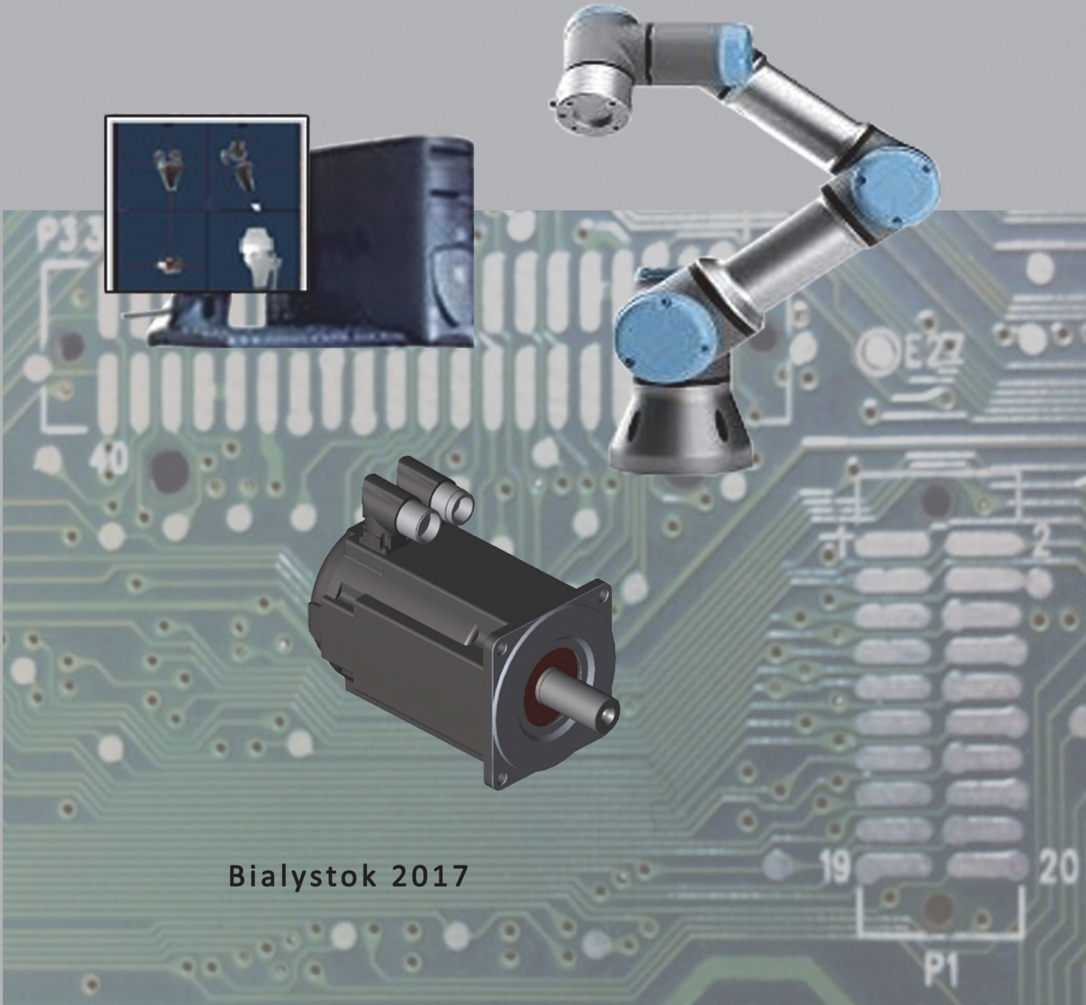
Kanstantsin Miatliuk

Conceptual Design of Mechatronic Systems



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CONTENT

Conventional signs and notions

Abbreviations and notions:

MS – mechatronic system,
MO – mechatronic object,
MP – mechatronic process,
CD – conceptual design,
DD – detailed design,
CS – conceptual studies,
PC – personal computer,
GO – geometric object,
CAD – computer aided design,
CAE – computer aided engineering,
CIM – computer integrated manufacturing,
KBE – knowledge based engineering,
MDE – model-driven engineering,
FBS – function-behavior-structure,
FCBS – function-cell-behaviour-structure,
FKC – functional knowledge cell,
B-R – boundary representation,
CSG – constructive solid geometry,
ACS – automated computer system,
HS – hierarchical system,
DE – differential equations,
CG – computational geometry,
SRS – surgical robot system,
TKA – total knee arthoplasty,
MCM – manhole cutting machine,
PCB – printed electronic circuit board,
Coordination – systems design and control processes,
Aed – formal analogous of two-level system with dynamic objects,

Sings and variables:

X – input,

Y – output,

S^{ℓ} – system of level *l*,

${}_{o}S^{\ell}$ – object of level *l*,

${}_{\pi}S^{\ell}$ – process of system S^{ℓ} of level *l*,

${}_{\varepsilon}S^{\ell}$ – environment of system S^{ℓ},

σ^{ℓ} – structure of system S^{ℓ},

S_0^{ℓ} – coordinator of system S^{ℓ},

ω^{ℓ} – aggregated dynamic presentation of system S^{ℓ},

ρ^{ℓ} – reaction of system S^{ℓ},

φ^{ℓ} – function of states transition of system S^{ℓ},

L^{S} – numerical positional system,

λ – output function of coordinator (coordination strategy),

tt' – time interval [*t*, *t'*],

γ – coordination signal and sub-systems connections,

w – feedback signal.

PREFACE

This book presents the systemic model and coordination technology of hierarchical systems for conceptual design of mechatronic and other engineering objects. The conceptual model creation of the mechatronic system (MS) being designed is the actual task which is performed in the frames of automation and robotics, mechatronics, engineering design, computer integrated manufacturing (CIM), computer aided design (CAD) and other related subject fields. Conceptual model of the designed object is usually created before generating concrete mathematical models necessary for design tasks performing at the detailed design phase of the object life cycle [1].

Among the widespread models and methods which are usually used at the conceptual and detailed design phases are the following ones. *Models of classical mathematics* – discrete and continuous – successfully used to model the dynamics or structure of the object being designed, but do not solve the basic problem of design: do not link the structure of the object and its function. *Models of artificial intelligence* are most commonly used when describing the structure of the designed object and design knowledge representation. Solving some specific design problems, these models remain one-level at its core and do not take into account the dynamics of the object being designed and therefore do not perform the general design task in one common theoretical basis. For the case of *logical-dynamical system*s, the dynamical elements of these systems are connected by logical systems in a higher-level structure, thus forming a higher-level system. Such systems are most useful in the design, but they take into account two levels of the designed objects only. *Hybrid systems* use the models of classic mathematics or artificial intelligence in that cases when they work most sufficiently, i.e. use the models of classic mathematics for description of systems dynamics or models of artificial intelligence for structure description of the object being designed. But it is impossible to represent both models of classic mathematics and artificial intelligence in the common formal basis. It complicates the performance of the general design tasks. Other methods and models are described in Chapter 1.

In the book presented, the conceptual formal model of mechatronic systems is created using construction and technology of hierarchical systems (HS) [2] with their standard element *aed* by S.Novikava and K.Miatluk [3-8], dynamic systems by M. Mesarovic and Y.Takahara [9, 10], numeric positional systems and hierarchical geometry [7, 11, 52]. This conceptual model allows the connected formal descriptions of a mechatronic system structure, its aggregated dynamic representation as a unit in its environment, the system environment, its process and system-environment interactions; the system coordinator and its coordination, i.e. design and control, processes. Besides, the conceptual model takes into account the connected descriptions of mechatronic subsystems of different nature, i.e. mechanical, electronic, electromechanical, and computer. Availability of HS coordinator allows the realization of inter-level connections of mechatronic system being designed.

In this manuscript, the objects under consideration are designed and controlled mecharonic systems described in the theoretical basis of hierarchical systems (HS). The design and control process of mecharonic system is realized by HS coordinator. Mechatronic system (3.1) can be presented according to HS model in forms of mechatronic object (3.2) and process (3.3) being coordinated, i.e. designed and controlled (see Section 3.1). Therefore, *mechatronic system* (MS) in this book can be also called a *mechatronic object* (MO) or *mechatronic process* (MP) depending on the way of its consideration.

At the beginning, an overview of modern approaches and methods of conceptual design and engineering design methods are presented in this book in Introduction – Chapter 1. An overview of wide-spread geometric models which are used in CAD systems is given in Chapter 2. The formal basis of the conceptual model developed with the help of hierarchical systems, dynamic systems, and numeric positional systems is presented in Chapters 3. First, the standard block of hierarchical systems – *aed* (ancient Greek word) – a formal analogue of a two-level system [3,4] is described in Chapter 3. Formal modes of mechatronic object being designed, its environment and their processes are described using dynamic systems [9,10]. The main element – coordinator – which performs design and control tasks on its strata is formally presented using its canonic model. Metrical characteristics of hierarchical mechatronic systems including numeric positional system are given after that. Chapter 4 presents the description of the proposed design method and hierarchical geometric representations of mechatronic objects being designed. The exemplary tasks of the conceptual design of methatronic objects are presented in Chapters 5-9. Among the tasks there are biomechatronic surgical robot system (SRS) design, conceptual design of dinosaur Bioloid robot, human motion design, design and testing of electronic printed circuit boards (PCB), technological manhole cutting machine (MCM) design and control. Conclusive remarks are finally presented in the book.

1. INTRODUCTION – OVERVIEW OF DESIGN METHODS AND MODELS

Conceptual model creation of an engineering, e.g. mechatronic system being designed is the actual task for automotive control, robotics and industrial production systems which usually operate with modern CAD/CIM systems [1, 12-15]**.** Nowadays, there are a lot of definitions of Conceptual Design (CD) and corresponding methods and models which are used at the conceptual design phase. The following definitions: “conceptual design or what some call ‘ideation’ defines the general description of the product” given by Paul Brown, “the early part of any design process, which can occur at any point in the product development cycle” given by Bob McNeel, and “conceptual design is about possibilities” given by Fielder Hiss are analyzed by Hudspeth [16]. Hudspeth considers conceptual design to be more about what a product might be or do, how it would meet the expectations of the manufacturer and the customer. M.J. French defines the conceptual design as the phase of the design process where the statement of the problem and generation of broad solution to it in the form of schemes is performed [17]. In any case before giving a definition to Conceptual Design and Conceptual Model of an engineering (e.g. mechatronic) object being designed it seems to be reasonable to define the place of Conceptual Design in general design process and object’s life cycle. To make such a definition, the general design scheme is presented by French [17] in the form of a block diagram given in Figure 1.1.

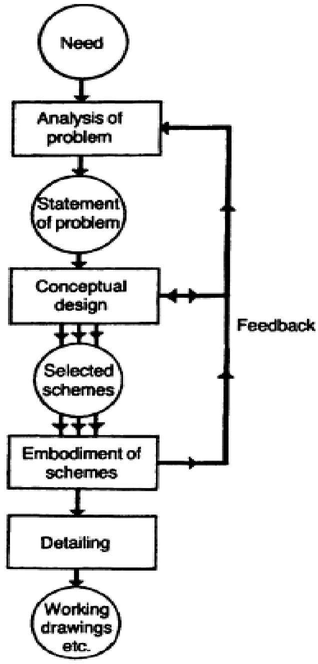


Fig. 1.1. Block diagram of the design process [17]

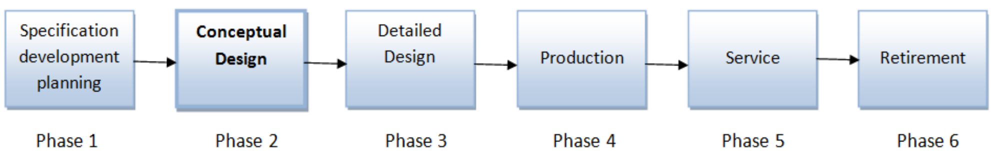


Fig. 1.2. Mechatronic object life cycle [1]

Here circles represent phases and rectangles represent work which is not completed. In this diagram things that are important but outside of the scope of the work, have been omitted by French. For examples, interactions with such activities as research and development, inputs of information, etc., were not under consideration. The 'evaluation' box was also omitted in the scheme because French believes it should be going on continuously in all the rectangles of the diagram [17]. According to Ullman [1], in the object life cycle (Fig. 1.2) the Conceptual Design phase is just before the phase of the Detailed Design (DD) where the object's concrete mathematical model is created and numeric calculations are performed. Another similar scheme which contains both conceptual and detailed design phases in the total design core is presented by P. Childs in [13], see Figure 1.3.

