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3. MODAL ANALYSIS OF VIBRATORY MICROGYROSCOPES

The MEMS rotational velocity sensor is a well-known inertial device that consists of two main vibrating systems: resonator (drive direction) and accelerometer (sense direction). These crucial vibratory subsystems and their fit influences performance, accuracy and measurement scope of the sensor. As kinetic energy is transferred between both vibratory directions, vibratory modes must be analyzed to obtain the optimal response of the device. As MEMS gyroscope is a complex device whose operation depends on several structural details, the crucial stage of the whole design process is CAD modeling of geometry taking into consideration desired effects, simulations of designed model and analysis of results. MEMS gyroscopes are well-known devices that find their usage and applications in many commercial, military and medical devices for measurement purposes [1, 2, 3, 4] due to low technology and fabrication price. The major disadvantage is the slow design process and development and its costs. The advantage, however, is that the final product has low power comsumption, and fast application to many PCB equipment which includes ASIC to process signal obtained from MEMS [5]. The simple output sensor signal processing makes these inertial sensors interesting in today's market.

Although MEMS gyroscope is a complex device, it consists of simple shapes that play an important role due to its influence on the performance parameters. In the case of vibratory gyroscopes, where frequency is one of the main physical quantities, vibratory adjustments decide its usage and application. Therefore, among a wide spectrum of analysis, modal analysis stands out as the main reference point for structure optimization.

The aim of this paper is to bring up modal analysis of MEMS gyroscope operation with COMSOL and Matlab/SIMULINK software. Two different geometries are assumed and simulation results are presented. The other objective of the paper is to present modal analysis results by taking into consideration spring constants and variation in the damping coefficients as an effect of temperature variation.

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3.1. INTRODUCTION

The MEMS vibratory gyroscope is a mechanical 2-degrees of freedom (DOF) spring-mass-damper system. Such devices consist of inertial mass suspended on springs that are anchored to the substrate. Depending on the type of gyroscope, all springs are anchored to the substrate or few of them only. Remaining ones are anchored to external inertial mass called inertial frame. Either internal mass or external, one is directly or indirectly connected to comb structures which are an integral part of sensor and due to its displacement, can measure the specific physical quantity for further transfer to the other one with integrated circuit support. The inertial mass is a combined electrode fingers (combs) and the movable part of gyroscope [6]. These movable fingers are extruded from all 4 sides of the mass [1, 7]. Resonator comb structures (drive direction) loaded with voltage causes vibrations, comb structures for accelerometer (sense direction) loaded with voltage, are used to detect capacitance changes in time, which in turn can be transformed in corresponding ASIC to angular velocity [8]. Some interesting geometries and simulation results can be found in papers [9, 10].

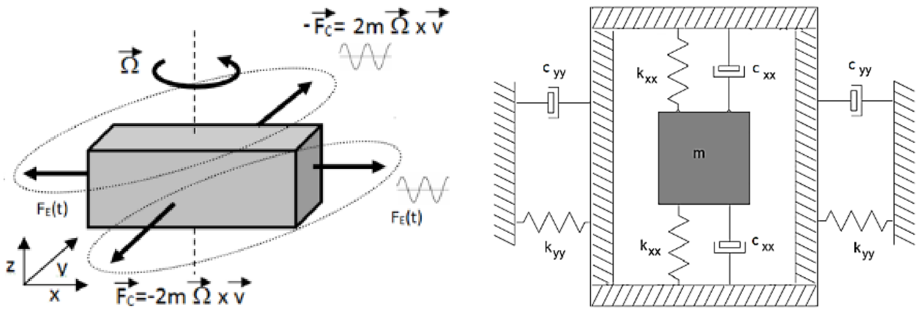

Fig. 3.1. General gyroscope and MEMS vibrating gyroscope operation principal (left) and general model of decoupled gyroscope (right)

