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Chapter 5
The efficiency of Wireless Power Transfer in periodic systems composed of circular coils

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Abstract: The article presents an analysis of the efficiency of the periodic Wireless Power Transfer (WPT) system. In the analysis circular coils were used. Also, considerations were given for changeability in the number of turns and the distance between the transmitting-receiving coils. The influence of variable system geometry and the frequency on system efficiency was analysed. The Finite Element Method (FEM) with the use of periodic boundary conditions was used for the analysis. Based on the obtained results, it was verified at which system parameters wireless power transfer of the system is possible.

Keywords: wireless power transfer (WPT), numerical analysis, magnetic fields, FEM.

Introduction

The power obtained from the power station in the form of electricity is very costly and the transmission efficiency is very low (~30%), because high tension conductors due to a high energy loss in a high resistive wire. Also, the power station that is running on coal, gas or nuclear materials, uses a lot of these resources to make electricity, is not cost-effective and has trouble associated with environmental issues (e.g. pollution). Due to infrastructure power disruptions are frequent. The indicated problems lead to find alternate solutions connected with transmit and distribute power (electricity). The other problem is the wires for powering home appliances at e.g. house, workplace, schools, because of a reduced range of machines that are prone to failure. Also, while changing the battery of any appliance, the circuit interface can be disturbed and may stop working. Considering the above, the method of a wireless charging system would be beneficial in both economic and social aspects. Many industries are trying to explore the use of electric vehicles or cars in order to the reduce fuel consumption [1, 2, 3]. Also, the WPT system would make a huge impact on the medical field (e.g. pacemaker) [4, 5]. The main focus of researchers connected with the WPT system is to develop a wireless powering mechanism useful for the private sector (e.g. medical, consumer use, industrial) and also for the public sector (e.g. transfer power on a large scale at a low cost and without pollution).

Wireless power transmission (WPT) has become one of the most important research points in this century. Portability is the main motivation for WPT as the number of portable devices is enormously increasing and wired chargers will limit their portability.

Several review works [6, 7, 8] briefly explained the WPT theory, system overview with circuit structure and applications. However, the progress in the resonant coupled system, including the multi-coil WPT structure [9, 10, 11, 12], effects of couplings and frequency splitting on efficiency, as well as human exposure issues [13, 14], has not been extensively studied so far.

In the late 20th century, the near-field inductive power transfer (IPT) [15, 16] became popular because it attained the charging of portable consumer devices. The IPT system can effectively transmit power from a source to a device using the principle of EM induction and is also non-radiative. Inductive chargers, such as those commonly found in electric toothbrushes, charging pads for cell phone [7], operate on this same principle. For the IPT applications of a few kilowatts (kW), like the charging of electric vehicles, almost 90% of transmission efficiency can be achieved by increasing its operating frequency, and over 70% of efficiency is also possible to achieve for low-power (maximum 5 W) mobile phone charging. For low-power industrial and domestic applications, the operating frequency range of the inductive coupled technique is generally from 20 kHz to several MHz.

Nowadays, the emerging application and a growing market of portable mobile appliances, remote charging and powering of these portable devices, the demand for contactless RFID (Radio-Frequency Identification) for security applications and transportation, and power harvesting of battery-free CMOS devices for biomedical engineering – they all are playing the major role in the push-forward of the resonant coupled WPT system.

After the first experiment of a glowing bulb through resonant coils without a barrier and also with a barrier made by MIT (Massachusetts Institute of Technology) in 2007, there have been new advances in the resonant coupled system to make it suitable for commercial applications [17]. In 2008, Intel explored the resonant coupled WPT by using flat coils, which are much easier to fit in mobile devices than the helix coils used in [18]. In [19] an advanced contactless approach simultaneous powering of multiple receiving devices (e.g. laptops, cell phones) was presented. For some research works on resonant coupled WPT, the operating frequency range from 10 kHz to nearly 200 MHz [20] was used.

