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4. INFLUENCE OF ORTHOGONAL METHODS ON DESIGN PROCESS OF VIBRATION CONTROL SYSTEM FOR CANTILEVER BEAM WITH NON-COLLOCATED PIEZO-ELEMENTS

To design vibration control system for flexible structures, their mathematical model should be reduced. In this paper, we consider the influence of the model reduction on the dynamics of the real closed-loop system. A simple cantilever beam is the object of consideration since we are able to formulate the exact analytical model of such structure. As a result of reduction, the model with low frequency resonances is usually separated from the high frequency dynamics because high frequency part of the model is naturally strong damped. In order to estimate dynamical system for control purposes in the paper, we applied a few orthogonal methods such as: modal and Schur decompositions. As it is shown, all methods well calculate resonances frequencies but generate different anti-resonances frequencies. In the vibration control systems, the anti-resonances play essential role. They influence the stability and dynamics of the closed-loop systems. The controllers designed for different reduced models were applied to real full plant. Dynamics behavior of the closed-loop systems with such controllers were analyzed and compared. The theoretical considerations were confirmed by experimental investigations. In conclusion, we should carefully choose model reduction methods in the design process of the vibration control system.

4.1. INTRODUCTION

Increase of the dimensions and reduction of the masses of the mechanical structures lead to their higher flexibility and compliance to internal and external excitations. Therefore, the active vibration control systems of flexible structures gain more and more popularity. The reduction of structure mathematical models is an important problem when we want to design the active vibration control system. As we know, a model reduction simplify

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procedure of design control system but in many cases can amplify many problems like *spillover* effect [1], research for optimal distribution of sensors and actuators [2], influence of non-collocation of sensor/actuator [3, 4], or looking for unknown damping components of the structure [5].

Some of aforementioned problems are solved by using Finite Element Method which allows to predict dynamics of the designed structure. Such solutions are used in the papers [5, 6, 7] where vibrations of flexible rotors, trusses or plates are described by the mass and stiffness matrices. This way obtained numerical model (with many eigenvectors and eigenvalues) is very helpful to describe the dynamic behavior of the investigated structures, but from control strategy point of view, it is too complicated to design a proper control law. Thus, in order to simplify the control system design process the model with large number of degree of freedoms (DOF's) should be reduced by well-known orthogonal methods [6, 7].

The modal decomposition is a classical method for the linear model reduction. According to this method, the eigenvectors and eigenvalues are used to separate the dynamics description of the particular modes and modes are divided into controlled and non-controlled ones. Apart from aforementioned modal method, often to describe the vibration of structure especially beam and bar structures, the Rayleigh-Ritz method [8] or Schur decomposition [7] are used. In all reduction methods, the eigenvalue problem of consider structure is solved to determine an approximation functions of the mode shapes and natural frequencies based on assumed boundary and initial conditions.

The influence of the model reduction on the dynamics of real closed-loop system is considered in the paper. For beam with non-collocated piezo-elements (sensor, actuator), the model is determined in process of identification procedure and next compared with others reduced order models obtained by using aforementioned orthogonal methods. Then, each obtained model is transformed to the partial-fraction form which then allows the calculation of the sum of static residuum R_0 of the model. In case of modal decomposition, the obtained model according to [6] can be expressed as

$$G(s) = \sum_{i=1}^{N} \frac{\varphi_i(k)\varphi(l)}{m_i(s^2 + 2\alpha_i s + \alpha_i{}^2 + \omega_i{}^2)} + R_0, \tag{4.1a}$$

$$G(s) = \sum_{i=1}^{N} \frac{1}{m_i} \frac{R_{i1}}{(s + \alpha_i + j\omega_i)} + \sum_{i=1}^{N} \frac{1}{m_i} \frac{R_{i2}}{(s + \alpha_i - j\omega_i)} + R_0, \tag{4.1b}$$

where $\varphi_i(k)$, $\varphi_i(l)$ – the modal amplitudes at the actuator (k) and sensor (l) locations, m_i - the i-th modal mass, $j\omega_i$ - the imaginary parts of the poles of the transfer function, α_i - the real parts of the poles, N - considered amount of mode shapes, R_{i1}, R_{i2} – residues of the transfer function for i-th mode shape.