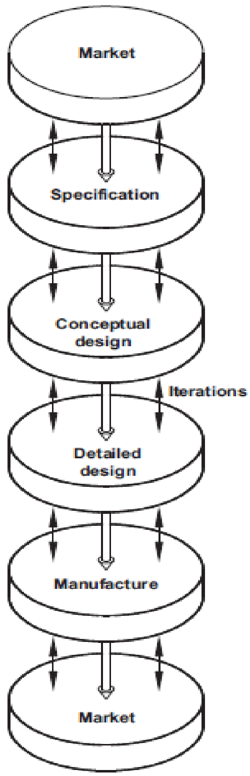


Fig. 1.3. The total design core [13]

The main design tasks performed at both CD and DD phases (Fig. 1.3) are synthesis and analysis tasks. The places of these tasks in the general design process are presented by the scheme in Figure 1.4 given in [13]. Synthesis is defined by Childs as the process of combining the ideas developed into a form or concept, which offers a potential solution to the design requirement. Analysis task is recognized as involvement of the application of engineering science. In this case, subjects are explored extensively in traditional engineering courses, such as statics and dynamics, mechanics of materials, fluid flow and heat transfer. These engineering 'tools' and techniques are used in analysis tasks to examine the design to give quantitative information such as whether it is strong enough or will operate at an acceptable temperature [13]. Analysis and synthesis invariably go together.

In the frames of the conceptual design model presented in this book and described below in *aed* formal basis of HS (see Chapters 3-4), the synthesis and analysis tasks are also performed together and defined as coordinator tasks of creating (*synthesis*) and changing (after *analysis*) mechatronic system construction and technology by selecting units of lower levels and settling their interactions to make the state and activity of the system on higher levels (environment) best coordinated with environmental aims.

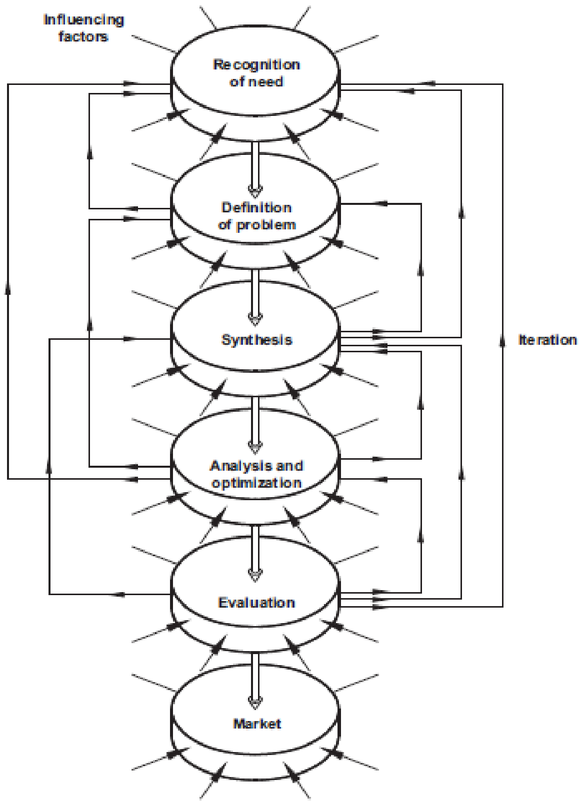


Fig. 1.4. Synthesis and analysis design steps in the general design process [13]

Another definition of the *conceptual design* process was given in the Overview section of the work titled “Analyzing Requirements and Defining Microsoft.net Solution Architectures” [18] where MSF (Microsoft Solutions Framework) process model was presented. It was pointed out here that the planning phase of the Model involves three design processes: conceptual, logical, and physical. The conceptual design starts during the envisioning phase of the MSF process model and continues throughout the planning phase. Because the MSF design process is evolutionary as well as iterative, conceptual design serves as the foundation for both logical and physical design. The following three steps of conceptual design and associated baselines are formulated. Conceptual design is an iterative process and the steps are repeated as required: 1) research, 2) analysis, 3) optimization. The optimization baseline leads to the baseline of the conceptual design. Correspondent Figure 1.5 illustrates these steps in the conceptual design.

During the research step of conceptual design, the team gathers more information to refine and validate data collected during the envisioning phase. Typically, the information gathered during the envisioning phase is high level and lacking in detail. During the first step of the conceptual design, the design team needs to collect detailed information. For example, the team first identifies questions raised by the first iteration of information gathering; the team then continues to clarify the tasks, business processes, and workflow. As greater detail is discovered, the results are incorporated in the use cases and draft requirements.

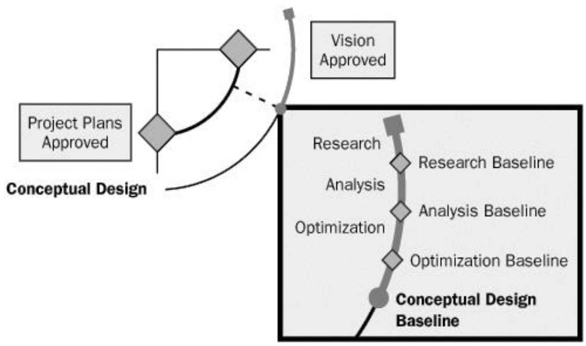


Fig. 1.5. Steps in conceptual design [18]

US Department of Transportation [19] formulated that conceptual studies (CS) are typically initiated as needed to support the design planning and programming process. CS phase identifies, defines and considers sufficient courses of action (i.e., engineering concepts) to address the design needs and deficiencies initially identified during the planning process. This phase advances a project proposed in the program to a point where it is sufficiently described, defined and scoped to enable the preliminary design and technical engineering activities to begin. The CS and preliminary design phases are performed in conjunction and concurrently with the environmental process which evaluates environmental impacts of the engineering proposals resulting from the conceptual studies and preliminary design phases.

Environmental systems and processes are also taken into account in knowledge based engineering (KBE) approach proposed by Pokojski and Szustakiewicz in [20]. The main structures for extended KBE application are design process and design models. The models contain specific aspects such as product structure as a whole and its fragments, engineering calculations and analysis with ability of integration with external systems, design requirements and decision making processes. This object-oriented approach makes it possible to speed up the process of generating the source code of design models from the extended KBE application. KBE approach is also used in [132] to support multidisciplinary design optimization. Another approach where the decision support tools in the domain of conceptual design are sufficiently developed is model-driven engineering (MDE) described in [21]. Here, a prototype software is presented that allows the user to specify functional requirements for the designed buildings, and hierarchical graphs and graph grammars serve as knowledge representation tool.

In *systematic* respect, designing is defined in [22] as the optimization of given objectives within partly conflicting constraints. Requirements can be changed with time, so that a particular solutions can be optimized for a particular circumstances set. In *organizational* respect, design is recognized as a part of the product life cycle. This cycle is trigged by a market need or a new idea. Life cycle starts with product planning and ends with recycling or environmentally safe disposal. The activity on engineering design is placed at the center of two interacting technical and cultural streams (Fig. 1.6) in [22].

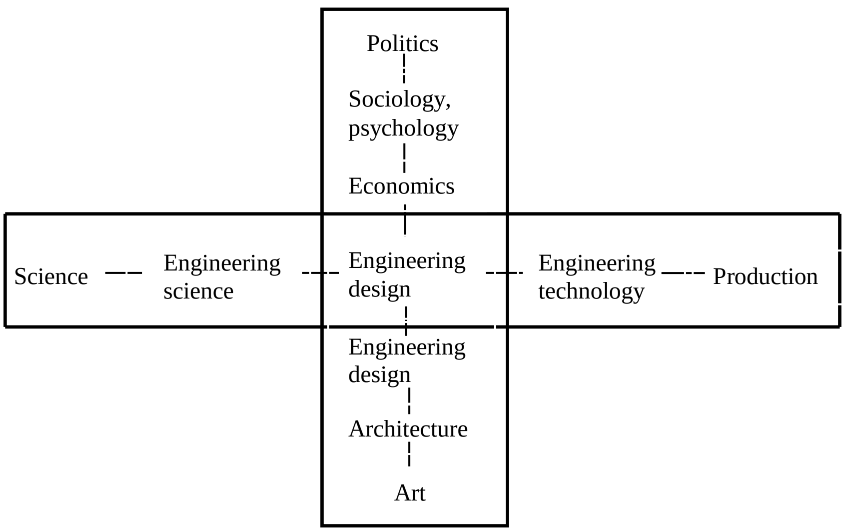


Fig. 1.6. The central activity of engineering design [22]

From the point of view of Hierarchical Systems (HS) method proposed in this manuscript, both streams belong to the general process of level increasing of the object being designed in its life cycle. This process and the life cycle for the case of Tractor and Automobile Industry (TAI) were described in [23, 24] and are presented in Fig. 1.7 below.

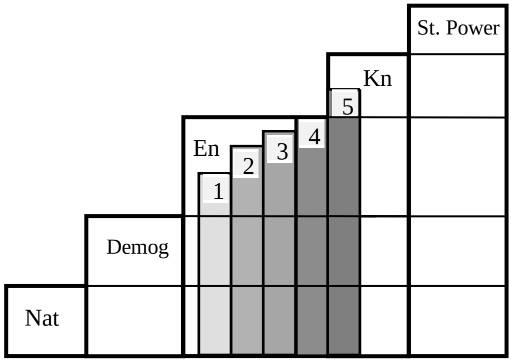


Fig. 1.7. Main phases (1-5) of product life cycle and their places in State levels space [24]

Figure 1.7 presents the main phases of product life cycle for the case of articles of Tractor and Automobile Industry (TAI) and their places in State levels. States levels are (Nat) natural (physical, chemical, biological), (Demog) demographical (social), (En) engineering, (Kn) knowledge and the level of the State power (St.Power). The phases of TAI product life cycle are: (1) manufacturing of articles and equipment for TAI; (2) transport networks, the delivering of raw materials, details, equipment; (3) trade; (4) field of activity and technical service of TAI articles; (5) design of new things (innovations) in TAI, financial and juridical service, scientific maintenance. The place of the conceptual design phase where HS design technology and correspondent conceptual model (presented and described below in this book) are implemented is in the fifth block – design of new things, financial and juridical service, scientific maintenance – of the scheme (Fig. 1.7).

The connection of the Life Cycle phases (Fig. 1.7) as well as State levels – natural, demographic, engineering, knowledge, State power – is realized by *aed* technology of hierarchical system partially described by A^{λ} scheme given in [8]. The scheme connects elements of symbol construction of hierarchical system presented in A^{λ} form of *aed* and reflects both the design mechanism and the law of object's level increasing in its life cycle.

To represent concept design knowledge, the technology of functional modelling was researched and applied by Bryant et al. [25]. Gero and Kannengiesser [26] presented an function–behavior–structure (FBS) ontology representation process for concept design in different domains, and emphasized the reasoning mechanism with the FBS ontology for knowledge representation. Borgo et al. [27] suggested an ontological characterization of artefact behavior and function to capture the informal meanings of these concepts in the engineering practice and to characterize them as part of a foundational ontology. As for a mechatronic design process, Yan and Zante stated that "mechatronic system design is normally considered to be a sequential process in which a design solution to a given design problem is generated, explored and evaluated following a series of prescribed steps" [28]. A mechatronic approach towards designing inspection robots using rapid prototyping and real-time simulation is also described by Giergiel et al. in [29].

The design methods based on the abstract modelling of PALMERA and the DMC model, Design Flow for Reconfigurable Architectures (INDRA) has been developed to guide the designer through different implementation steps to create a concrete dynamic reconfigurable system architecture. These design methods and the design flow are described in [133, 134].

A formal definition of the concept design and a conceptual model linking concepts related to design projects are presented by Ralph P. and Wand Y. [30]. Their definition of design incorporates seven elements: agent, object, environment, goals, primitives, requirements and constraints. The design project conceptual model is based here on the view that projects are temporal trajectories of work systems that include human agents who work to design systems for stakeholders, and use resources and tools to accomplish this task. Ralph and Wand demonstrate how these two conceptualizations can be useful by showing that 1) the definition of design can be used to classify design knowledge, and 2) the conceptual model can be used to classify design approaches. The analysis performed by Ralph and Wand [30] led to defining the design as follows in Fig. 1.8.

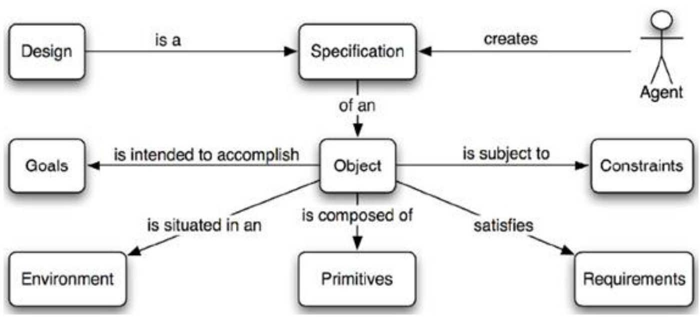


Fig. 1.8. Conceptual model of Design as a noun [30]

Here the conceptual model of Design presented as a *noun* in Fig. 1.8 is recognized as a specification of an object, manifested by an agent, intended to accomplish goals in a particular environment using a set of primitive components, satisfying a set of requirements, subject to constraints. The Design recognized as a *verb* is presented to create a design in an environment where the designer operates.