In opposition to the traditional 2-coil system [21], the 4-coil wireless powering approach is designed by placing two intermediate multi-turn coils between two loop coils. Each loop coil is a form of impedance matching mechanism and acts as a non-resonator to exchange energy between the circuits and intermediate coils [18]. The works [22, 23] connected with multi-coil linked resonant coupled WPT technique have raised interest in 4-coil system for better performance and impedance matching, compared to 2-coil system. The advantage of this approach is that the two intermediate coils are physically set free from circuits, but the main disadvantage is that it requires bigger space than any other transmission structure.

Wireless transmission of electricity to multiple receivers using a single source coil has been described and analysed in [24]. The disadvantage of this approach is the fact that the resonant frequency of coils splits when two receivers are in close enough proximity, and hence lowers efficiency. More difficulty arises with multiple receivers due to the high complexity of the circuit model and a lack of interaction among the coils.

Resonators form an array of coils as a domino [9], and linear resonator arrays [10] are considered, where in the intermediate space between the transmitter and the receiver energy transfer is assisted using several resonators. However, a detailed analysis was performed for a series configuration of resonators, while parallel-series topology of planar coils, acting as a group of energy transmitters and receivers, are still not fully developed.

The power transmission efficiency of the WPT declines at any coupling greater or lesser than its critical value. Therefore, a system to maintain high efficiency without shifting the resonant frequency at coupling distance variation is required. Several approaches, e.g. like optimum frequency adjustment [25, 26], coupling manipulation [27], adaptive matching using multi-loop coils [28] and LC circuits [29], adjusting resonant parameters [30], and use of antiparallel resonant loop [31], have been discussed to deliver power at improved efficiency in the resonant coupled system. Some of the above articles also presented the effect of axial and angular misalignment of the coil, which is the major issue in WPT implementation for portable devices, such as electric vehicle charging [32].

Energy supply or charging of many devices located in close range to each other may be simplified using WPT systems as a grid of periodically arranged coils, which form surfaces for transmitting or receiving the energy. This solution increases density of transferred power and also enables simultaneous powering (using single power source) of many devices (e.g. for charging many electric cars in one parking). The proposed solutions can be used to power either one or multiple independent loads and, in some cases, replace conventional IPT systems. The developed periodic WPT system allows for the simultaneous supply/charging of many low-power receivers, such as mobile devices or sensors repeatedly distributed over hard-to-reach areas.

The article presents a wireless charging system with periodically arranged planar coils. The proposed analysis of the unit cell with periodic boundary conditions does not require full 3D model with many coils [33], where the number of degrees of freedom is huge. A simplified model in the form of the well-known T-type equivalent circuit is an alternative for more extensive matrix formulation [9, 10, 34], where a large coefficient matrix with lumped parameters has to be known. The main purpose of this work is to introduce and study the model, which can be applied to analyze power transfer conditions in the discussed systems.

The article presents a system of periodically arranged transceivers and receivers coils. This proposed system could be used to load mobile devices as the wireless power transfer system. Numerical approach reduces the size and complexity of typically utilized models. By the proposed appropriate selection of load resistance, it was possible to determine the maximum efficiency of the WPT system. Calculations of the exemplary periodic WPT system were performed over a frequency range from 0.1 MHz to 1 MHz. The analysis of the influence of the number of turns and the distance between the transmitting and receiving coils on the efficiency of the system was performed.

Proposed periodic model of the Wireless Power Transfer System (WPT)

A system consisting of a plane of transmitting coils and a plane of receiving coils between which energy was transmitted was considered (Fig. 5.1).