Then, the residues of the model for each considered mode shape can be express as

$$R_{i1} = \frac{j\varphi_i(k)\varphi_i(l)}{2\omega_i}, \quad R_{i2} = \frac{-j\varphi_i(k)\varphi_i(l)}{2\omega_i}, \tag{4.2}$$

Taking into account eq. (4.1) and eq. (4.2), we can consider the most favorable indicator during design control law is a value of static residuum R_0 since

$$R_{11} + R_{12} + R_{21} + R_{22} + R_{31} + R_{32} + \cdots = 0, \tag{4.3}$$

and for $N \to \infty$ we have $R_0 \to 0$.

In order to check how this value influence the control system, the obtained models (estimated and reduced order ones) are investigated. With the help of root-locus method, the boundary gain of feedback loop is determined for each model and next verified during experimental investigations.

4.2. THE BEAM AS A CONTROL PLANT

The cantilever beam with non-collocated piezo-elements located on opposite sites of the beam, as shown in Fig. 4.1, is an object of the considerations. The piezo-stripes used during investigations work as an actuator and a sensor respectively. From strategy control point of view, the considered beam represents non-collocated system because the control and measurement places are different. The main disadvantage of such system is that the control system can lead to their unstable behavior. Therefore, in order to counteract such behavior, the control law should be carefully designed.

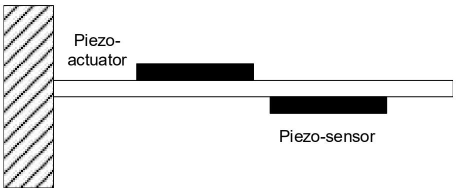


Fig. 4.1. The steel beam with the piezo-stripes as a actuator and a sensor:
scheme (left), photo (right)

4.2.1. IDENTIFICATION OF THE CONTROL PLANT

The determination of the mathematical model of the smart beam with the use of identification procedure [9, 10] was the first point of investigation. To do this, the beam as a control plant is excited by signal generated from Digital Signal Analyzer (DSA) chirp signal in form of $u(t)$=5sin(ωt) in the selected frequency range 10 - 410 Hz. Next, such obtained signal is amplified by the bipolar amplifier SVRbip3/150 and applied to the piezo-actuator. At the same time, the amplitude of vibrations of the beam are measured by piezo-sensor and transformed to voltage by piezo-charge amplifier Kistler 5018A1000. As a result, the frequency response function of the beam is achieved and recorded by DSA (see Fig. 4.2b). The photo of test rig during identification procedure is shown in Fig. 4.2a.

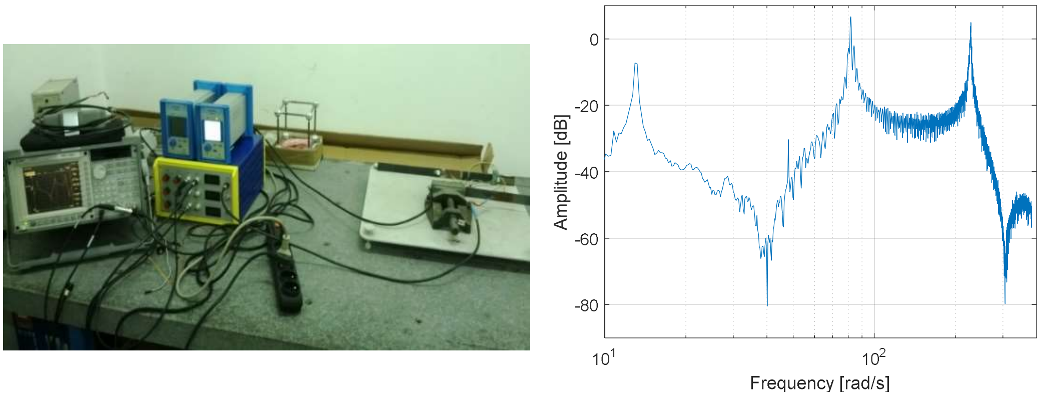


Fig. 4.2. The photo of test rig during identification procedurę (left), the frequency response function of the smart beam in the selected frequency range 10-410 Hz (right)