Both noun and verb design definitions correspond to the object and its process respectively as the elements of *aed* model of HS conceptual design method suggested in this book, see *aed* scheme in Fig. 3.1, Chapter 3.

A systematic approach for the competency-oriented development of learning factories integrating the conceptual design levels ‘learning factory’, ‘teaching module’ and ‘learning situation’ is given by Tisch et al. in [129]. The presented approach enables “an effective competency development in learning factories by addressing problems of intuitively designed learning systems. As a result learning factories, teaching modules and single teaching–learning situations meeting industries’ requirements can be realized with less effort and an increased success in applied competencies in real situations”. A systematic analysis of mechatronic objects is also presented by Gawrysiak in [104].

The function–cell–behaviour–structure (FCBS) model for better comprehending representation and reuse of design knowledge in conceptual design was presented by Gu et al. [31]**.** Hierarchical two-layer concept is given here, i.e. two knowledge representing layers – the principle layer and the physical layer – are presented in the FCBS model. The principle layer is utilized here to represent the principle knowledge. Case modelling is employed in the physical layer to integrate the structural information and behavioural performances of the existing devices, which applies the design principles represented by the functional knowledge cells (FKCs).

FCBS model presented by Gu et al. [31] and a formal definition of the concept design and a conceptual model given by Ralph P. and Wand Y. [30] are close to Hierarchical Systems design technology which is described below in this book and was chosen in this work as the formal basis of the conceptual model creation.

In the presented book the Conceptual Design (CD) is recognized as a process of creation of a systemic model of the object being designed on the early phase of its life cycle. To define the conceptual model of a mechatronic system in systemic bases it is necessary to describe: mechatronic system structure; its dynamic representation as a unit in its environment; the system environment, its process and system-environment interactions; the system coordinator and its coordination (design&control) processes; processes executed by mechatronic subsystems and a general process. Besides, the conceptual model should take into account the connected descriptions of mechatronic subsystems of different nature, i.e. mechanical, electromechanical, electronic and computer.

Furthermore, conceptual model of the mechatronic system being designed and its coordination (deisgn&control) technology should meet the *requirements of the general design system* which must allow performing the design – synthesis and analysis – and control tasks under condition of any information uncertainty, i.e. (1) to create and change mechatronic system construction and technology by selecting units of lower levels and settling their interactions to make the state and activity of the system in higher levels (environment) best coordinated with environmental aims (selection stratum); (2) to change the ways (strategies) of the design task performing when the designed unit is multiplied and the knowledge uncertainty is removed (learning stratum); (3) to change the above mentioned strata when new knowledge is created (self-organization stratum).

The coordination technology must also cohere with traditional forms of information representation in mechatronics, i.e. numerical and geometrical systems. The theoretical basis of the design process in agreement with these requirements must be a hierarchical construction connecting any level unit with its lower and higher levels.

The above mentioned methods used in the conceptual design and well as models of mathematics based on set theory and methods of artificial intelligence do not meet the above formulated requirement. They do not allow the description of mechatronic subsystems with their specific characteristic features in common formal basis. At the same time they do not describe the mechanism of interlevel dynamics of the mechatronic object (MO) being designed since the set theory describes one-level world outlook only.

To solve the problem, the symbol construction and coordination technology of Hierarchical Systems (HSs) [2-8] was chosen in the work as the formal basis of the conceptual model creation. HS formal model was constructed with the help of *aed* (Fig. 3.1) – formal analogue of two-level system [3-8], and general dynamic systems by Mesarovic and Takahara [9,10], numeric positional system codes, geometry and cybernetics methods [32].

Among the first attempts of HS technology and *aed* model application in design are the results presented in [3-4, 33]. The current version of the conceptual model and design technology of mechatronic objects based on the Hierarchical Systems approach is partially described in [49] and presented below in this book.

2. GEOMETRIC MODELS IN DESIGN SYSTEMS

Geometric models of modern CAD systems which are used for solving the problems of mechatronics, robotics and automation are overviewed and analyzed in this Chapter. The overview covers recent decades, when almost all theoretical geometric models were realized in design systems and some new ideas were formulated. The main attention is paid to the models implemented in the systems which have their commercial realization.

There are two main classes of geometric representations in computer aided design (CAD) systems and other automated systems – Boundary Representation, B-R [34] and Constructive Solid Geometry, CSG [35].

An object being constructed is presented by expressions of CSG type in the form of a group of connected elementary objects (primitives) – cylinders, prisms, spheres, cones etc. The connection can be formally described by algebraic sum, i.e. some objects take part in this group with negative signs. CSG expression is graphically presented as a tree, whose root is the whole object. The object unites the details at each level, where the terminal tops (leaves) are primitives (Fig. 2.1).

Objects in B-R are represented by the set of parts of surfaces (sides) and the connections between the sides by their boundaries (edges) may be indicated (Fig. 2.1).

It is easy to see that B-R in this case contains the set of boundary elements of primitives which are used for uniting in CSG representation. At the same time, B-R contains only such boundary elements which are connected with the environment directly, and are not connected with each other in the whole object construction. Therefore, after defining each primitive in CSG model by its boundary representation and erasing common parts of their boundaries we obtain B-R of the whole object. It means that there is a simple transition from CSG to B-R but the reverse process is not so simple.

Primitives and more complex objects can be presented in algebraic form, which include patent, parametric [36], polynomial representations [37] and different types of splines [37, 38], and also in the form of sequences of Euler operators [39]. Geometric representation and *parametric model* of the object were proposed in the mid 80s of the 20th century. For geometric representation [40] the set of notions is introduced:

$$\{\text{surface, point, vectors, scalars}\}\ .$$

Each of the notions can have a set of meanings in accordance with Table 2.1. It is easy to see that the meaning of variable "surface" in Table 2.1 plays the role of key and to each meaning of key there corresponds a strictly defined set of other variables in the same line in the table.

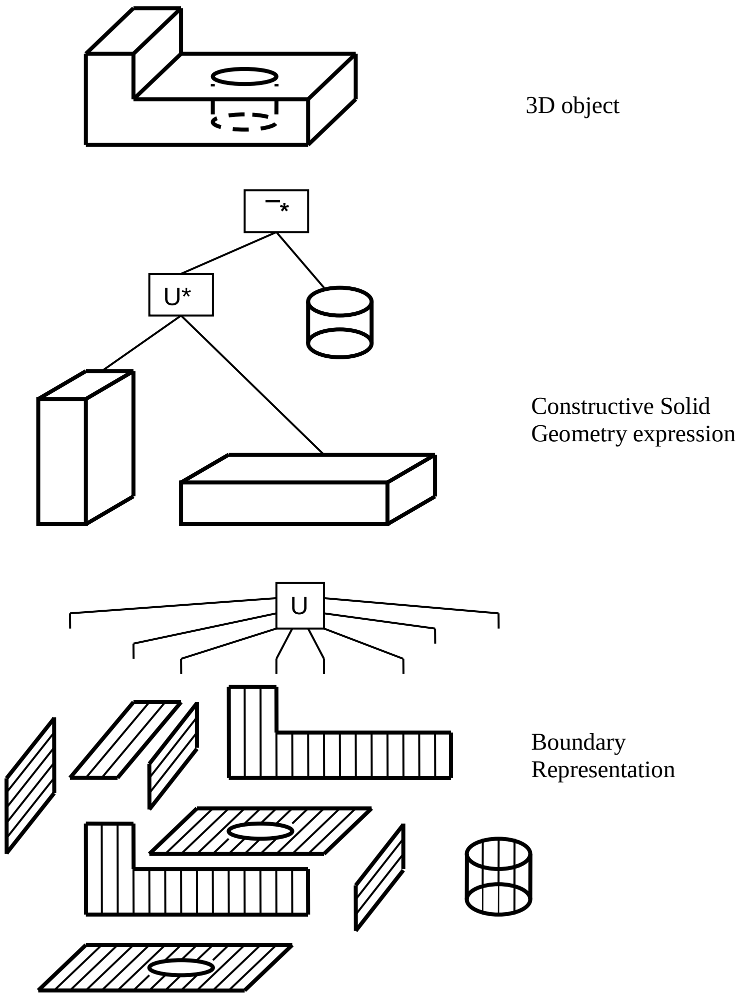


Fig. 2.1. Known basic geometric representations of object in CAD [140]

Table 2.1. Multitude of geometric characteristics meanings

Surfaces	Points	Vectors	Scalars
Plane	Any point on the plane	Unitary normal	No
Sphere	Center	No	Radius
Rectilinear cycle cylinder	Any point on the axis	Unitary vector parallel to axis	Radius, length of axis
Rectilinear cycle cone	Top	Unitary vector parallel to axis	Top angle, radius

There is a certain mutual concordance between the representation of primitive in the space of geometric characteristics and its algebraic definition in form:

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Jy + K = 0, \tag{2.1}$$

where: x, y, z are coordinates in 3-dimension Euclidean space. That means that there are algorithms for transformation of geometric representation to algebraic one and vice versa. Calculations presented in [40] show that geometric representations require the minimal computer memory in comparison with other forms of representation. Besides, it is well connected with the natural language, convenient for enquiring about searching for elements of the object and forming directives for its change. It is sometimes useful to calculate some metrical characteristics, such as volumes and squares of surfaces using geometric representation directly. But it is necessary to convert the geometric representation to other forms to calculate physical characteristics, to form graphic images and to determine the technological process.

Objects and their primitives can be presented by the sequences of *Euler operators* [39]. Therefore, objects are described by the sequence of constructing operators and inverse sets of destructing operators of geometric objects of a higher (in the case of destruction – lower) dimension. For instance, if we have an image of polyhedron we can erase one of its apexes *(vertex)* and save this operation on computer. After that we can erase another top which is located on the same edge. These two operations correspond to the operation of the edge erasing. The sequence of operations of erasing of all edges which form one polyhedron side corresponds to the operation of removal of the whole side and finally, the erasing of all the sides destroys the object.

The repetition of the chain of all the operations in the inverse sequence when destructing operations are changed by constructing ones allows the object to be rebuilt. Such direct and inverse chains are called Euler operators. Their name arose from Euler's formula:

$$v–e+f=2, \tag{2.2}$$

where: v – number of apexes, e – number of edges, f – number of sides.

Euler's formula allows correctness of the object construction to be controlled. General formula which corresponds to the object of any connectivity is presented in the form of the expression:

$$v–e+f=2(s–h)+r, \tag{2.3}$$

where: s – general number of untied components, h – number of through holes in object, r – general number of cavities in sides.

For the formation of graphic images Euler operators are one of the best expressions. At the same time their most effective application to computer raster terminal units is achieved by the hybrid connection of Euler operators with Octrees [41]. Such trees finally include the white and black leaves only (i.e. empty and filled elements of the object corresponding to cube primitives) but initially and in the middle state they have the grey leaves as well. Grey leaf corresponds to the case when one primitive simultaneously contains empty and filled areas. It is decomposed according to the general rule for each level till only black and white leaves remain.

Octrees is the most acceptable geometric model for raster graphics. The transition from Euler operators to Octrees is one of the way of transition from vector (which is most useful for the output of boundary expressions) to raster representation [42]. It is necessary to convert Euler operators to other forms in order to calculate functional characteristics of objects and to define technological processes.

The algebraic representations of geometric objects are well investigated [40, 43].

They are not practically used in requests to search the objects and are hard for their formation with participation of the user. There are some possibilities of connection of created in this way primitives in CAD systems with CSG representation. But connection algorithms of objects of various forms are very complex and require a lot of computer memory and time for calculation.

At the same time such representations are suitable for computer graphics tasks (in vector forms), for generation of some programs for automated production and are the most effective for calculation of many functional characteristics in many tasks of engineering analysis.

The most convenient way of forming an algebraic expression is the case when one of the input units of a computer is scanning a physically realized (or simply graphically written) object. In this case the unit forms a range of observations during a definite period of time which is changed later by corresponding mathematical definitions. It allows the information to be compressed and the initial range of observations to be restored when necessary.

Rational parametrical B-spline [38, 44, 45] gives the general form of any object's surface representation in homogeneous, i.e. descriptive, coordinates.

The spline is obtained from common parametric *B-spline* in the following way. $C(t)$ is polynomial B-spline curve in 3D Euclidian space, i.e.

$$C(t) = \sum_{i=1}^{n} B_{i,k}(t) P_i \;, \tag{2.4}$$

where: P_i – 3-dimention control points, t – parameter, such as $a \le t \le b$ & a, b are fixed and $0 \le a \le b$; $B(t)$ – polynomial of variable t of k order (power k-1). $B_{i,n}(t)$ – basic functions defined by node vector $\{t_j\}_{j=1}^{n+k}$ and $a = t_1 = t_2 = ... = t_k < {}_+t_{k1} \le t_{k+2} \le$
$\le ... \le t_n \le t_{n+1} = ... = t_{n+k} = b$.