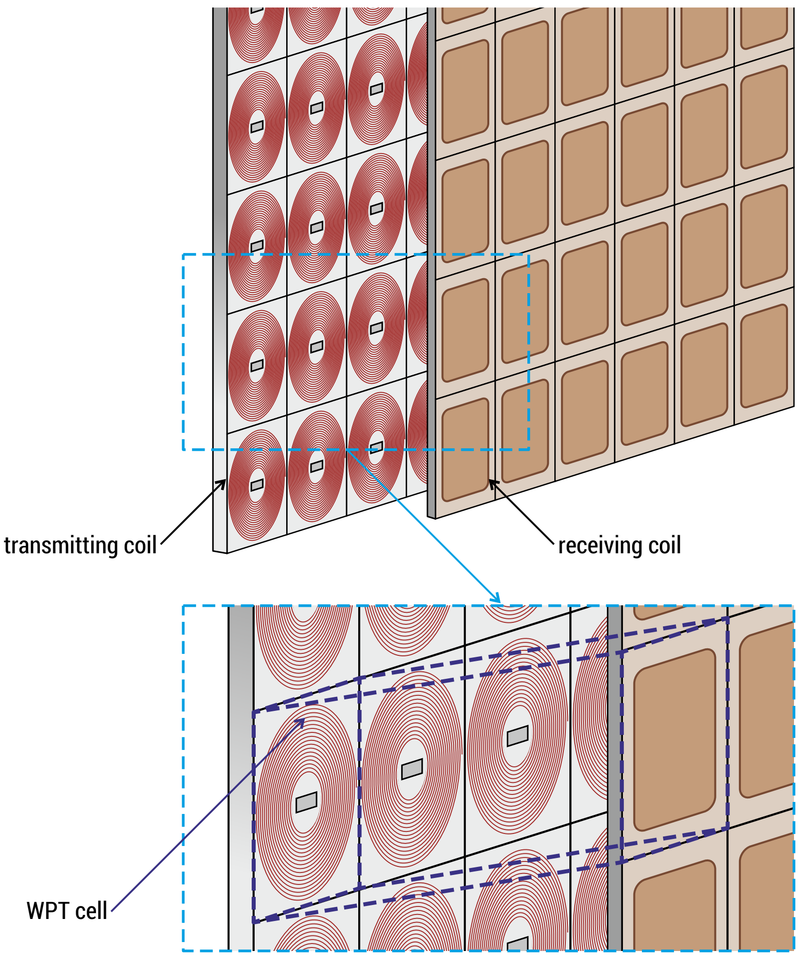

FIGURE 5.1. A three-dimensional view on the WPT system

The WPT cell presented in Fig. 5.1 has external dimensions $d \times d$. The transmitting and receiving coils are placed at a distance (h). The turns are placed on a plastic carcass in which the compensating capacitor connected in series with the coil is located. The three-dimensional distribution of WPT cells leads to the creation of a periodic network, which includes transmitting and receiving surfaces. The WPT cell consists of a transmitter-receiver pair constituting an arrangement of identical coils with a radius r and the number of turns n (Fig. 5.2).

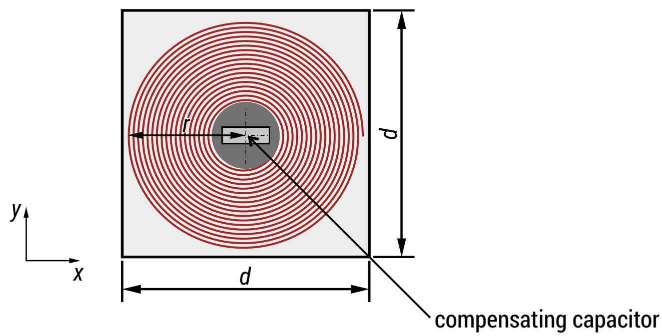

FIGURE 5.2. One circular coil used in the proposed WPT system

The transmitting surface is powered so that each transmitter is connected in parallel with a sinusoidal voltage source with the effective value U. The coils creating the receiving surface are connected directly to the load.

The presented system gives an increase in the density of transmitted power in the area between the receiving and the transmitting surface. The article presents the WPT system, in which it is possible to power many independent receivers, where a set or each WPT cells is assigned a separate load. Each receiver is connected to the load $\underline{Z}$.

Numerical analysis of the WPT system

In the analysis of the WPT system, the FEM method was used. The accuracy of the solution depends on the size of the model, e.g. the number of degrees of freedom (N_{DOF}). A greater number of degrees of freedom allows for obtaining a greater accuracy of the solution. Unfortunately, it causes longer calculation time.