Principle of operation of MEMS vibratory gyroscope is based on the well-known Coriolis phenomena which appears in rotating objects (Fig. 3.1) with non-zero linear velocity [4, 11]

$$F_C = -2m\left(\vec{\Omega} \times \vec{v}\right), \tag{3.1}$$

where, Ω is angular velocity, v – linear velocity, and m – mass of object. Fundamental equations governing MEMS gyroscopes can be expressed with second order differential equations

$$m_x \frac{d^2x}{dt^2} + c_x \frac{dx}{dt} + k_x x = F_D \sin(\omega t) + 2m_y \frac{dy}{dt} \Omega, \tag{3.2a}$$

$$m_y \frac{d^2y}{dt^2} + c_y \frac{dy}{dt} + k_y y = -2m_x \frac{dx}{dt} \Omega \tag{3.2b}$$

Elements $2m_y\Omega\dot{y}$ and $2m_x\Omega\dot{x}$ are related to Coriolis force components induced by rotation. Coriolis force terms are induced by dynamic coupling between resonator and accelerometer. Ignoring the crosstalk interference element of the equation $-2m_y\Omega\dot{y}$, both equations can be written in the following form

$$m_x\frac{d^2x}{dt^2}+c_x\frac{dx}{dt}+k_xx=F_D\,sin(\,\omega t), \tag{3.3a}$$

$$m_y\frac{d^2y}{dt^2}+c_y\frac{dy}{dt}+k_yy=-2m_y\frac{dx}{dt}\Omega \tag{3.3b}$$

Often we can meet these equations which include $\zeta=\frac{c}{m\omega}$, where ζ is a damping ratio,

$$\omega_x=\sqrt{\frac{k_x}{m_x}},\ \omega_y=\sqrt{\frac{k_y}{m_y}},\ Q_x=\frac{m_x\omega_x}{c_x},\ Q_y=\frac{m_y\omega_y}{c_y}, \tag{3.4}$$

Q factor describes the behaviour of the gyroscope during damping. High Q factors reflect oscillators with low damping – they vibrate longer. Q factor value of 0.5 is a threshold which changes meaningful damping during vibrations.

In many MEMS applications, it is crucial to analyze the sensitivity of a vibrating gyroscope eigenfrequencies in relation to temperature fluctuation. Based on the above considerations, MEMS vibrating gyroscope require frequency stability under changes taking place in the environment where the device operates.

Since in the case of MEMS vibratory gyroscope, mode-matching of both vibration directions is crucial, modal analysis of such device is one of the most important steps during the design.

3.2. DESIGN AND SIMULATIONS

Model of MEMS vibratory gyroscopes were prepared in COMSOL Multiphysics software and simplified models were prepared in Matlab/SIMULINK software. A model created in COMSOL reflects the real physical device shape, whereas model in Matlab/SIMULINK implements Newton's motion equations and its complexity comes directly from the need to reflect geometry details. FEM models were simulated in stationary and eigenfrequency studies.

Geometrical dimensions and physical properties are presented in Table 3.1, 3.2 and 3.3.

Eigenfrequency calculations of particular nodes were performed in two steps:

- stationary study - computes both displacement and stress,
- eigenfrequency study - stationary study step was applied to calculate natural frequencies of particular nodes.

To compute eigenfrequencies for different entry value (for example external force), parametric sweep was applied.

Two different fundamental cases of modal analysis should be considered for MEMS inertial devices: for constant, reference temperature and for various temperature.

Table 3.1. Geometrical details of gyroscope with one inertial mass

Quantity	Value
Proof mass length/height	$1000 \cdot 10^{-6}$ m
Edge spring length	$200 \cdot 10^{-6}$ m
Device thickness	$30 \cdot 10^{-6}$ m
Drive electrode count	30
Sense electrode count	30
Gap between fingers	$26.5 \cdot 10^{-6}$ m

Table 3.2. Geometrical details of gyroscope with two inertial masses

Quantity	Value
Proof mass height	$1000 \cdot 10^{-6}$ m
Proof mass length	$200 \cdot 10^{-6}$ m
Inertial frame height	$675 \cdot 10^{-6}$ m
Inertial frame thickness	$25 \cdot 10^{-6}$ m
Device thickness	$30 \cdot 10^{-6}$ m
Drive electrode count	30
Sense electrode count	30
Gap between fingers	$26.5 \cdot 10^{-6}$ m

Table 3.3. Physical properties of polysilicon

Quantity	Value
Young’s modulus	160 GPa
Poison ratio	0.22

3.3. MODAL ANALYSIS RESULTS FOR CONSTANT TEMPERATURE

In a constant temperature environment, it is assumed that the temperature has no influence on the device. It is assumed that it equals to reference temperature. Here, temperature is assumed to be 293.15 K.