The representation of $C(t)$ in homogeneous (descriptive) space with fourth coordinate h_i has the following form:

$$C(t) = \frac{\sum_{i=1}^{n} B_{i,k}(t) h_i P_i}{\sum_{i=1}^{n} B_{i,k}(t) h_i} \;, \tag{2.5}$$

or $$C^h(t) = \sum_{i=1}^{n} B_{i,k}(t) P_i^h \;, \tag{2.6}$$

where P_i^h is a control point in 4D space.

The corresponding *spline* representation for surface is defined by the expression:

$$S(s,t) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} B_{i,k}(s) B_{j,l}(t) h_{i,j} P_i}{\sum_{i=1}^{n}\sum_{j=1}^{m} B_{i,k}(s) B_{j,l}(t) h_{i,j}} \,. \tag{2.7}$$

This approach allows good representation of both the objects which are usually described by manifest definition (cycles, cones, primitive cubes) and the objects which are defined with the help of the parametrical polynomial forms (sculptural surfaces) [45]. The exemplary 2D path generated based on $p(x, y)$ points using Spline interpolation algorithm is presented in Fig. 2.2. Spline interpolation is used in this case to create the 3 degree polynomials for the set of measurement points of the cutting path. Technological process of the pipe cutting along the predefined path is realized by robot cutter or CNC machine in production environment [46].

First, 1D spline for ***x*** and ***y*** point is created independently. Then the 2D path is generated as the composition of 1D ***x*** and 1D ***y*** splines. The set of measurement points in this case is as follows: $x = [0 \quad -0{,}4 \quad -0{,}5 \quad 0 \quad 0{,}4 \quad 0{,}5 \quad 0{,}4 \quad 0]$ and $y = [0{,}5 \quad 0{,}4 \quad 0 \quad -0{,}4 \quad -0{,}5 \quad 0{,}4 \quad 0 \quad 0{,}5]$, see Fig. 2.2.

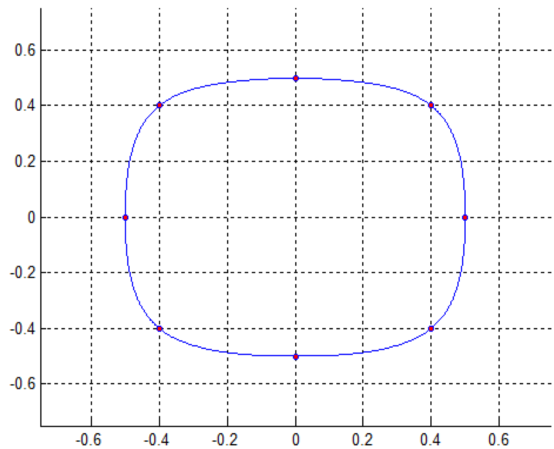
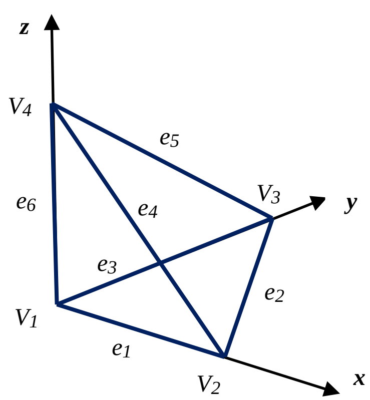


Fig. 2.2. 2D path generated based on $p(x, y)$ points using Spline interpolation algorithm

The manifest functions require more elementary actions and less computer memory for representation of primitive surfaces and lines. But the use of one general expression improves the characteristics of the system in general and allows the memory volumes for programs location to be reduced, the speed of operations to be increased and the dialogue with the engineer to be unified. At the same time the expenses of system development, exploitation and the amount of documentation are reduced.



Vertex list	Edge list
$V_1(0, 0, 0)$	$e_1[V_1, V_2]$
$V_2(1, 0, 0)$	$e_2[V_2, V_3]$
$V_3(0, 1, 0)$	$e_3[V_3, V_1]$
$V_4(0, 0, 1)$	$e_4[V_2, V_4]$
	$e_5[V_4, V_3]$
	$e_6[V_1, V_4]$

Fig. 2.3. Tetrahedron – an example of a wire-frame model

For the case of *wireframe modeling* [46], the model consists entirely of points, lines, arcs and circles, conics, and curves. In 3D wireframe model, an object is not recorded as a solid. Instead the vertices that define the boundary of the object, or the intersections of the edges of the object boundary are recorded as a collection of points and their connectivity (Fig. 2.3).

Wireframe models are most economical in term of time and memory requirements, easy to construct, used to model solid object and often used for previewing objects in an interactive scenario. At the same time the models have the following disadvantages. They do not allow for use of photo realistic rendering tools, have no ability to determine computationally information on mass properties (e.g. volume, mass, moment, etc.) and line of intersect between two faces of intersecting models and do not guarantee that the model definition is correct, complete or manufacturable.

To expand the potential possibilities of traditional boundary representation, the connections of dimensions (length, squares, volumes, angles) are added to algebraic expression in the process of forming the representation. It permits the process of parametric construction with the use of geometry of variations to be realized.

Parametric model, or which is often used for the representation of objects in data bases of design systems, is defined by set $\{T, D, O, V, R, C\}$, where: T – set of topologies, D – set of possible dimensions, O – set of operations, V – set of variables, R – set of relations between V and $T/O/D$, C – set of restrictions of the meanings of variables.

Any class of objects P_S can be presented in these terms in the following form:

$$P_S = \{t, d, o, v, r, c\}, \quad (2.8)$$

$$t_i \in T,\ d_i \in D,\ o^i \in O,\ v^i \in V,\ r^i \in R,\ c^i \in C.$$

It is possible to create B-R defining T and D only. To create CSG it is necessary to define O.

Therefore, parametric model of the object contains not only traditional representations (B-R in its most developed form based on geometry of variations) but also the relations between different forms of representations. Declarative definition of all the elements of set $\{T, D, O, V, R, C\}$ ensure flexibility of the system, possibility of its adaptation to different classes of objects and optimization of system parameters in its functioning process (for instance, finding more effective operations, defining new relations, etc.).

Having many characteristic features of systems of artificial intelligence, parametric model has a drawback – its relatively weak formalization. Six elements presented above are not strictly defined. That makes the creation of such models very complicated for the user. In other words, sets of possible meanings (states) of objects T, D, O, V, R, C are not defined. Possible connections of these states, i.e.

states of the whole system, and transition operations from one possible state to another are not defined either.

A more theoretically grounded from the point of view of realization simplicity is the approach (independent from technical realisation) of graphical standard development proposed by CGS Associates [47].

This approach is based on the application of mathematical means of formal system theory and algebra for the development of data bases. GSC Associates has adapted its technology in its commercial software products. Some of these products are Computer Graphics Metafile (CGM) generator used by Apple, and CGM interpreter used by Microsoft, Apple and Symantec [47].

Geometrical objects are considered in [47] as formal systems which have their inputs, outputs, states and actions of state transitions. A set of the actions also contains the ways of obtaining information about the object's state (states recognition).

Definite data structure of a data base corresponds to the formal system – abstract data type, which can be adapted to any concrete state (any data of this type) and the set of state transition functions, i.e. operations which do not take the elements out the frames of the abstract type being considered. For instance, the structure of group of integer numbers is defined on computer by a cell organised for representation of integer number as a positional number with a definite base and actions of adding and subtraction.

The cell can be in any of its states (can contain one of integer numbers presented in the computer numeric format). Each of the mentioned operations with the present number and some other number which goes to the cell input are performed with the help of sequence of elementary operations (with bits) and forms the output which is also an integer number, i.e. has the same abstract type.

Division operation does not belong to the group of integer numbers because it can take the elements out of the frames of this abstract type (it is necessary to have one additional cell for the division remainder). The fields of real and complex numbers, logical data are also realized in computers in the form of abstract data types.

With respect to geometric expressions, this approach means the representation of a geometric object in the form of abstract data type with a set of operations of states transition which do not take the elements out of the frames of this abstract type. For instance, the data structure "segment" can be defined by a set of axioms, limits and by the operations of motion on a straight line, rotation and change of length. The operation of arbitrary uniting of two segments is not included in the data structure "segment" because it can take elements out of the frames of this abstract type. For instance, it is possible to obtain the broken line or "+"-like object as a result of uniting two segments, i.e. lines.

Data structure of “segment” also contains the operations with it. The description of several connected data structures for vector computer graphics with similar organization is presented in [47].

In this case the analogy of parametric model set *T*, *D*, *O*, *V*, *R*, *C* is a set of notions of formal system theory which corresponds to the language of abstract data types. That means that the notions and their relations from this set are not only strictly defined and easy for programming but are realized in many cases as hardware.

So, *concluding* the overview of traditional ways of geometric information representations, it can be pointed out that the possibility of formal system theory and abstract data types application for the construction of expressions of geometric objects (models) opens new perspectives in CIM systems including robotics and CAD.

Both the CSG and B-R have their own advantages and drawbacks. So, the CSG models are simply constructed with the participation of a user. CSG model also requires less computer memory, is better adapted for definition of technological processes (for inst. assembling type) and for imitation of movements of object’s details.

On the other hand, B-R models can actually be built without human participation, allow the sculptural surfaces to be constructed, allow variations methods to be used, and are better adapted for calculation of technological processes of continuous deformation and for aerodynamic and hydrodynamic calculations.

The connection of these two approaches is possible, but sub-process of geometric constructions creating and changing for CSG differs in marked degree from this sub-process for B-R.

In the existing CAD-systems these are principally different design technologies. Integration of these approaches in one system can be possible in the case of existence of the general formal language which can contain both representations and transition operations from one to another. It looks like the hierarchical and general systems theory can give an opportunity to create such a formal language. In this book, the hierarchical geometry model given in Chapter 4 is used in design tasks of mechatronic objects. The exemplary design tasks where HS (*aed*) geometric model is used are presented is Sections 4.4.2 and 4.4.3, and Chapters 6-8.

3. THEORETICAL BASIS OF THE DESIGN METHOD

3.1. Formal model of hierarchical mechatronic system

Theoretical means of the both conceptual and detailed design method, as was stated in Chapter 1, must present the mathematical apparatus for formal description of the designed mechatronic system (MS) structure, its aggregated dynamic representation as a unit in its environment and the environment model. All the descriptions should be connected by the coordinator which performs the design&control tasks on its selection, learning and self-organization strata. Furthermore, theoretical means of the proposed mechatronic design method should allow presentation of the numerical and geometrical information in its formal basis and be coordinated with the main requirements to design systems which must:

– carry out the main design and control task for the systems of any level under the conditions of any initial knowledge uncertainty, i.e. to create or to change the system construction and technology, to make its activity in a higher level system (environment) most coordinated with the desired environment states on all its levels (*selection task*);

– change the ways (strategies) of fulfilment of the main design and control when designed constructions and technologies are multiplied and knowledge uncertainty is removed (*learning task*);

– change the selection and learning strata when new (higher level) knowledge constructions and technologies have been created (*self-organisation task*).

Traditional one-level mathematic, artificial intelligence and cybernetic models and methods used in conceptual and detailed design do not meet all the above mentioned requirements, because it is impossible to express interleave relations in their terms.

Therefore, the construction of hierarchical system (HS) with its standard block *aed* (formal analog of two-level system) has been chosen as the theoretical basis for conceptual and detailed mechatronic design performing. *Aed* – ancient Greek word – is a standard element of design systems [3-7, 33, 49], which realizes the general laws of systems organization on each level and the inter-level connections.

The first version o*f aed* S^{ℓ} model was described in [3-5, 33], and later in [6-8]. *Aed* contains all the characteristic features of two-level system [2] and also the following ones:

1) for inter-level connections modeling *aed* contains more levels, the block of environment ${}_{\varepsilon}S^{\ell}$, in particular;

2) two-level system in [2] is not sufficiently formalized for measuring quantitative characteristics and for investigating dynamic laws of hierarchical systems;

aed S^{ℓ} formal model is based mainly on the two particular cases – dynamic systems $(\overline{\rho},\overline{\varphi})$ [9, 10] and numerical positional systems Λ^{S} by H. Lebesque [48].

3) structural connections in two-level system were explored only for the method of the general systems synthesis [9, 10], but the nature and the dynamics of the connections were not revealed;

connections in S^{ℓ} are hierarchical systems and can be described by *aed* S^{ℓ} formal model; for S^{ℓ} structural connections and for the connections of system S^{ℓ} with environment ${}_{\varepsilon}S^{\ell}$ the laws of their coordinated dynamics, i.e. the dynamics of inter-level transition are defined;

4) the coordinator of two-level system does not change its own state; it is connected only with the subsystems being coordinated and cannot construct or change the formal model of two-level system;

coordinator S_0^{ℓ} of *aed* is constructed from the interactions of lower level systems coordinated and directly connected with the coordinator $S_0^{\ell+1}$ of higher level.