The numerical approach to the analysis of a system composed of many WPT cells requires taking into account: coil geometry, number of WPT cells and elements of the electrical circuit connected to each coil. The coils are wound from several dozen turns, which are made of ultra-thin wires with a diameter (w) and insulated from each other by an electrical insulator of thickness (i). A compensating capacitor can be modelled as a lumped element with capacity (C), attached to each coil.

The WPT system was modelled with periodic boundary conditions (PBC) [35, 36, 37]. This approach allows for simplifying model to a single WPT cell containing a pair of transmitting and receiving coils (Fig. 5.3). The perfectly matched layer (PML) is a place at the top and bottom of the model to imitate infinite dielectric background.

Each transmitting coil is connected to a voltage source with an effective value (U) and frequency (f) that forces the transmitter current ($\underline{I}_{tr}$) to flow. In the receiving coil the source is replaced by a linear load ($\underline{Z}$), which conducts the induced current ($\underline{I}_{re}$).

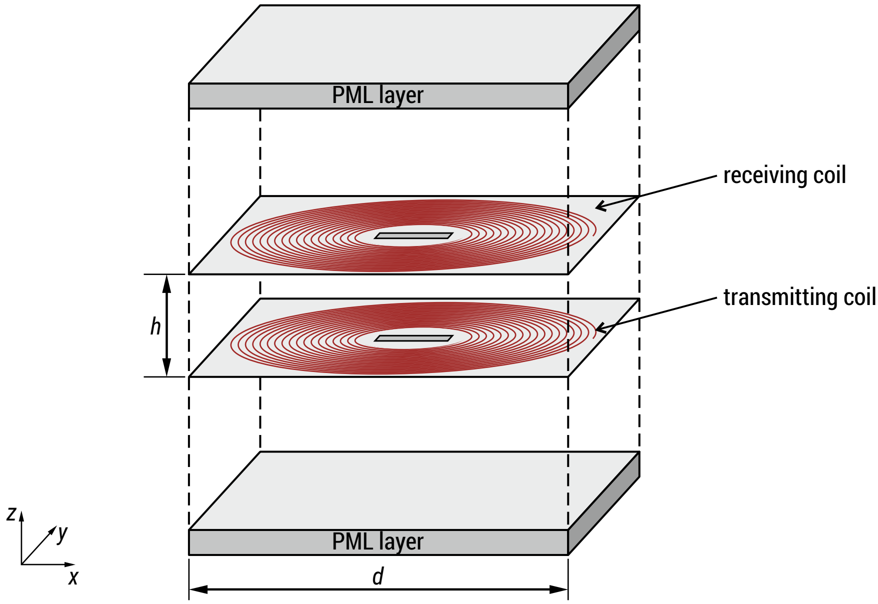

FIGURE 5.3. Numerical model of the periodic WPT system

The issue of energy transport in the presented WPT model can be solved using magnetic vector potential in the form:

$$\mathbf{A} = [\mathbf{A}_x \;\; \mathbf{A}_y \;\; \mathbf{A}_z]. \tag{5.1}$$

Also was used a description of magnetic phenomena in the frequency domain (Helmholtz equation):

$$\nabla \times \left(\mu_0^{-1} \nabla \times \mathbf{A}\right) - \mathrm{j}\omega\sigma\,\mathbf{A} = \mathbf{J}_{ext}\,, \tag{5.2}$$

where:
μ_0 – vacuum magnetic permeability [H/m],
ω – pulsation [rad/s],
σ – conductivity [S/m],
$\mathbf{J}_{ext}$ – external current density vector [A/m^2].

Periodicity conditions on four side surfaces are given in the form of magnetic isolation:

$$\mathbf{n} \times \mathbf{A} = 0, \tag{5.3}$$

where $\mathbf{n} = [\mathbf{1}_x\ \mathbf{1}_y\ \mathbf{1}_z]$ is a normal vector to a surface.

The voltage source U with frequency f determines the value of $\mathbf{J}_{ext}$, and taking into account the equation (3) allows to solve the relationship (2) and determine the spatial distribution of magnetic vector potential $\mathbf{A}(x,y,z)$. For this purpose, the FEM method can be used. The compensating capacitor capacity can be determined, e.g. based on parametric analysis of the system at different capacitance values (C). In the case when $\mathrm{Im}[\underline{I}_t] \approx 0$, it is assumed that the system has a resonance state and the selected C is the sought capacity.

