possible to linearize temperature dependence of the heat receiver on the voltage source by adjusting the shape of the element and using a proper resistive material with a positive temperature coefficient. Linear dependence of the receiver temperature of the source voltage occurs in a wide range of available voltages (i.e. from $0.2U_{max}$ to U_{max}), while the difference between the ideal linear function and the calculated one is less than 8.1%. Higher values of the resistance temperature coefficient and temperature of the heating layer have led to a more ideal linearization of T = f(U) characteristics, but at the same time also reduced the power (heat) dissipated in the conductor.

Streszczenie: W artykule zaproponowano budowę planarnej struktury grzewczej na elastycznym podłożu, przeznaczoną do zastosowań w układach regulacji temperatury. Scharakteryzowano wpływ budowy warstwy nagrzewanej elektrycznie na parametry termiczne układu cieplnego z odbiornikiem chłodzonym konwekcyjnie. Ustalono wpływ konstrukcji warstwy oporowej na termiczny i elektryczny punkt pracy systemu w stanie ustalonym. Dyskusji poddano rezultaty otrzymane dla trójwymiarowego modelu numerycznego, przykładowego układu grzewczego małej mocy z uwzględnieniem nieliniowego charakteru zmian rezystywności elementów grzewczych o dobieranej strukturze. Rozwiązanie sprzężonych zjawisk elektrycznych i termicznych otrzymano za pomocą metody elementów skończonych (MES). Zidentyfikowano czynniki kształtujące charakterystykę temperaturowo-napięciową zadanego odbiornika ciepła. (Kształtowanie charakterystyk niejednorodnych elementów planarnych przeznaczonych do efektywnych układów nagrzewających).

Słowa kluczowe: niejednorodne materiały kompozytowe, maty grzewcze, efektywność nagrzewania, analiza numeryczna.

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