The dynamics of S_0^{ℓ} states was explored for four periods of time: T_{ψ}^{ℓ}, T_{χ}^{ℓ}, T_{φ}^{ℓ} and T_{λ}^{ℓ} (T_{ψ}^{ℓ}, T_{χ}^{ℓ}, $T_{\varphi}^{\ell}, T_{\lambda}^{\ell} \subset T^{\ell}$, T^{ℓ} – time of level ℓ). The initial moment for each concrete system S^{ℓ} and its coordinator S_0^{ℓ} is the moment $t_{\varphi}^{\ell} = 0^{\ell} \subset T_{\varphi}^{\ell}$. Moment t_{φ}^{ℓ} is fixed when the elements $\overline{S}_{\varphi}^{\ell}$ of level $\ell_{\varphi}(\ell_{\varphi} < \ell)$ of system S^{ℓ} have arisen. For time T_{φ}^{ℓ} the formal model $S^{\ell} \mid T^{\ell}$ allows the experimental checking of its coherence only with the systems of lower levels $\tilde{\ell} \le \ell - \tau_{\varphi}$ $(\tau_{\varphi} \ne 0)$ and cannot be coordinated with the synthesis laws of level ℓ systems. It causes improvement of the model. During the process of system S^{ℓ} synthesis its coordinator S_0^ℓ begins to form and contain new system law $S^\ell | T^\ell$. Control of system S^ℓ coherence with its model $S^\ell | T^\ell$ and $S^\ell | T^\ell$ improvement are the tasks of the self-organization layer of coordinator S_0^ℓ.

In addition, *aed* S^ℓ has some less important differences in comparison with two-level system. For instance, formal models of objects and their processes in S^ℓ are not coincide, entities of coordinating signals and feedback signals were revealed, etc.

Graphical image of any level ℓ *mechatronic system* S^ℓ as *aed* is presented in Figure 3.1, where:

$\ell \in L^s$, L^s – numerical positional system, defined on the multitude $L_\beta = \{\ell_\beta : \ell_\beta = (\ell, I^\ell)\}$, $\ell \in L$, L – multitude of levels indices, I^ℓ – finite multitude of indices on level $\ell (i, \tau, \nu \in I^\ell)$;

${}_oS^\ell$ is *mechatronic object*, X^ℓ is input, Y^ℓ is output of object ${}_oS^\ell$;

${}_\omega\gamma^\ell \leftrightarrow \{_k \{X, C, Y\} : k \in_k L\}^\ell$ – interactions of mechatronic system S^ℓ with other systems from its environment ${}_\varepsilon S^\ell$, and $k \in_k L \leftrightarrow \{o, o\pi, \pi\varepsilon, \varepsilon\}$ are indexes of mechatronic object (o), its process $(o\pi)$, process performer by MS environment $(\pi\varepsilon)$, and environment itself (ε) respectively.

$\{{}_{o\pi}S^\ell, {}_{\varepsilon\pi}S^\ell\}$ are processes, and ${}_{o\pi}S^\ell$ is the *mechatronic process* which object ${}_oS^\ell$ performs in its environment ${}_\varepsilon S^\ell$, ${}_{\pi\varepsilon}S^\ell$ are actions, which environment ${}_\varepsilon S^\ell$ executes with object ${}_oS^\ell$;

$\bar{S}^{\ell-1} = \{S_i^{\ell=1} : i \in I^\ell\}$ and ${}_\pi\bar{S}^{\ell-1} = \{_\pi S_i^{\ell-1} : i \in I^\ell\}$ – families of subsystems, connected in S^ℓ by interconnections ${}_\sigma\gamma^\ell \supset_\omega \bar{\gamma}^{\ell-1} = \{_\omega \gamma_i^{\ell-1} : i \in I^\ell\}$ and correspondent sub-processes;

$C^\ell \supset \bar{C}^{\ell-1} = \{C_i^{\ell-1} : i \in I^\ell\}$ – control signals from ${}_o\bar{S}^{\ell-1}$ to ${}_{o\pi}\bar{S}^{\ell-1}$;

$\bar{Z}^{\ell-1} = \{Z_i^{\ell-1} : i \in I^\ell\}$ – feedback from ${}_{o\pi}\bar{S}^{\ell-1}$ to ${}_o\bar{S}^{\ell-1}$; $\bar{Z}^{\ell-1} \subset Z^\ell \subset X^\ell$;

S_0^ℓ – coordinator;

$\gamma^\ell \in \Gamma^\ell$ – coordinating signals from S_0^ℓ to subsystems $\bar{S}^{\ell-1}$;

$W^\ell \supset \{W_i^\ell : i \in I^\ell\}$ – feedback from subsystems $\bar{S}^{\ell-1}$ to coordinator S_0^ℓ;

$\gamma_\omega \in \Gamma^{\ell+1} \subset X^\ell$ – coordinating signals for system S^ℓ;

$w_\omega \in W^{\ell+1} \subset Y^\ell$ – feedback from S_0^ℓ to coordinator $S_0^{\ell+1}$ of higher level.

Aed S^ℓ is a system which has the following characteristic features:

– environment ${}_\varepsilon S^\ell$ contains a set of systems of level $\ell \in L$;

– it is possible to present S^ℓ as object ${}_o S^\ell$, its interactions with ${}_\varepsilon S^\ell$ create a higher level system;

– process ${}_\pi S^\ell$ contains both the actions ${}_{o\pi} S^\ell$ which the object ${}_o S^\ell$ performs in environment ${}_\varepsilon S^\ell$ and actions ${}_{\varepsilon\pi} S^\ell$ of environment system ${}_\varepsilon S^\ell$ with object ${}_o S^\ell$;

– dynamic realization of S^ℓ in ${}_\varepsilon S^\ell$ is defined by aggregated model $\omega^\ell \leftrightarrow \{\tilde{\omega}, S_0\}^\ell$, $\tilde{\omega}^\ell \leftrightarrow \{\{{}_o\,\omega, {}_{o\pi}\,\omega\}, {}_\omega\gamma, \{{}_{\pi\varepsilon}\,\omega, {}_\varepsilon\,\omega\}\}^\ell$, where ${}_o\omega$, ${}_\varepsilon\omega$ and ${}_{o\pi}\omega, {}_{\pi\varepsilon}\omega$ are aggregated dynamic models of object ${}_o S^\ell$, environment ${}_\varepsilon S^\ell$ and their processes ${}_{o\pi} S^\ell$, ${}_{\varepsilon\pi} S^\ell$ respectively;

– structure σ^ℓ contains the lower level systems $\bar{S}^{\ell-1}$ with their interactions ${}_\sigma\gamma^\ell$ which create coordinator S_0^ℓ and in this way connect σ^ℓ with ω^ℓ: $\sigma^\ell \leftrightarrow \{S_0, \tilde{\sigma}\}^\ell$, $\tilde{\sigma}^\ell \leftrightarrow \{\{\{\omega_i : i \in I\}^{\ell\pm\tau} : \tau \in L^s\}, {}_\sigma\gamma\}^\ell$;

– the task of coordinator S_0^ℓ is to create not only systems S^ℓ but also (partly) higher levels systems; for solving this task coordinator S_0^ℓ constructs the informational models of concrete systems $\bar{S}^\ell (\ell \in L)$; the models are constructed from the elements of interconnections of S^ℓ with other systems $\bar{S}^\ell$ of level ℓ and must be coordinated both with $\bar{S}^\ell$ and higher levels systems;

– incoherence of the models with concrete systems causes a change of the abstract informational model of S^ℓ by coordinator S_0^ℓ.

Definition 3.1. Mechatronic system S^ℓ is expressed by the next *aed* formal system:

$$S^\ell \leftrightarrow \{\omega, S_0, \sigma\}^\ell, \qquad (3.1)$$

where ω^{ℓ} is an aggregated dynamic realization of any level $\ell \in L$ mechatronic system in its environment ${}_{\varepsilon}S$, σ^{ℓ} is a model of the system S^{ℓ} structure, S_0^{ℓ} is coordinator, ℓ – index of level, $\ell \in L^S$, see Fig.3.1.

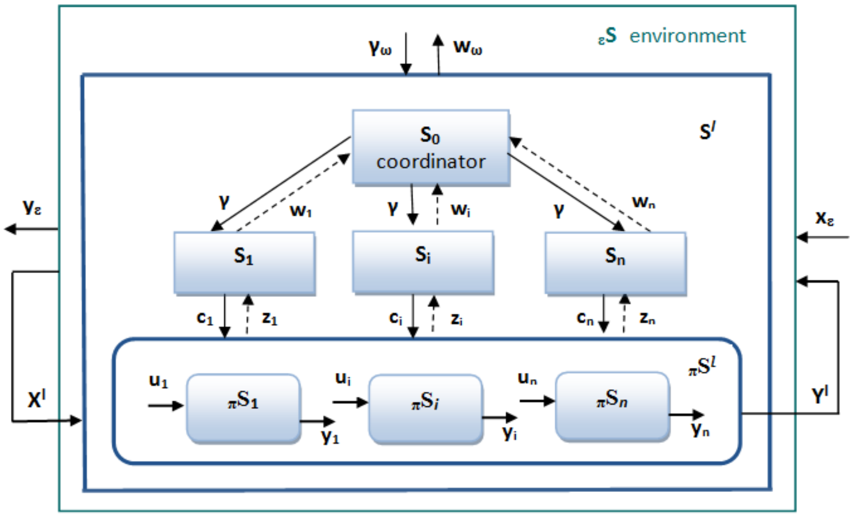


Fig. 3.1. Structure diagram of *aed* – standard block of HS. S_0 is the coordinator, ${}_\varepsilon S$ is the environment, S_i are subsystems, ${}_\pi S_i$ are subprocesses, ${}_\pi S^l$ is the process of level ℓ, X^l and Y^l are the input and output of mechatronic system S^l; c_i, z_i, γ, w_i , u_i, y_i are interactions

3.1.1. Aggregated dynamic realization

Taking into account the necessity of creation of the external functional representation of any mechatronic object being designed, the aggregated dynamic realizations for the elements of *aed* construction are built in this Section. Reaction ${}_oR^{\ell} : C^{\ell} \times X^{\ell} \to Y^{\ell}$ and dynamic realization ${}_o(\overline{\rho},\overline{\varphi})^{\ell}$ of *mechatronic object* (MO) ${}_oS^{\ell}$ which is described by the ratio ${}_oS^{\ell} \subset X^{\ell} \times Y^{\ell}$ are constructed by the method proposed in [9]:

$$
{}_o(\overline{\rho},\overline{\varphi})^{\ell} : {}_o\overline{\rho}^{\ell} = \{ {}_o\rho_t : C_t \times X_t \to Y_t \;\&\; t \in T \}^{\ell}, \tag{3.2}
$$

$$
{}_o\overline{\varphi}^{\ell} = \{ {}_o\varphi_{tt'} : C_t \times X_{tt'} \to C_{t'} \;\&\; t,t' \in T \;\&\; t' > t \}^{\ell},
$$

where T^{ℓ} is the time of level ℓ, reaction ρ, state transition function φ, and time interval *tt'*.

Model of actions, i.e. processes ${}_{o\pi}S^{\ell}$ which object ${}_{o}S^{\ell}$ performs in environment ${}_{\varepsilon}S^{\ell}$ changing its own state $c^{\ell} \in C^{\ell}$ differs from the model ${}_{o}(\overline{\rho},\overline{\varphi})^{\ell}$ of the object:

– object of initial states ${}_{\pi}C^{\ell}$ of process ${}_{o\pi}S^{\ell}$ coincides with the object of inputs X^{ℓ};

– input ${}_{\pi}X^{\ell}$ of process ${}_{o\pi}S^{\ell}$ is defined by states $c^{\ell} \in C^{\ell}$, their alterations are control signals to process ${}_{o\pi}S^{\ell}$.

Taking into account the differences, dynamic realization ${}_{o\pi}(\overline{\rho},\overline{\varphi})^{\ell}$ of *mechatronic process* ${}_{o\pi}S^{\ell}$ connected with ${}_{o}(\overline{\rho},\overline{\varphi})^{\ell}$ is presented in the following way:

$$
\begin{aligned}
&{}_{o\pi}(\overline{\rho},\overline{\varphi})^{\ell} :\ {}_{o\pi}\overline{\rho}^{\,\ell} = \left\{ {}_{o\pi}\rho_t : X_t \times C_t \to Y_t \ \&\, t \in T \right\}^{\ell}, \\
&{}_{o\pi}\overline{\varphi}^{\,\ell} = \left\{ {}_{o\pi}\varphi_{tt'} : X_t \times C_{tt'} \to X_{t'} \ \&\, t, t' \in T \ \&\, t' > t \right\}^{\ell}.
\end{aligned}
\tag{3.3}
$$

The expression for ${}_{o\pi}(\overline{\rho},\overline{\varphi})^{\ell}$ shows that the input of object ${}_{o}S^{\ell}$ is defined not only by the environment ${}_{\varepsilon}S^{\ell}$ but also (partially) by system S^{ℓ} in process ${}_{o\pi}S^{\ell}$, i.e. S^{ℓ} can form its inputs by itself.