Results of simulation performed in FEM software are depicted in Fig. 3.2 (for device including one common mass for both drive and sense directions) and Fig. 3.3 (for device including two inertial masses) respectively. These figures present two first modes for drive and sense directions separately, which are important in point of view vibratory gyroscope, because they are distinctly related with the particular drive and sense directions. According to the figures presented, it can be seen that the displacement vibration modes are directed towards particular motion axis. Therefore, applying forces with vectors coinciding with these directions allow the control of displacement amplitude: force controls amplitude – amplification of amplitude (taking into consideration natural frequencies). It is obvious that this considers both sensors (adjust displacement amplitude that may cause an increase in the measurement range, sensitivity and accuracy) and actuators (change displacement amplitude).

Particular natural frequencies were obtained in COMSOL with sweeping simulation use. These natural frequencies are presented later as a part of the discussion of simulation results. To ensure accurate computation of the natural frequencies, they are compared with results obtained from Matlab/SIMULINK simulations.

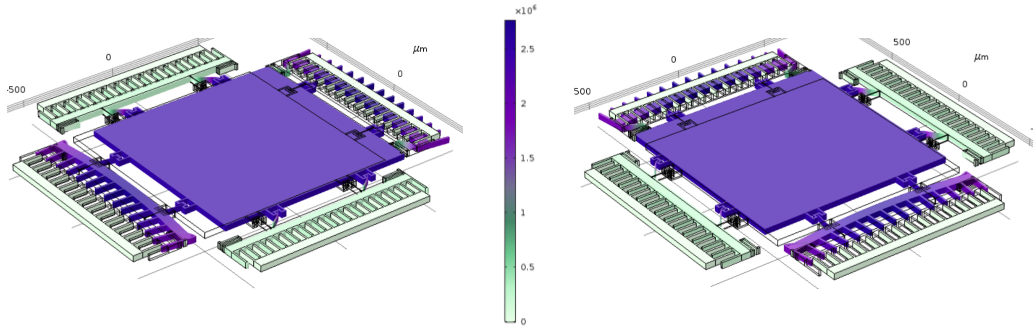

Fig. 3.2. Modal analysis of MEMS gyroscope
with one mass (left) and double (right) central springs on each side

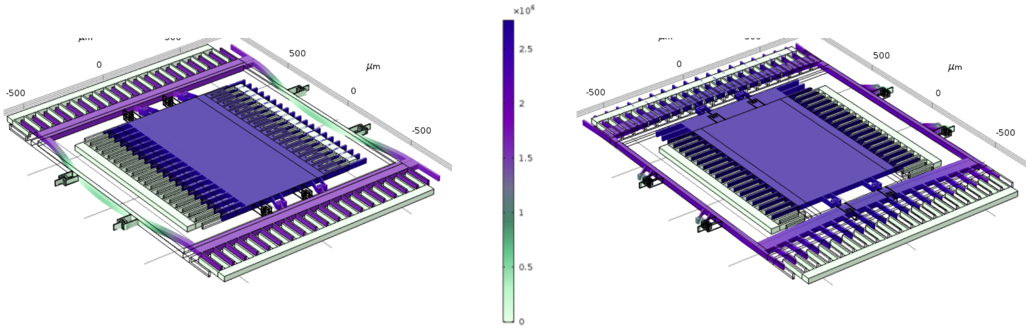

Fig. 3.3. Modal analysis of MEMS gyroscope
with inertial frame (left) and central springs (right)