Taking into account Fig. 4.2b, we can see that the lowest three natural frequencies of the plant are $f_1 = 13.6$ Hz, $f_2 = 85$ Hz, $f_3 = 237$ Hz and the lowest two anti-resonances frequencies are $f_{1A} = 41.1$ Hz and $f_{1A} = 321$ Hz which will be a base for further investigations. In order to determine the mathematical model of this beam, the estimated model with different equation orders is tested. For each case, the good fitting of estimated model to experimental data was an indicator chosen to find the best order model. As it can be seen in Fig. 4.4, the best results are achieved for model with order $p = 60$. Of course from strategy control point of view, the assumed order is too high. So on further calculations, the order of this model is maximal reduced to $p = 21$ with the use of the well-known balance method. The obtained reduced order estimated model still has a good convergence between model and experimental data (Fig. 4.3b).

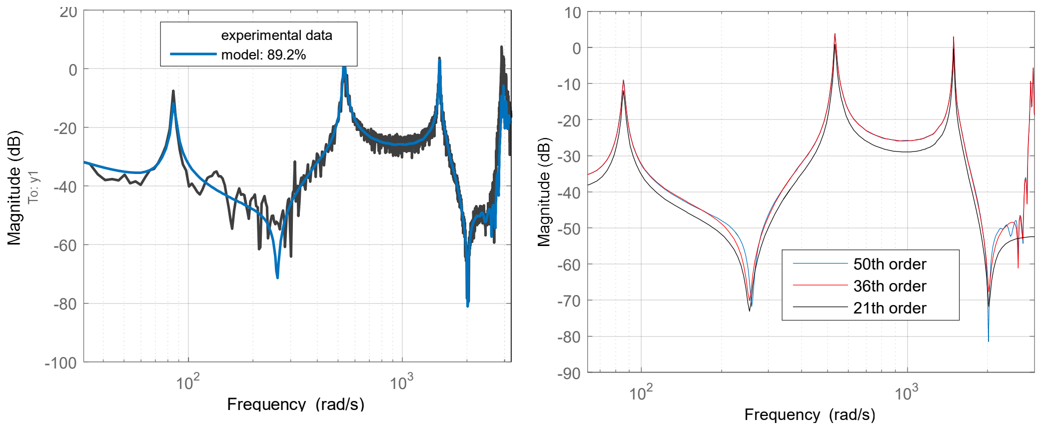


Fig. 4.3. The comparison of amplitude plot of:
experimental data and the estimated model (left), estimated reduced order models (right)

The finally estimated model is described in partial-fraction that sum of residues equals -0.0002. For this model, with help of root locus method the Evans plot are determined and shown in Fig. 4.4. Taking into account obtained plots, we can see four zeros of this model located on right half-plane which are due to two lowest frequency anti-resonances. Such location of these zeros causes that consider system with proportional control law may be unstable especially in low frequency range. Thus, in order to ensure stability of control system for the estimated model, the boundary gain of the controller should not cross over a value of 35.2.

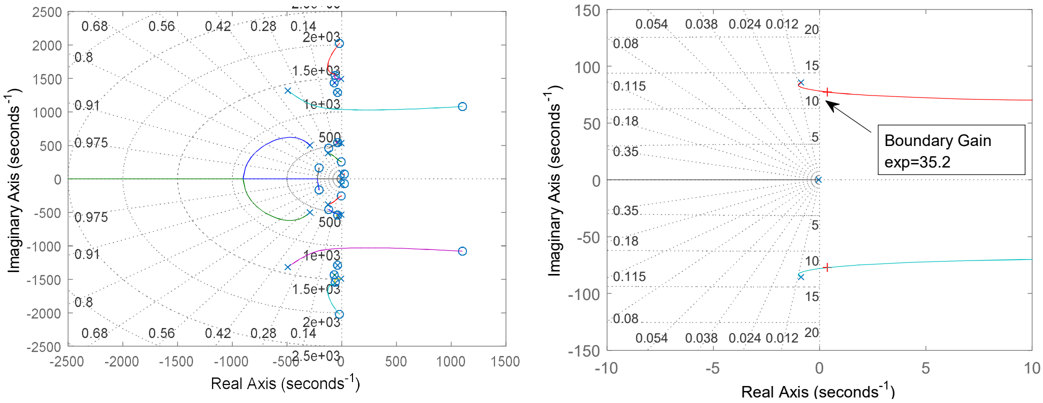


Fig. 4.4. The Evans plot for the estimated model obtained from experimental identification