Actions ${}_{\varepsilon\pi}S^{\ell}$ of environment system ${}_{\varepsilon}S^{\ell}$ with the object ${}_{o}S^{\ell}$ are also formalized by dynamic system ${}_{\varepsilon\pi}(\overline{\rho},\overline{\varphi})^{\ell}$:

$$
\begin{aligned}
&{}_{\varepsilon\pi}(\overline{\rho},\overline{\varphi})^{\ell} :\ {}_{\varepsilon\pi}\overline{\rho}^{\,\ell} = \left\{ {}_{\varepsilon\pi}\rho_t : Y_t \times {}_{\varepsilon}C_t \to X_t \ \&\, t \in T \right\}^{\ell}, \\
&{}_{\varepsilon\pi}\overline{\varphi}^{\,\ell} = \left\{ {}_{\varepsilon\pi}\varphi_{tt'} : Y_t \times {}_{\varepsilon}C_{tt'} \to Y_{t'} \ \&\, t, t' \in T \ \&\, t' > t \right\}^{\ell},
\end{aligned}
\tag{3.4}
$$

where ${}_{\varepsilon}C^{\ell}$ is both the state of environment ${}_{\varepsilon}S^{\ell}$ and inputs of process ${}_{\varepsilon\pi}S^{\ell}$; Y_t^{ℓ} is the state of environment process ${}_{\varepsilon\pi}S^{\ell}$, *tt'* is a time interval.

Aggregated dynamic models of actions ${}_{\pi}S^{\ell} = \{ {}_{o\pi}S^{\ell}, {}_{\varepsilon\pi}S^{\ell} \}$ of systems ${}_{o}S^{\ell}$ and ${}_{\varepsilon}S^{\ell}$ are defined as ${}_{\pi}(\overline{\rho},\overline{\varphi})^{\ell}$:

$$_{\pi}(\overline{\rho},\overline{\varphi})^{\ell} = \{ {}_{o\pi}(\overline{\rho},\overline{\varphi})^{\ell},\ {}_{\varepsilon\pi}(\overline{\rho},\overline{\varphi})^{\ell} \}. \tag{3.5}$$

The model of environment $_{\varepsilon}S^{\ell}$ of system S^{ℓ} is presented as follows:

$$_{\varepsilon}(\overline{\rho},\overline{\varphi})^{\ell} :\ {}_{\varepsilon}\overline{\rho}^{\ell} = \{{}_{\varepsilon}\rho_t : {}_{\varepsilon}C_t \times {}_{\varepsilon}X_t \to {}_{\varepsilon}Y_t \ \&\, t \in T\}^{\ell}, \tag{3.6}$$

$$_{\varepsilon}\overline{\varphi}^{\ell} = \{{}_{\varepsilon}\varphi_{tt'} : {}_{\varepsilon}C_t \times {}_{\varepsilon}X_{tt'} \to {}_{\varepsilon}C_{t'} \ \&\, t,t' \in T \ \&\, t' > t\}^{\ell}.$$

The coherence condition of constructed models with concrete mechatronic systems were regarded. As stated in [9], for reaction family $\overline{\rho} = \{\rho_t : C_t \times X_t \to Y_t \ \&\, t \in T\}$ coherence with the time system $S \subset X \times Y$ it is necessary and enough the execution of the following conditions [9]:

$$(\Psi 1)\quad (\forall c_0)(\forall x^t)(\forall x_t)(\exists c_t)[\rho_t(c_t, x_t) = \rho_0(c_0, x^t \circ x_t)|T_t], \tag{3.7}$$

$$(\Psi 2)\quad (\forall c_t)(\forall x_t)(\exists c_0)(\exists x^t)[\rho_t(c_t, x_t) = \rho_0(c_0, x^t \circ x_t)|T_t].$$

Time system is called a dynamic system only when it is possible to find both the reaction family $\overline{\rho}$ and the family of the state transitions $\overline{\varphi}$ coordinated with the system, and all functions $\varphi_{tt'}$ satisfy the conditions (i)–(iii) [9]:

$$\text{(i)}\quad \rho_t(c_t, x_t)|T_{t'} = \rho_t(\varphi_{tt'}(c_t, x_{tt'}), x_{t'}),\quad x_t = x_{tt'} \circ x_{t'}; \tag{3.8}$$

$$\text{(ii)}\quad \varphi_{tt'}(c_t, x_{tt'}) = \varphi_{t''t'}(\varphi_{tt''}(c_t, x_{tt''}), x_{t''t'}),\quad x_{tt'} = x_{tt''} \circ x_{t''t'};$$

$$\text{(iii)}\quad \varphi_{tt}(c_t, x_{tt}) = c_t.$$

The coherence conditions similar to $(\Psi 1)$ and $(\Psi 2)$ (3.7) for the object reaction $_{o}\overline{\rho}^{\ell}$ are presented in the following way:

$$({}_{o}\Psi 1)\quad (\forall c_0^{\ell})(\forall x^{\ell t})(\forall x_t^{\ell})(\exists c_t^{\ell})[{}_{o}\rho_t^{\ell}(c_t^{\ell}, x_t^{\ell}) = {}_{o}\rho_0^{\ell}(c_0^{\ell}, x^{\ell t} \circ x_t^{\ell})|T_t^{\ell}], \tag{3.9}$$

$$({}_{o}\Psi 2)\quad (\forall c_t^{\ell})(\forall x_t^{\ell})(\exists c_0^{\ell})(\exists x^{\ell t})[{}_{o}\rho_t^{\ell}(c_t^{\ell}, x_t^{\ell}) = {}_{o}\rho_0^{\ell}(c_0^{\ell}, x^{\ell t} \circ x_t^{\ell})|T_t^{\ell}].$$

The conditions for $_{o\pi}\overline{\rho}^{\ell}$, $_{\varepsilon\pi}\overline{\rho}^{\ell}$, $_{\varepsilon}\overline{\rho}^{\ell}$ are constructed similarly taking into account the meanings of states, inputs and outputs in the dynamic representations of processes $_{o\pi}S^{\ell}$, $_{\varepsilon\pi}S^{\ell}$ and environment $_{\varepsilon}S^{\ell}$ (3.3-3.6). The meanings are presented in Table 3.1, where ${}_{\tau\varepsilon}X^{\ell}$ and ${}_{\tau\varepsilon}Y^{\ell}$ are parts of ${}_{\varepsilon}X^{\ell}$ and ${}_{\varepsilon}Y^{\ell}$ immediately not dependant on S^{ℓ}.

Reaction families ${}_{o}\overline{\rho}^{\ell}$, ${}_{o\pi}\overline{\rho}^{\ell}$, ${}_{\varepsilon\pi}\overline{\rho}^{\ell}$, ${}_{\varepsilon}\overline{\rho}^{\ell}$ are coordinated with ${}_{o}S^{\ell}$, ${}_{o\pi}S^{\ell}$, ${}_{\varepsilon\pi}S^{\ell}$ and ${}_{\varepsilon}S^{\ell}$ only when corresponding conditions $({}_{o}\Psi 1, {}_{o}\Psi 2)$, $({}_{o\pi}\Psi 1, {}_{o\pi}\Psi 2)$, $({}_{\varepsilon\pi}\Psi 1, {}_{\varepsilon\pi}\Psi 2)$ and $({}_{\varepsilon}\Psi 1, {}_{\varepsilon}\Psi 2)$, which are called *coherence conditions* of ω^{ℓ} and S^{ℓ}, are realized.

Conditions (i)-(iii) (3.8) for function ${}_{o}\overline{\varphi}^{\ell}$ of states transition are presented as follows:

$$\text{o(i)}\quad {}_{o}\rho_{t}^{\ell}(c_{t}^{\ell}, x_{t}^{\ell})\Big|T_{t'}^{\ell} = {}_{o}\rho_{t'}^{\ell}({}_{o}\varphi_{tt'}^{\ell}(c_{t}^{\ell}, x_{tt'}^{\ell}), x_{t'}^{\ell}), \tag{3.10}$$

$$x_{t}^{\ell} = x_{tt'}^{\ell} \circ x_{t'}^{\ell};$$

$$\text{o(ii)}\quad {}_{o}\varphi_{tt'}^{\ell}(c_{t}^{\ell}, x_{tt'}^{\ell}) = {}_{o}\varphi_{t''t'}^{\ell}({}_{o}\varphi_{tt''}^{\ell}(c_{t}^{\ell}, x_{tt''}^{\ell}), x_{t''t'}^{\ell}),$$

$$x_{tt'}^{\ell} = x_{tt''}^{\ell} \circ x_{t''t'}^{\ell};$$

$$\text{o(iii)}\quad {}_{o}\varphi_{tt}^{\ell}(c_{t}^{\ell}, x_{tt}^{\ell}) = c_{t}^{\ell}.$$

Table 3.1. Connections of states, inputs and outputs for ${}_{o}S^{\ell}$, ${}_{o\pi}S^{\ell}$, ${}_{\varepsilon\pi}S^{\ell}$ and ${}_{\varepsilon}S^{\ell}$

${}_{\omega}\gamma^{\ell}$	C^{ℓ}	X^{ℓ}	Y^{ℓ}
${}_{o}S^{\ell}$	C^{ℓ}	X^{ℓ}	Y^{ℓ}
${}_{o\pi}S^{\ell}$	X^{ℓ}	C^{ℓ}	Y^{ℓ}
${}_{\varepsilon\pi}S^{\ell}$	${}_{\tau\varepsilon}X^{\ell}\times Y^{\ell} = {}_{\varepsilon}X^{\ell}$	${}_{\varepsilon}C^{\ell}$	${}_{\tau\varepsilon}Y^{\ell}\times X^{\ell} = {}_{\varepsilon}Y^{\ell}$
${}_{\varepsilon}S^{\ell}$	${}_{\varepsilon}C^{\ell}$	${}_{\tau\varepsilon}X^{\ell}\times Y^{\ell} = {}_{\varepsilon}X^{\ell}$	${}_{\tau\varepsilon}Y^{\ell}\times X^{\ell} = {}_{\varepsilon}Y^{\ell}$

The conditions for ${}_{o\pi}\overline{\varphi}^{\ell}$, ${}_{\varepsilon\pi}\overline{\varphi}^{\ell}$, ${}_{\varepsilon}\overline{\varphi}^{\ell}$ are constructed similarly taking into account the meanings of states, inputs and outputs (Tab. 3.1) in the dynamic representations of ${}_{o\pi}S^{\ell}$, ${}_{\varepsilon\pi}S^{\ell}$ and ${}_{\varepsilon}S^{\ell}$ (3.3-3.6).

So, the following qualities of the model being created and correspondent definition were formulated. If the coherence conditions of ω^{ℓ} with S^{ℓ} and conditions

${}_{o}$((i)-(iii)) $-_{\omega}$((i)-(iii)) are realized, systems ${}_{o}(\overline{\rho},\overline{\varphi})^{\ell}$, ${}_{o\pi}(\overline{\rho},\overline{\varphi})^{\ell}$, ${}_{\varepsilon\pi}(\overline{\rho},\overline{\varphi})^{\ell}$ and ${}_{\varepsilon}(\overline{\rho},\overline{\varphi})^{\ell}$ are *dynamic*.

Definition 3.2. Set of dynamic systems $\{ {}_{o}(\overline{\rho},\overline{\varphi})^{\ell}, {}_{\pi}(\overline{\rho},\overline{\varphi})^{\ell}, {}_{\varepsilon}(\overline{\rho},\overline{\varphi})^{\ell} \}$ connected by connections ${}_{\omega}\gamma^{\ell}$ and the creating system S_0^{ℓ} (coordinator) are called *dynamic realization* ω^{ℓ} of hierarchical system of any level $\ell \in L$:

$$\omega^{\ell} \leftrightarrow \{ \{\{{}_{o}\,\omega, {}_{o\pi}\,\omega\}, {}_{\omega}\gamma, \{{}_{\pi\varepsilon}\,\omega, {}_{\varepsilon}\,\omega\}\}^{\ell}, S_0^{\ell} \} \leftrightarrow \{ \tilde{\omega}^{\ell}, S_0^{\ell} \} . \tag{3.11}$$

Where $\tilde{\omega}^{\ell}$ is presented in the following form:

$$\tilde{\omega}^{\ell} \leftrightarrow \{\{{}_{o}\,\omega, {}_{o\pi}\,\omega\}, {}_{\omega}\gamma, \{{}_{\pi\varepsilon}\,\omega, {}_{\varepsilon}\,\omega\}\}^{\ell} \leftrightarrow \tag{3.12}$$

$$\leftrightarrow \{\{\omega^{\ell\pm 0}, {}_{\omega}\gamma^{\ell}, \{\omega^{\ell\pm\tau} : \tau \in L^{s}, \tau \neq 0\}\} \leftrightarrow \{\{\omega^{\ell\pm\tau} : \tau \in L^{s}\}, {}_{\omega}\gamma\}^{\ell},$$

and ${}_{k}\,\omega^{\ell}$ is as follows: ${}_{k}\,\omega^{\ell} \leftrightarrow {}_{k}(\overline{\rho},\overline{\varphi})^{\ell}$, $k \in_{k} L \leftrightarrow \{o, o\pi, \pi\varepsilon, \varepsilon\}$.