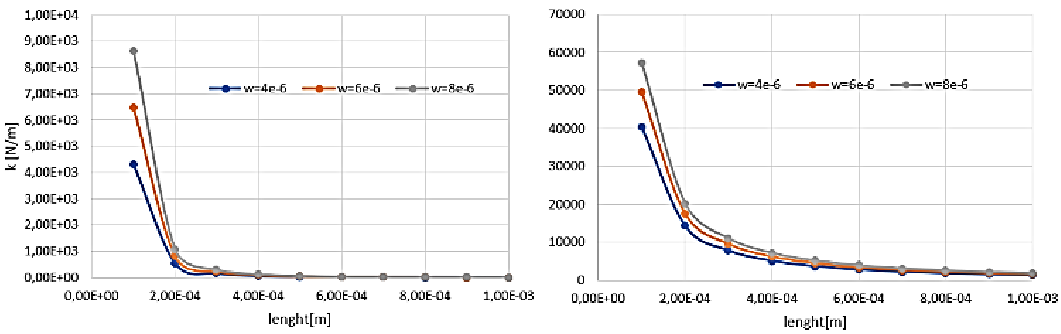

Fig. 3.4. Results of modal analysis of MEMS gyroscope
with one mass for different length of suspensions

Fig. 3.4 and 3.5 show the dependencies of eigenfrequency on the two crucial geometrical dimensions: width and length of particular suspensions. We observe, that the spring constant falls rapidly (for small spring length) and then the drop is gradual as the suspension length increases. Similarly, modal analysis shows strong dependency of eigenfrequency on the change in this dimension. In the case of increase in the width, we observe that the eigenfrequency grows, however, for shorter suspension, this dependency is stronger than for longer suspension.

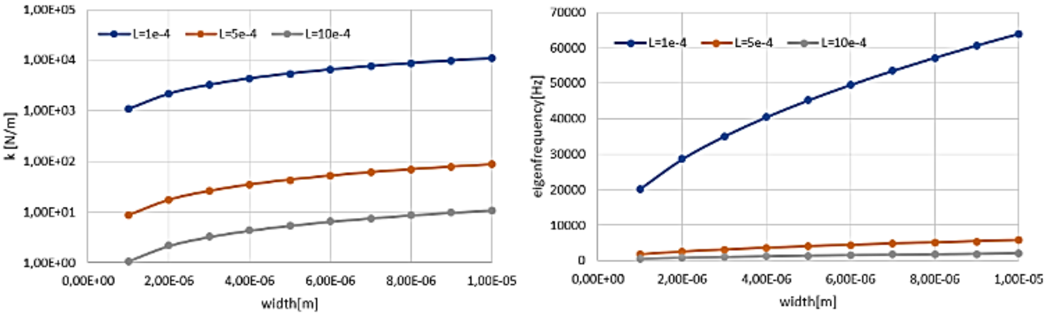

Fig. 3.5. Results of modal analysis of MEMS gyroscope
with one mass for different width of suspensions

Fig. 3.6 shows the magnitude and phase of transfer function for the first considered gyroscope (with one mass). It can be seen that this type of gyroscope is almost mode-matched - frequency difference Δf is very tiny. This difference (in case of fabricated device) is caused by inaccuracies in dimensions or fabrication imperfections. In an ideal situation - when vibrating gyroscope is mode-matched (the natural frequency of a drive axis exactly matches the natural frequency of sense axis), the primary benefit is the improvement of performance and increased sensitivity (key parameter of each microsensor - it grows in the case of drop in the frequency difference). For this kind of gyroscope, mode-matching strongly depends on the sameness of spring constants.

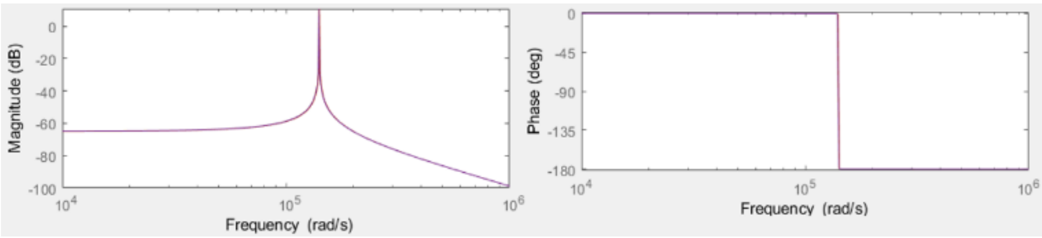

Fig. 3.6. Magnitude (left) and phase (right) of transfer function for gyroscope with one inertial mass