Assumption to the analysis

The WPT system was built of identical circular coils. Coil sizes $r = 25$ mm and a different number of turns $n \in \{90, 100\}$ and the distance $hr \in \{12.5, 25\}$ were analysed. Tab. 1 presents parameters of the wire and other values used in the analysis.

TABLE 5.1. Parameters used in the analysis

Parameter	Symbol	Value
Diameter of the wire	w	200 μm
Thickness of wire insulation	i	5 μm
Conductivity of the wire	σ	$5.6 \cdot 10^7$ S/m
Voltage source	U	1 V
Frequency domain	$fmin \div fmax$	100 ÷ 1000 kHz

On the basis of the obtained results for several exemplary periodic WPT systems, the correctness of the proposed numerical model was verified by comparing the active power of the receiver:

$$P_o = Z\left|\underline{I}_{re}\right|^2, \tag{5.4}$$

where: $\underline{I}_{re}$ is a current flowing through the receiving coil. Transmitter power is represented by:

$$P_z = U\underline{I}_{tr}, \tag{5.5}$$

where: $\underline{I}_{tr}$ is a current flowing through the transmitting coil. Using equations (4) and (5), the power transfer efficiency was described by:

$$\eta = \frac{P_o}{P_z} 100\% \, . \tag{5.6}$$

The results were based on the correct selection of Z_e (optimal load impedance) for maximum power transfer efficiency [11]:

$$Z_e = \sqrt{R_c^2 + (2\pi f M_{tr})^2} \, , \tag{5.7}$$

where:
M_{tr} is mutual inductance,
R_c is resistance of an inductor.

Calculation results

The numerical model (Fig. 5.3) was created in the *Comsol Multiphysics program*, using boundary conditions (PML and PBC), and then solved using the Finite Element Method. In order to determine the maximum efficiency transmitted to the receiver, the values of load impedance were calculated taking into account the number of turns and the distance between the coils; and the transmitter power (P_z) (Figs. 5.4, 5.7), receiver power (P_o) (Figs. 5.5, 5.8), power transfer efficiency (η) (Figs. 5.6, 5.9) were presented on this basis.

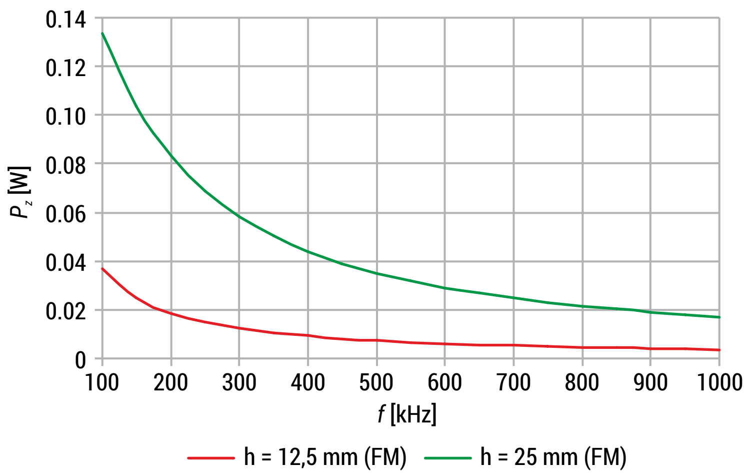

FIGURE 5.4. Results of transmitter power (*Pz*) dependent on the distance at number of turns (*n* = 90)