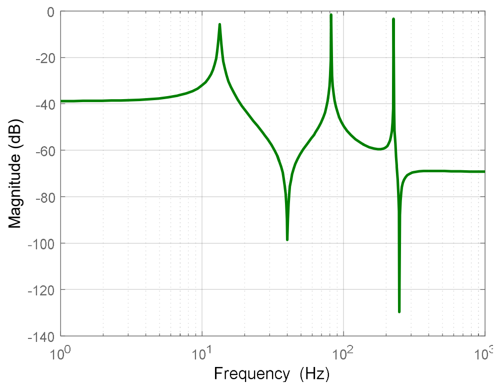
4.3. INFLUENCE REDUCTION ORDER METHODS TO VALUE OF BOUNDARY GAIN

Further investigations need to check how the choice of reduction order methods influence the value of boundary gain in case of considerations of damped system. As a first method, the modal numerical decomposition method is chosen. With the help of this method, the reduced order model is determined in the frequency range of 10 - 410 Hz that contains only the first three lowest frequency resonances. Taking into account equation (4.1) and modal amplitudes of piezo-actuator and piezo-sensor to consider mode shapes, the model of the beam can be written as

$$\begin{aligned} G_{MODAL}(s) = {} & \frac{0.0782j}{s-(-0.1+1419.4j)} + \frac{-0.0782j}{s-(-0.1-1419.4j)} \\ & + \frac{0.3433j}{s-(-0.4+513.5j)} + \frac{-0.3433j}{s-(-0.4-513.5j)} \\ & + + \frac{0.4407j}{s-(-0.9-83.8j)} + \frac{-0.4407j}{s-(-0.9+83.8j)} - 3.3931\cdot 10^{-4}. \end{aligned} \tag{4.4}$$

As it can be seen, the static residue of this model equals $-3.39\cdot 10^{-4}$. It is a value close to zero, so in the next step, there is a need to check how this value influence the value of boundary gain of feedback loop. Again with the help of Evans plot, a value of this gain is estimated and results are shown in Fig. 4.5b.

As we can see in Fig. 4.5b, the obtained feedback gain is 46.5 and it is a value far from the gain obtained for estimated model. Analyses of magnitude plot (see Fig. 4.5a) also shows small differences in comparison to amplitude plot from Fig. 4.3b. Especially, it is visible in vicinity of third anti-resonance frequency that it is closer to the third natural frequency than it is in case of plots obtained from experimental identification.



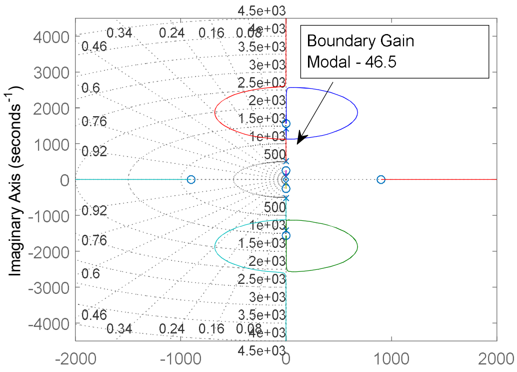


Fig. 4.5. The plots of reduced-order model obtained from modal numerical decomposition: amplitude plot (left), Evans plot (right)

4.4. MODAL ANALYTICAL MODEL

Modal analytical approach is the next method considered in the paper. The state space model of the beam with non-collocated piezo-stripes is once again used as in paper [11] with such difference that the values of particular damping coefficients for the first three lowest natural frequencies are considered in the state matrix $\boldsymbol{A}$. As a result, the damped model of smart beam can be written as