Thanks to the strong connections of $\tilde{\omega}^{\ell}$ components through their inputs, outputs and states, the ability of ω^{ℓ} reconstructing from its parts as well as calculating unknown parts from its known ones was obtained.

The following relations are revealed. Process ${}_{\pi}S^{\ell} = \{{}_{o\pi}S^{\ell}, {}_{\varepsilon\pi}S^{\ell}\}$ bonds the systems ${}_{o}S^{\ell}$ and ${}_{\varepsilon}S^{\ell}$. Dynamic realization ${}_{\pi}(\overline{\rho},\overline{\varphi})^{\ell} = \{ {}_{o\pi}(\overline{\rho},\overline{\varphi})^{\ell}, {}_{\varepsilon\pi}(\overline{\rho},\overline{\varphi})^{\ell} \}$ as well as ${}_{\omega}\gamma^{\ell}$ are the relations of models ${}_{o}(\overline{\rho},\overline{\varphi})^{\ell}$ and ${}_{\varepsilon}(\overline{\rho},\overline{\varphi})^{\ell}$ in $\tilde{\omega}^{\ell}$. The difference of ${}_{o}(\overline{\rho},\overline{\varphi})^{\ell}$ and ${}_{o\pi}(\overline{\rho},\overline{\varphi})^{\ell}$ models (similar to ${}_{\varepsilon}(\overline{\rho},\overline{\varphi})^{\ell}$ and ${}_{\varepsilon\pi}(\overline{\rho},\overline{\varphi})^{\ell}$) is caused by the fact that the processes, i.e. actions, have their own sense in the coordination tasks described below.

With regard to the constructed dynamic realization of the system being designed, the main three tasks, which coordinator S_0^{ℓ} performs in ω^{ℓ} are singled out:

on *selection layer* S_0^{ℓ} receives the coordination signals $\gamma^{\ell+1}$ from higher level and forms feedback signals $w^{\ell+1}$ by the way which takes into account the running level of information uncertainty in ω^{ℓ} model;

on *learning* layer S_0^ℓ adapts models ω^ℓ to concrete system S^ℓ and makes concrete the parameters of abstract system $\tilde{\omega}^\ell$ components (removes the uncertainty);

on *self-organization* layer S_0^ℓ can change model ω^ℓ and both ways of its adaptation to concrete systems and feedback signal $w^{\ell+1}$ forming, i.e. the ways of solving the tasks of choice and learning layers.

Models ${}_o(\overline{\rho},\overline{\varphi})^\ell$, ${}_\pi(\overline{\rho},\overline{\varphi})^\ell$ and ${}_\varepsilon(\overline{\rho},\overline{\varphi})^\ell$ are constructed from the information about interactions ${}_\omega\gamma^\ell$ of system S^ℓ with environment ${}_\varepsilon S^\ell$ and can be obtained by coordinators S_0^ℓ, $S_0^{\ell+1}$ and S_{0i}^ℓ, $i \in I^{\ell+1}$ ($S_{0i}^\ell \subset {}_\varepsilon S^\ell$). Therefore, coordinator S_0^ℓ of each system of level ℓ can predict changes in system $S^{\ell+1}$ and in part controls them, performing in this way the functions of coordinator $S_0^{\ell+1}$ of higher level $\ell+1$.

Aggregated dynamic realization ω^ℓ of each system S^ℓ is constructed from the elements of this system by the laws of level ℓ. On the other hand, ω^ℓ is formed from connecting elements of system S^ℓ with environment ${}_\varepsilon S^\ell$, i.e. from the elements of other systems of level ℓ, and contains information about the individual characteristics of these systems. Therefore, ω^ℓ begins to reflect the general laws of higher level systems $S^{\ell+1}$ and can, therefore, be regarded as a part of coordinator $S_0^{\ell+1}$. In this way coordinator S_0^ℓ is connected with $S_0^{\ell+1}$ and continuous connection of discrete level is obtained.

3.1.2. Structure

The model of mechatronic system structure is introduced for the construction description of an object being designed. The structure is defined as follows.

Definition 3.3. The next formal system is called a structure σ^ℓ :

$$\sigma^\ell \leftrightarrow \{S_0^\ell,\ \tilde{\sigma}^\ell\}, \tag{3.13}$$

and $\tilde{\sigma}^{\ell}$ is as follows:

$$\tilde{\sigma}^{\ell} \leftrightarrow \{\{\{\omega_i : i \in I\}^{\ell \pm \tau} : \tau \in L^s\}, {}_{\sigma}\gamma\}^{\ell} \leftrightarrow \{\{\sigma^{\ell \pm \tau} : \tau \in L^s\}, \omega^{\ell \pm 0}\}^{\ell}, \quad (3.14)$$

where:

S_0^{ℓ} is coordinator,

$\overline{\omega}^{\ell \pm \tau}$ are aggregated dynamic models of subsystems,

$\overline{S}^{\ell \pm \tau} = \{S_i^{\ell \pm \tau} : i \in I^{\ell \pm \tau}\}, \ \tau \in L^s$,

${}_{\sigma}\gamma^{\ell}$ are connections of the subsystems,

${}_{\sigma}\gamma^{\ell} \supset {}_{\omega}\overline{\gamma}^{\ell \pm t} = \{{}_{\omega}\gamma_i^{\ell \pm \tau} : i \in I^{\ell \pm \tau}\}$.

$\tilde{\sigma}^{\ell}$ is the connection of dynamic systems $\overline{\omega}^{\ell \pm \tau}$ and their interactions ${}_{\sigma}\gamma^{\ell}$ coordinated with ${}_{\omega}\gamma^{\ell} = {}_{\sigma}\gamma^{\ell \pm \tau}\big|S^{\ell}$. The dynamics of structural interconnections ${}_{\sigma}\gamma^{\ell}$ illustrates the dynamics of system S^{ℓ} organization from moment $0^{\ell} \in T_{\psi}^{\ell}$.

Aggregated dynamic models $\overline{\omega}^{\ell-1}$ are formed both by coordinators $\overline{S}_0^{\ell-1}$ of systems $\overline{S}^{\ell-1}$ and coordinator S_0^{ℓ} of a higher level, i.e. $\overline{\omega}^{\ell-1}$ are the interlevel connections of coordinators S_0^{ℓ} and $\overline{S}_0^{\ell-1}$.

3.1.3. Coordinator

For the description of the design system and its functions in the design process the model of coordinator is constructed in this section. The first version of the model was initially described in [3] and later improved and presented in [4-8,49].

Coordinator S_0^{ℓ} is the main element of hierarchical systems which realizes the processes of mechatronic systems design and control. It is defined in the following form in accordance with *aed* representation (3.1).

Definition 3.4. The following system S_0^ℓ is called a coordinator:

$$S_0^\ell \leftrightarrow \{\, \omega, S_0 \,, \sigma\}_0^\ell \;, \tag{3.15}$$

where: ω_0^ℓ is aggregated dynamic realization of S_0^ℓ,

σ_0^ℓ is the structure of S_0^ℓ,

S_{00}^ℓ is coordinator control element.

That is S_0^ℓ has own aggregated dynamical realization ω_0^ℓ and structure σ_0^ℓ . S_0^ℓ is defined recursively. Coordinator constructs its aggregated dynamic realization ω_0^ℓ and structure σ_0^ℓ by itself. The development of coordinator formal model on each level $\ell \in L$ goes from its initial state $S_0^\ell \big| T_\psi^\ell$ from moment $0^\ell \in \overline{T}_\psi^{\,\ell} = T_{t_\psi t_\chi}^\ell \subset T^\ell$ simultaneously with *aed* formal model $S^\ell \big| T_\psi^\ell$ development for level ℓ .

Dynamic realization ω_0^ℓ of coordinator S_0^ℓ is constructed from the information about its interactions ${}_\omega\gamma_0^\ell$ with coordinator environment ${}_\varepsilon S_0^\ell$:

$${}_\omega\gamma_0^\ell = \{\, X_0^\ell ,\; {}_\sigma\gamma_0^\ell ,\, Y_0^\ell \;\} = {}_\sigma\gamma_0^{\ell \pm \tau} ,\;\; \tau \in L^s ,\;\; \tau \neq 0 \tag{3.16}$$

where X_0^ℓ is input of S_0^ℓ, Y_0^ℓ is output of S_0^ℓ, ${}_\sigma\gamma_0^\ell$ are structural connections of S_0^ℓ.

With this aim, the inputs, outputs, both structural ${}_\sigma\gamma_0^\ell$ and external ${}_\omega\gamma_0^\ell$ connections of coordinator were defined on the sets of its coordination signals G^ℓ and feedbacks W^ℓ as follows:

$$X_0^\ell = \{\, G^{\ell+1}, W^\ell \,\}, \quad Y_0^\ell = \{\, G^\ell , W^{\ell+1} \},\tag{3.17}$$

$${}_\sigma\gamma_0^\ell = \{\, W^\ell ,\, \sigma^\ell ,\, G^\ell \,\} = {}_\omega\gamma_0^{\ell | \ell-1} \;,\; \{\, G^{\ell+1} ,\, \sigma^\ell ,\, W^{\ell+1} \} \;=\; {}_\omega\gamma_0^{\ell | \ell+1} ,$$

where: G^{ℓ} – coordination signals for systems $\bar{S}^{\ell-1}$, W^{ℓ} – feedback from $\bar{S}^{\ell-1}$, $W^{\ell+1}$ – feedback from S_0^{ℓ} to coordinator $S_0^{\ell+1}$ of higher level, $G^{\ell+1}$ – coordination signals from $S_0^{\ell+1}$ to S_0^{ℓ}.

The meanings of γ^{ℓ} and w^{ℓ} were revealed. For each moment $t \in T^{\ell}$ *coordination signals* $\gamma_t^{\ell} \in G^{\ell}$ contain the forecast of meanings of structure σ^{ℓ} parameters for period T_t^{ℓ}:

$$\gamma_t^{\ell} = \{\sigma^{\ell} \mid T_t^{\ell}\}^{\ell}. \tag{3.18}$$

Feedback signal $w^{\ell t} \in W^{\ell}$ transfers to coordinator S_0^{ℓ} the actual values of all the parameters of structure $\sigma^{\ell}\Big|\bar{T}^{\ell t}$ as well as predictions of $\sigma^{\ell}\Big|T_t^{\ell}$ made by the coordinators of systems $\bar{S}^{\ell-1}$ of lower level:

$$w^{\ell t} = \{\sigma^{\ell}\Big|\bar{T}^{\ell t}, \sigma^{\ell}\Big|T_t^{\ell}\}^{\ell-1}. \tag{3.19}$$

It is shown, that feedback w^{ℓ} contains the analogue of coordination signals γ^{ℓ}, i.e. each coordinator $S_0^{\ell-1}$ of lower level ℓ-1, being the part of coordinator S_0^{ℓ} can fulfil its functions, as stated above.

Signals $\gamma_t^{\ell+1}$ and $w^{(\ell+1)t}$ of higher level $\ell+1$ were presented just as in (3.16-3.17):

$$\gamma_t^{\ell+1} = \{\sigma^{\ell+1}\Big|T_t^{\ell+1}\}^{\ell+1}, \quad w^{(\ell+1)t} = \{\sigma^{\ell+1}\Big|\bar{T}^{(\ell+1)t}, \sigma^{\ell+1}\Big|T_t^{\ell+1}\}^{\ell}. \tag{3.20}$$

The availability of coordinator control element S_{00}^{ℓ} allows evaluation and changing of coordinator S_0^{ℓ} by itself. Let:

$$\lambda \leftrightarrow \ell \pm \tau_{\lambda}, \quad \varphi \leftrightarrow \lambda \pm \tau_{\varphi}, \quad \chi \leftrightarrow \varphi \pm \tau_{\chi}, \tag{3.21}$$

$$\psi \leftrightarrow \chi \pm \tau_{\psi}, \quad ? \leftrightarrow \psi \pm \tau_{?}, \qquad \dots,$$

$${}^{\beta}L \leftrightarrow \{\lambda, \varphi, \chi, \psi, ?\dots\}.$$

Then $S_{00}^{\ell} \leftrightarrow {}^{\beta}S_{0}^{\ell}$ & $\beta \in L^{\beta}$; ${}^{\beta}S_{0}^{\ell}$ is contraction of system S^{β} on the S^{ℓ}:${}^{\beta}S_{0}^{\ell} \leftrightarrow S^{\beta}/S^{\ell}$ and

$$
\begin{aligned}
&{}^{\lambda}S_{0}^{\ell} \leftrightarrow \{\, {}^{\lambda}\omega, {}^{\varphi}S_{0}, {}^{\lambda}\sigma\}_{0}^{\ell}, \\
&{}^{\varphi}S_{0}^{\ell} \leftrightarrow \{\, {}^{\varphi}\omega, {}^{\chi}S_{0}, {}^{\varphi}\sigma\}_{0}^{\ell}, \\
&{}^{\chi}S_{0}^{\ell} \leftrightarrow \{\, {}^{\chi}\omega, {}^{\psi}S_{0}, {}^{\chi}\sigma\}_{0}^{\ell}, \ldots
\end{aligned}
\tag{3.22}
$$

Systems ${}^{\beta}S_{0}^{\ell}$ are strata of coordinator S_{0}^{ℓ} and β is the outlook in the level space. Structure of coordinator S_{0}^{ℓ} is presented as a multi-strata hierarchical system in agreement with [2]. Functions of each stratum are formulated as presented below.