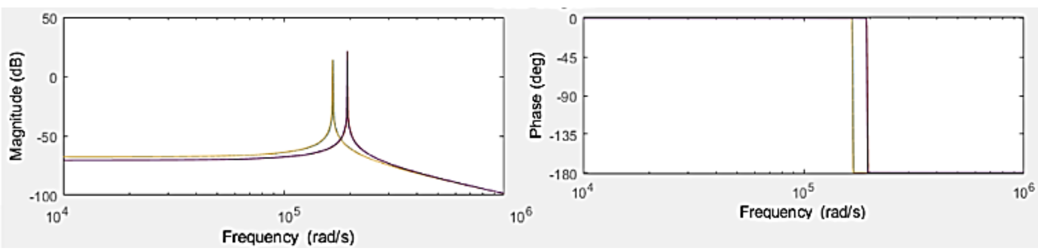

Fig. 3.7. Magnitude (left) and phase (right) of transfer function for gyroscope with two masses

Fig. 3.7 shows the magnitude and phase of the transfer function for gyroscope with two inertial masses. It is seen, that larger difference between drive and sense modes - angular frequency difference is about $0.3 \cdot 10^5$ rad/s (4777 Hz). Therefore, in the case of such structure, mode-matching is worse than in the case of single-mass configuration.

3.4. TEMPERATURE INFLUENCE ON MODAL ANALYSIS RESULTS

As the inertial sensors or actuators are mechanical structures built of solid materials, they are very susceptible to some environmental factors, which can play significant role in commercial applications. One such factors is temperature, considered by some as the crucial factor having a destructive effect on the device performance.

Basically, temperature variation has a significant influence on the material properties, including those in used modern electronics and micromechanics disciplines. These materials are thermo-sensitive in small or large extent, being potentially a source of some response to physical quantity variations, which, in turn, may be crucial in sensing or actuating process and accuracy of measurement or may be just avoided. Table 3.4 shows the thermal properties of polysilicon used in this study.

Table 3.4. Thermal properties of polysilicon

Quantity	Value
Thermal coefficient α (for ΔT=0)	$2.6 \cdot 10^{-6}$ [1/K]
Thermal coefficient of Young's modulus β (for ΔT=0)	$-80 \cdot 10^{-6}$ [1/K]

Since each solid material expands under temperature, its geometrical dimensions also change, and can be expressed by

$$l(T) = l_0(1 + \alpha\Delta T), \tag{3.5}$$

where l_0, l, α, ΔT are initial length (in T_0 temperature), length in T temperature, thermal expansion coefficient and temperature difference $T - T_0$ respectively. The next parameter which is temperature dependent is elastic modulus. When it changes, it causes variation of the gyroscope stiffness and also the resonant frequency [5, 6]

$$E(T) = E_0(1 + \alpha\Delta T), \quad k(T) = k_0(1 + \alpha\Delta T), \tag{3.6}$$

where $E(T)$, E_0 are elastic modulus for the polysilicon material at a temperature T and T_0 respectively. Similarly, k and k_0 are stiffness at temperature T and T_0 respectively. Based on gyroscope inertial mass m and stiffness k, resonant frequency ω_0 may be calculated with the following equation

$$\omega_0(T) = \sqrt{\frac{k(T)}{m}} = \sqrt{\frac{k_0(1 + \alpha\Delta T)}{m}}. \tag{3.7}$$

From the above formula, we see that the resonant frequency varies with temperature variation.

Results presented in Fig. 3.8 show how spring constant is affected by temperature variations. Spring constant was obtained in two ways, with the use of both COMSOL (FEM) and separately with SIMULINK (analytical) simulation based on the models presented here. Calculation of spring constant with FEM required model to be body loaded with specified object like Thermal Expansion, a sub-object of the Linear Elastic Material. As a result, it was observed that both modeling methods gave similar results of spring constants (for analytical calculations (one-mass gyroscope configuration): 1470 N/m, FEM simulations: 1540 N/m), however, results from FEM software are more reasonable as they provide more device accuracy and sensitiveness – and consequently better mode matching of both actuator and sensor.