$$\begin{gathered}\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\varepsilon(U_s), \\ U_{sensor}(t) = \boldsymbol{C}\boldsymbol{x}(t),\end{gathered} \tag{4.5}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -{\omega_1}^2 & 0 & 0 & -2\xi_1\omega_1 & 0 & 0 \\ 0 & -{\omega_2}^2 & 0 & 0 & -2\xi_2\omega_2 & 0 \\ 0 & 0 & -{\omega_3}^2 & 0 & 0 & -2\xi_3\omega_3 \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ W[-U_1(x_1) + 2\cdot U_1(x_2) - U_1(x_3)] \\ W[-U_2(x_1) + 2\cdot U_2(x_2) - U_2(x_3)] \\ W[-U_3(x_1) + 2\cdot U_3(x_2) - U_3(x_3)] \end{bmatrix},$$

$$\boldsymbol{C} = [k_u\varepsilon_1(t) \quad k_u\varepsilon_2(t) \quad k_u\varepsilon_3(t) \quad 0 \quad 0 \quad 0],$$

k_u - factor of electro-mechanical coupling in the piezo-sensor, $W = \frac{k_k a_b}{\rho_b A_b}$ - constant, k_k - coefficient of factor which depends on the type of piezo-actuator.

Again, taking into account equation (4.1), the obtained model from equation (4.5) is expressed in partial-fraction form as

$$
\begin{aligned}
G_{EXACT}(s) = {} & \frac{0.3088j}{s-(-1.6-1489.1j)} + \frac{-0.3088j}{s-(-1.6+1489.1j)} \\
& + \frac{0.3895j}{s-(-4.3-534.1j)} + \frac{-0.3895j}{s-(-4.3-534.1j)} \\
& + \frac{0.0024j}{s-(-0.2-85.5j)} + \frac{-0.0024j}{s-(-0.2-85.5j)}.
\end{aligned} \tag{4.6}
$$

Analysis of this model indicates that sum of their residue equals zero. So, taking into account this fact, we can suppose that value of feedback gain calculated for this model should approach value of 35.2. The obtained results proved that it is true, because such value is 42.2.

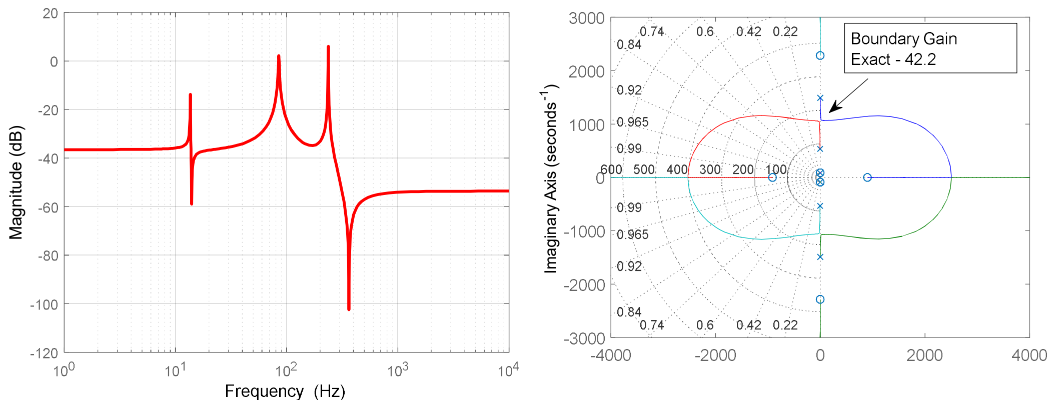


Fig. 4.6. The plots of reduced order model obtained from modal analytical decomposition: amplitude plot (left), Evans plot (right)

4.5. SCHUR DECOMPOSITION

Schur decomposition is the last orthogonal method considered in the paper. For this purpose, the state space matrix of damped model is decomposed to orthogonal matrix $\boldsymbol{U}$ which represents mode shapes and upper triangular matrix $\boldsymbol{T}$. In results of such decomposition, the order of the model is reduced according to equation (4.7) and once again express in partial-fraction form (4.8)

$$\boldsymbol{U} \cdot \boldsymbol{T} \cdot \boldsymbol{U}^{\boldsymbol{T}} = \boldsymbol{A}, \tag{4.7}$$

where: $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix}$ and $\boldsymbol{C}$ is the Rayleigh damping.