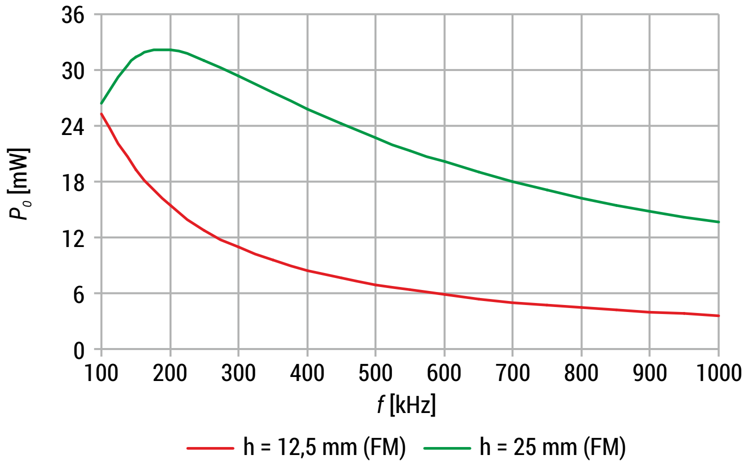

FIGURE 5.5. Results of receiver power (*Po*) dependent on the distance at number of turns (*n* = 90)

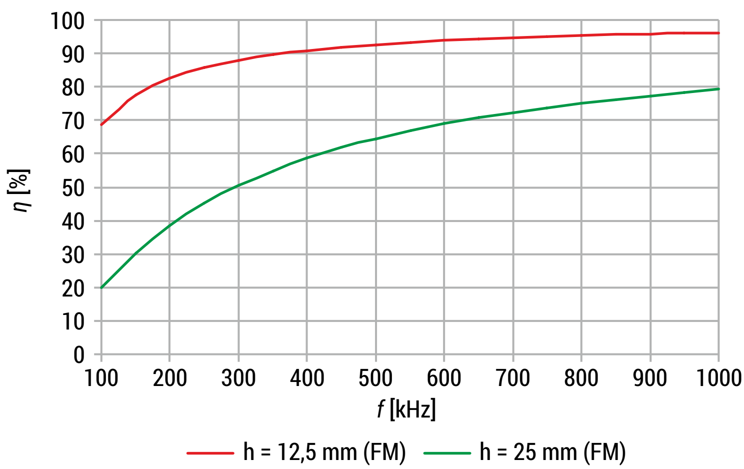

FIGURE 5.6. Results of power transfer efficiency dependent on the distance at number of turns (*n* = 90)

The transmitter power P_z decreases over the entire frequency range, regardless of the number of turns n and the distance h (Figs. 5.4, 5.7). The power P_z is higher at the distance h = r = 25 mm than at the distance h = r/2 = 12.5 mm.

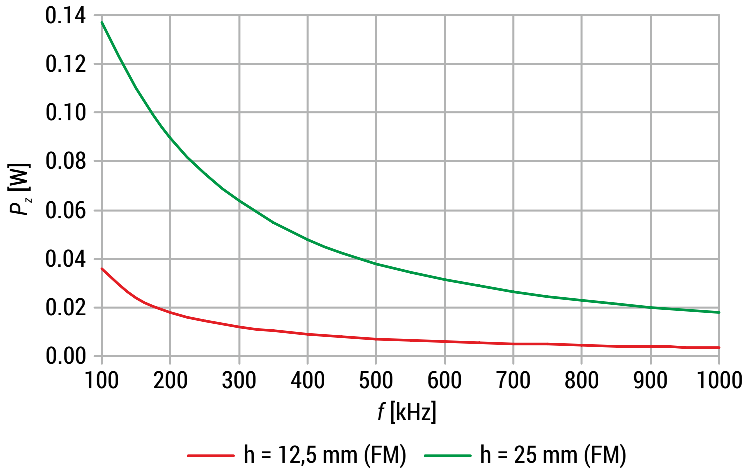

FIGURE 5.7. Results of transmitter power (P_z) dependent on the distance at number of turns (n = 100)