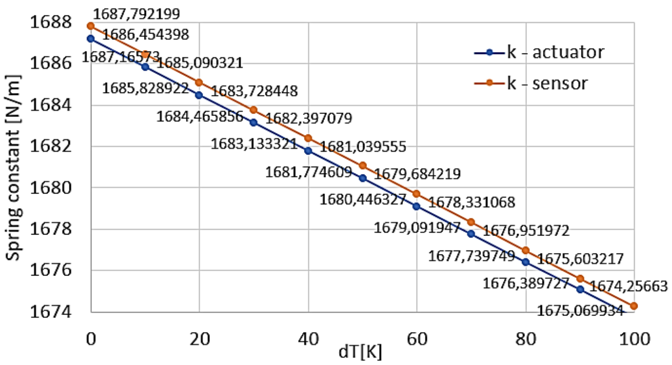

Fig. 3.8. Spring constant dependency on temperature for MEMS actuator and sensor in vibratory gyroscope with one common central mass

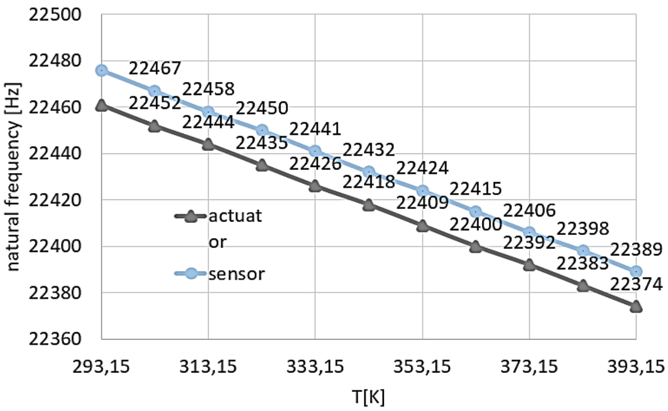

Fig. 3.9. Natural frequency dependency on temperature for MEMS actuator and sensor in vibratory gyroscope with one common central mass

Results of the sweeping simulations in terms of natural frequency dependency on temperature are shown in Fig. 3.9 and 3.10. The model also took into consideration the Young's modulus temperature dependency. For one mass decoupled MEMS device, results are presented in Fig. 3.9. It can be seen that natural frequency depends linearly on temperature, however, frequency values for particular temperatures drop rapidly with temperature - almost 100 Hz with ΔT =100 K.

For MEMS structure with inertial frame including edge serpentine suspensions (Fig. 3.10), the results meaningfully differ. In case of actuator, natural frequency drops with temperature, whereas in the case of sensor, it increases. In contrast to structure with one mass – the natural frequencies for resonator and sensor are nonlinear.

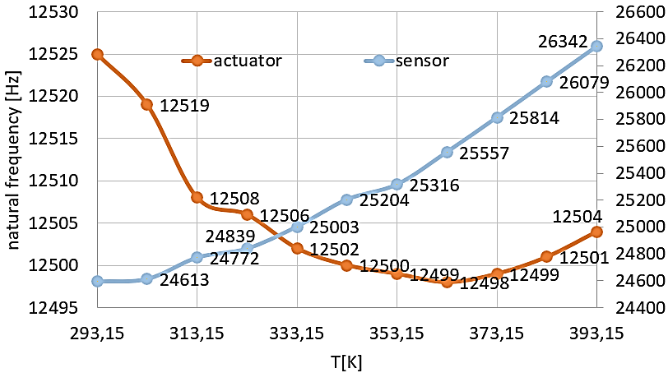

Fig. 3.10. Natural frequency dependency on temperature for MEMS actuator and sensor in vibratory gyroscope with inertial

3.5. CONCLUSIONS

Results of modal analysis are very important part of MEMS design process. First of all, it allows the detection (and fit) of frequency range of the device to operate. Secondly, it allows the matching of both drive and sense modes to obtain the maximum amplitude for sense direction (recall that this amplitude is 1000 less than for drive direction) and the resonance effect that is advisable to use. Results show that through manipulation of dimensions and geometry choice, we can decrease or increase eigenfrequency, however, the option is to manipulate suspension dimensions as the numerator of the equation (3.4) changes more than the denominator (mass of suspension changes slightly in comparison to inertial mass).

Results of simulations performed for different temperatures further show that the particular modes are temperature sensitive and have influence on natural frequency values. According to the principle of operation of vibratory gyroscope, such temperature variation will degrade the performance of the device.

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