$$
\begin{aligned}
G_{SCHUR}(s) = {} & \frac{0.0195 - 0.723j}{s - (-5 + 1419.5j)} + \frac{0.0195 + 0.723j}{s - (-5 - 1419.5j)} + \frac{-0.0197 + 4.575j}{s - (-13.2 + 513.5j)} \\
& + \frac{-0.0197 - 4.575j}{s - (-13.2 - 513.5j)} + \frac{0.1382j}{s - (-0.4 + 83.8j)} \\
& + \frac{-0.1382j}{s - (-0.4 - 83.8j)}.
\end{aligned}
\tag{4.8}
$$

Similar to previous models, also in this case, the amplitude plot (Fig. 4.7a) and root-locus curves (Fig. 4.7b) are plotted. As it is shown in Fig. 4.7b, the obtained value of boundary gain is the worst, because difference between this value and the gain obtained for estimated model equals even 14.5. Such divergence in results of course may lead to instability of the whole control system during experimental tests.

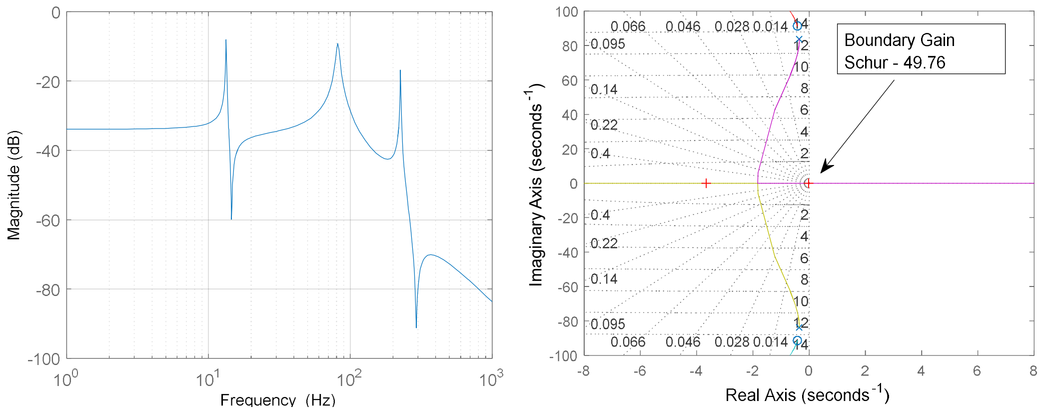


Fig. 4.7. The plots of reduced order model obtained from Schur decomposition: amplitude plot (left), Evans plot (right)

4.6. ANALYSIS OF THE CLOSED-LOOP SYSTEM WITH BOUNDARY GAIN

Next step of investigation was the analysis of designed control law in the laboratory. In this order, firstly, there is a need to prepare laboratory stand to test by proper connection of all equipment such as: DSA, DSP controller, bipolar amplifier and charge-amplifier. Next, with the help of Matlab Simulink software with inbuilt toolbox of DSP controller, there is a need to design control system (Fig. 4.8) to test this system.

Experimental investigations in the frequency domain are carried out once again using DSA that allow generation of a similar signal as it was during identification procedure. Such generated signal in the form of $u(t)=5\sin(\omega t)$ in the selected frequency range 10 – 410 Hz according to Fig. 4.9 is added to the control signal derived from proportional controller. The new signal obtained in this manner is applied to the piezo-actuator by D/A card located on board of DSP controller. Vibrations of the beam are measured by the piezo-sensors and transmitted to A/D DS2002 card and also mounted on the DPS board. The experimental amplitude plot of the closed-loop system are obtained by using two channels analyzer HP35670A. The first channel is connected to control signal with chirp signal but the second, to signal from charge amplifier. The amplitude plots as a result of the experiment are recorded and shown in Fig. 4.9.