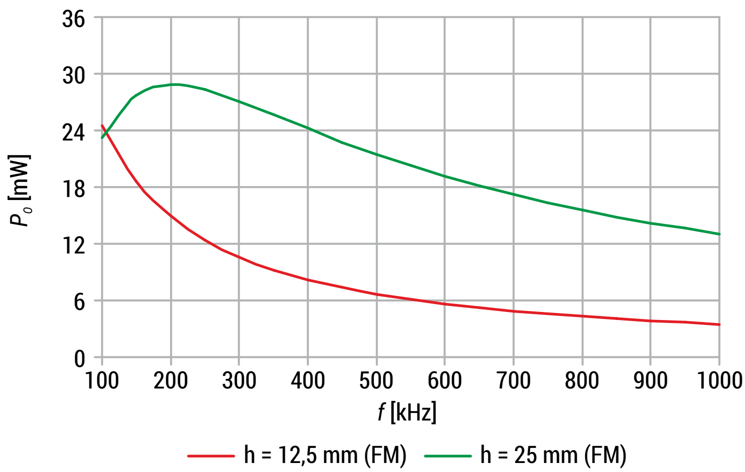

FIGURE 5.8. Results of receiver power (P_o) dependent on the distance at number of turns (n = 100)

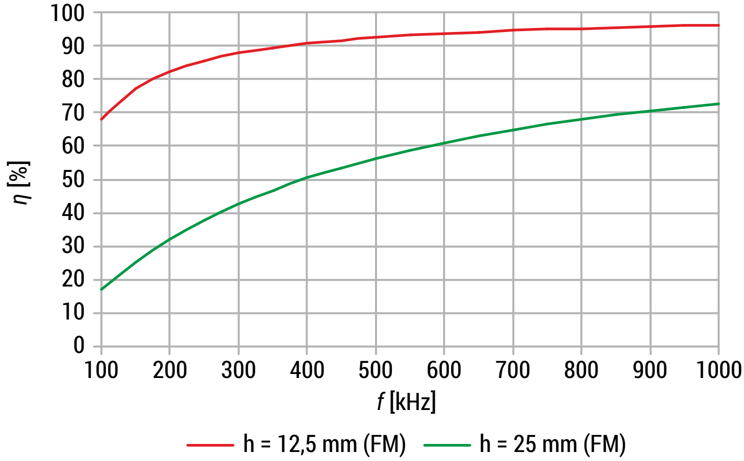

FIGURE 5.9. Results of power transfer efficiency dependent on the distance at number of turns (n = 100)

At the distance h = 12.5 mm the receiver power P_o decreases with increasing frequency. At the distance h = 25 mm the power P_o increases first, and decreases with increasing frequency. By increasing the number of turns, the receiver power will fall. The shape of the receiver power characteristics for both numbers of turns is very similar. As the frequency increases, the efficiency of the system increases and reaches almost 95%. Doubling the distance between the coils reduces the efficiency by up to 50%.

Conclusions

The article presents the numerical approach of solving models of periodic WPT. The maximum power transfer efficiency in periodic WPT systems was analyzed based on exemplary structures with many magnetic couplings between constituent inductors. A different number of turns and distances were taken into account.

The numerical model of the WPT system is an alternative to experimental research. It allows for quick calculations of the efficiency of the WPT system with different geometry of coils. The proposed numerical model makes it possible to estimate the influence of the construction of the coil system and the coil itself on the efficiency of power transmission. By regulating the number of turns and increasing the frequency of the current, it was possible to obtain high power transmission for the loads supplied using the proposed system, without the use of intermediate coils. By proper selection of load impedance, it was possible to determine the power transferred to the receiver and the corresponding efficiency.

Doubling the distance between the coils reduces efficiency by up to 50%. However, increasing the number of turns by 10 results in an increase in the efficiency of the system by almost 5%.

Streszczenie: W artykule przedstawiono analizę wydajności układu periodycznego WPT (Wireless Power Transfer). Do analizy wykorzystano cewki okrągłe. Również uwzględniono zmienność liczby zwojów oraz odległość między cewkami (nadawczą a odbiorczą). Analizowano wpływ zmiennej geometrii układu oraz częstotliwości na sprawność układu. Do analizy wykorzystano metodę elementów skończonych (FEM) z zastosowaniem periodycznych warunków brzegowych. Na podstawie uzyskanych wyników sprawdzono, przy jakich parametrach układu możliwy jest bezprzewodowy transfer energii. (Sprawność bezprzewodowego przesyłu energii w periodycznych układach złożonych z cewek okrągłych).

Słowa kluczowe: bezprzewodowa transmisja energii (WPT), numeryczna analiza, pole magnetyczne, FEM.

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