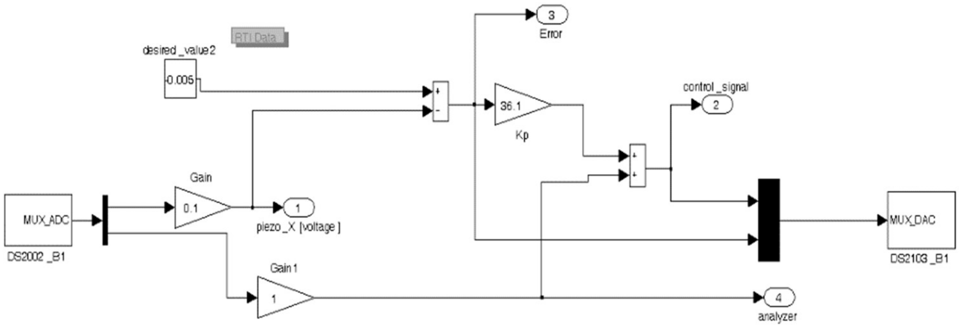


Fig. 4.8. The system designed in Simulink software with A/D and D/A cards of Dspace controller

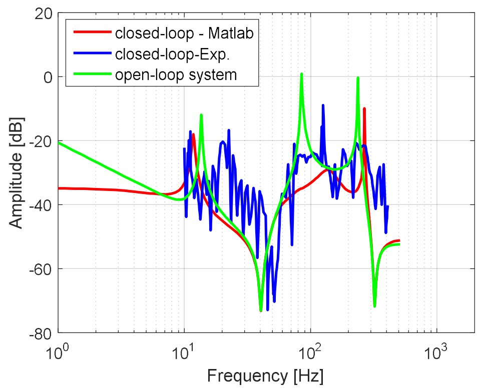


Fig. 4.9. The comparison of the amplitude plot of the open-loop system and closed-loop systems (simulation and experiment) in the selected frequency range

Analysis of the experimental amplitude plot of the closed-loop system with simulation results indicate that there exist a good fitting between this plots. Especially, it is visible in vicinity of the first peak of natural frequency, where amplitudes of both systems are similar and also their values are the same. Additionally, comparison of these results with amplitude plot of the open-loop system show us that amplitude of all natural frequencies of the considered closed loop systems are lower, about 8 - 10 dB in comparison to amplitude of peaks of the open-loop system. It means, that in the system there is an occurrence of an additional damping which damp the vibration of the considered smart beam.

4.7. CONCLUSIONS

The modern and real mechanical structures used in many applications are structures described by multi-degree of freedom mathematical models. A large number of DOF's caused that their control is a very difficult task and requires determination of a suitable mathematical model for these structure and in some cases, also reduce order model. As it is known from literature, the choice of model reduction methods has a significant influence on the dynamics of the real closed-loop system. Thus, in order to check how orthogonal method influence the stability of the model, a simple cantilever beam with piezo-stripes actuator/sensor located in two different planes was chosen as an object of considerations.

The starting point of investigations determined estimated model of the beam from identification procedure and compared this model with others reduced order models (modal and Schur decomposition). The obtained results show that each considered reduced order methods well calculate natural frequencies, but generate different anti-resonance frequencies. Also, the obtained results (Figs. 4.5b, 4.6b, 4.7b, 4.8b) has shown, that closed-loop systems with reduced order models can be unstable in the selected frequency range, because some zeros are located on right side of Evans plots. Of course, from control point of view, such results are very dangerous. Thus, in the next step, there is a need to consider such control law that ensure stability of the whole system.

Investigations described in the paper show that a good indicator during design control law may be a sum of residue of particular orthogonal models, especially when the control system is designed using root-locus method. Then, the obtained results unequivocally indicated that the best method that gives the best result is modal analytical method. Both obtained values of the sum of residues (-3.39e-4) and boundary gain of feedback loop (42.2) are the closest to values obtained for model obtained from identification (-2.1e-4; 35.2), respectively.

Experimental investigations carried out in the lab stand confirm the above conclusion. Taking into account control scheme from Fig. 4.8, it can be seen that the set up value of gain controller ($k_p = 36.1$) during test ensures similar Bode plot as in the case of simulation analysis. For both closed-loop models (experimental and simulation), the value of first natural frequency is slightly decreasing in comparison to the first natural frequency of the open-loop system. Moreover, the amplitudes of these control systems are also decreasing, about 8 - 10 dB in comparison to the amplitude of the open-loop system especially in vicinity of resonance peaks. This proves the occurrence of additional damp in the control system.

Finally, it can concluded that analysis of the sum of residues of damped open-loop system is a good method during the design of simply control law, especially for SISO models as a cantilever beam with non-collocated piezo-elements.

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