The main design task is performed on the *selection stratum* ${}^{\lambda}S_{0}^{\ell}$ when the strategies of ${}^{\lambda}S_{0}^{\ell}$ (processes ${}_{o\pi}^{\lambda}S_{0}^{\ell}$) connect the structure dynamics $\sigma^{\ell-\tau}$ and $\sigma^{\ell+\tau}$ with using ω^{ℓ}. Coordinator S_{0}^{ℓ} produces actual values of coordination signals $G^{\ell+1}$, G^{ℓ} and receives feedback signals W^{ℓ}, $W^{\ell+1}$ on its *selection stratum*. The ways of generation and receptions of signals (*coordination strategies*) depend on coordinator state C_{0}^{ℓ} and state ${}_{\varepsilon}C_{0}^{\ell}$ of its environment ${}_{\varepsilon}S_{0}^{\ell}$. For each way there are several levels of information uncertainty and system S^{ℓ} organization.

The information uncertainty of S_{0}^{ℓ} increases with the distance from ℓ. Every level ${}_{\beta}\tau$ of uncertainty on every strata ${}^{\beta}S_{0}^{\ell}$ has its own coordination strategy. The change of strategies ${}_{o\pi}^{\lambda}S_{0}^{\ell}$ is executed by *learning layer* ${}^{\varphi}S_{0}^{\ell}$ (the moment it can do it) and it is controlled by the following (*self-organization*) ${}^{\chi}S_{0}^{\ell}$ stratum. At the same time the outlook in the level space extends from $\lambda \leftrightarrow \ell \pm \tau_{\lambda}$ to $\varphi \leftrightarrow \lambda \pm \tau_{\varphi}$. Uncertainty removal in S_{0}^{ℓ} outlook is equivalent of system organization increasing (the increasing of interactions level), when ${}^{\lambda}S_{0}^{\ell}$ realizations are united and multiplied by ${}^{\beta}S^{\ell}$ $(\beta > \lambda)$, which realize the level increasing process in hierarchic space S^{ℓ} .

On the *learning stratum* coordinator S_0^ℓ changes the coordination strategies removing in this way uncertainty and improving the organization level of system S^ℓ. In case when the formal model of *aed* $S^\ell \big| T^\ell$ for level ℓ is known, the task of uncertainty removing is reduced to the task of selection of a concrete variant of the formal model which is used for generating signals G^ℓ, $W^{\ell+1}$ and receiving information $G^{\ell+1}$, W^ℓ.

Formal model $S^\ell \big| T^\ell$ is constructed on *self-organization stratum* ${}^\chi S_0^\ell$ of the coordinator, i.e. this stratum can change the state of both learning and selection strata.

Therefore, the coordination strategies on selection stratum correspond to the coordinator S_0^ℓ reactions for different states, and the functions of learning and self-organization strata correspond to the functions of states transition.

Aggregated dynamic representation ω_0^ℓ of coordinator S_0^ℓ is presented by analogy with (3.11) as follows:

$$\omega_0^\ell \leftrightarrow \{\ \{\{{}_o\,\omega_0, {}_{o\pi}\,\omega_0\}, {}_\omega\gamma_0, \{{}_{\pi\varepsilon}\,\omega_0, {}_\varepsilon\,\omega_0\}\}^\ell, S_0^\ell\} \leftrightarrow \{\ \tilde{\omega}_0^\ell, S_0^\ell\}\ . \qquad (3.23)$$

Where $\tilde{\omega}_0^\ell$ is presented in the following form:

$$\tilde{\omega}_0^\ell \leftrightarrow \{\{{}_o\,\omega_0, {}_{o\pi}\,\omega_0\}, {}_\omega\gamma_0, \{{}_{\pi\varepsilon}\,\omega_0, {}_\varepsilon\,\omega_0\}\}^\ell \leftrightarrow \qquad (3.24)$$

$$\leftrightarrow \{\{\omega_0^{\ell\pm0}, {}_\omega\gamma_0^\ell, \{\omega_0^{\ell\pm\tau} : \tau \in L^s, \tau \neq 0\}\} \leftrightarrow \{\{\omega_0^{\ell\pm\tau} : \tau \in L^s\}, {}_\omega\gamma\}^\ell,$$

and ${}_k\,\omega_0^\ell$ is as follows: ${}_k\,\omega_0^\ell \leftrightarrow {}_k(\bar{\rho},\bar{\varphi})_0^\ell$, $k \in {}_k L \leftrightarrow \{o, o\pi, \pi\varepsilon, \varepsilon\}$. Here ${}_o(\bar{\rho},\bar{\varphi})_0^\ell$, ${}_\pi(\bar{\rho},\bar{\varphi})_0^\ell$ $=\{$ ${}_{o\pi}(\bar{\rho},\bar{\varphi})_0^\ell$, ${}_{\varepsilon\pi}(\bar{\rho},\bar{\varphi})_0^\ell$ $\}$ and ${}_\varepsilon(\bar{\rho},\bar{\varphi})_0^\ell$ are dynamic realizations of coordinating element ${}_o S_0^\ell$, coordination process ${}_\pi S_0^\ell$ $=\{$ ${}_{o\pi} S_0^\ell$, ${}_{\varepsilon\pi} S_0^\ell$ $\}$ which is decomposed into actions ${}_{o\pi} S_0^\ell$ which element ${}_o S_0^\ell$ performs in its environment ${}_\varepsilon S_0^\ell$ and actions ${}_{\varepsilon\pi} S_0^\ell$ of ${}_\varepsilon S_0^\ell$ with coordinating element ${}_o S_0^\ell$, and the coordinator environment ${}_\varepsilon S_0^\ell$ respectively. Each component of the dynamic representation (3.24) are described just as in (3.2-3.6):

$$(\bar{\rho},\bar{\varphi})_0^\ell : \bar{\rho}_0^\ell = \{\rho_{0t} : C_{0t} \times X_{0t} \to Y_{0t}\ \&\, t \in T\}^\ell, \qquad (3.25)$$

$$\bar{\varphi}_0^\ell = \{{}_o\varphi_{0tt'} : C_{0t} \times X_{0tt'} \to C_{0t'}\ \&\, t, t' \in T\ \&\, t' > t\}^\ell.$$

The connections of states, inputs and outputs of coordinating element ${}_{o}S_0^{\ell}$, processes ${}_{\pi}S_0^{\ell}$ and coordinator environment ${}_{\varepsilon}S_0^{\ell}$ are shown in Table 3.2.

Table 3.2. Connections of states, inputs and outputs of coordinating element ${}_{o}S_0^{\ell}$, processes ${}_{\pi}S_0^{\ell}$ and environment ${}_{\varepsilon}S_0^{\ell}$ of coordinator

${}_{\omega}\gamma_0^{\ell}$	C^{ℓ}	X^{ℓ}	Y^{ℓ}
${}_{o}S_0^{\ell}$	C_0^{ℓ}	C_0^{ℓ}	Y_0^{ℓ}
${}_{o\pi}S_0^{\ell}$	X_0^{ℓ}	C_0^{ℓ}	Y_0^{ℓ}
${}_{\varepsilon\pi}S_0^{\ell}$	Y_0^{ℓ}	${}_{\varepsilon}C_0^{\ell}$	X_0^{ℓ}
${}_{\varepsilon}S_0^{\ell}$	${}_{\varepsilon}C_0^{\ell}$	${}_{\varepsilon}X_0^{\ell}$	${}_{\varepsilon}Y_0^{\ell}$

Dynamic realization ω_0^{ℓ} of coordinator is defined on its structure σ_0^{ℓ} which contains a set of coordination strategies $\{\bar{\rho}_0^{\ell}\}$ and their connections $\{\bar{\varphi}_0^{\ell}\}$, i.e. coordination subsystems $(\bar{\rho},\bar{\varphi})_0^{\ell}$ in S_0^{ℓ}.

Coordination strategies are realized through the elements of coordinator subsystems $S_0^{\ell-\tau}, \tau \in L^s, \tau \neq 0$ of lower levels which are connected by ${}_{\sigma}\gamma_0^{\ell}$. In other words each strategy has its own structure which is improved in accordance with the changes of the organization level of system S^{ℓ} and of the level of information uncertainty of its coordinator S_0^{ℓ}.

Dynamic model ω_0^{ℓ} is the initial stage of future investigation of coordinator S_0^{ℓ} and is still not concrete enough.

The next step is the introduction of the states space structure on the objects of initial states of model ω_0^{ℓ} components and the transition to the canonical model of the coordinator.

Canonical model $\hat{\omega}_0^{\ell}$:

$$\hat{\omega}_0^{\ell} \leftrightarrow \{\{{}_{o}(\bar{\hat{\varphi}},\bar{\hat{\lambda}})_0^{\ell}, {}_{\pi}(\bar{\hat{\varphi}},\bar{\hat{\lambda}})_0^{\ell}, {}_{\varepsilon}(\bar{\hat{\varphi}},\bar{\hat{\lambda}})_0^{\ell}\}, S_0^{\ell}\} \leftrightarrow \{\tilde{\hat{\omega}}_0^{\ell}, S_0^{\ell}\}, \tag{3.26}$$

is constructed on the basis of multi-layer system conception [2], where functions $\{ \bar{\hat{\varphi}}_0^{\ell} \}$ correspond to *learning* and *self-organization* strata and functions $\{ \bar{\hat{\lambda}}_0^{\ell} \}$ correspond to coordination strategies on *selection* stratum. The run of the process of canonical model constructing is described below in Section 3.2.

3.2. Canonical model of coordinator

To describe the dynamics of both mechatronic object being designed and its design system, the different states of system S^{ℓ} and its coordinator S_0^{ℓ} canonical model are defined in this Section. The descriptions of the states are presented for different moments of time, which correspond to the consequent stages of mechatronic system creation and elimination of information uncertainty of its coordinator. For each state of the system the most rational design and control strategies of coordinator are formulated as presented below.

The initial moment of both system S^{ℓ} of level ℓ and its coordinator S_0^{ℓ} arising is $t = 0^{\ell} = t_{\psi}^{\ell} \in T^{\ell}$ (T^{ℓ} is time of level ℓ), when at least one system $S^{\ell_{\psi}}$ of level ℓ_{ψ} ($\ell_{\psi} \in L,\ \ell_{\psi} < \ell$) appeared. Index ψ corresponds to the lowest organization of systems of level ℓ .

The initial period of S^{ℓ} and S_0^{ℓ} is the period of time $T_{\psi}^{\ell} = \bar{T}_{t_{\psi} t_{\chi}}^{\ell} \subset T^{\ell}$, where t_{χ}^{ℓ} is the moment, when the set of elements $\bar{S}^{\ell}$ are united in system $S^{\ell_{\chi}}$ of level ℓ_{χ} , $\ell_{\chi} = \ell_{\psi} + 1$.

For the *moment of time* $t \in T_{?}^{\ell} \subset T^{\ell}$:

formal model of *aed* is defined by contraction $S^{\ell} \Big| T_{\psi}^{\ell} = S^{\ell \left| \ell_{\psi} \right.}$ and its conformity with concrete systems is approved experimentally for levels $\tilde{\ell} \leq \ell_{\psi}$ only;

concrete bearers of formal model $S^{\ell \left| \ell_{\psi} \right.}$ are systems $\bar{S}^{\ell_{\chi}}$;

synthesis laws of level ℓ system are unknown but can be defined by the extrapolation $S^{\ell|\ell_\psi}\Big|T^{\ell} = {}_{\psi}S^{\ell}$;

the form of representation of coordination signals $\gamma^{\ell} \in \Gamma^{\ell}$ and feedback signals $w^{\ell} \in W^{\ell}$ is system ${}_{\psi}S^{\ell}$;

system $S^{\ell}\Big|T_{\psi}^{\ell}$ is in the state of the weakest organization (the difference $\xi_{\psi}^{\ell} = \ell - \ell_{\psi}$ between system level and level of its elements runs up to maximum);

coordinator $S_0^{\ell}\Big|T_{\psi}^{\ell}$ is in the state of the highest uncertainty of information;

signals $\gamma^{\ell+1} \in \Gamma^{\ell+1}$ and $w^{\ell+1} \in W^{\ell+1}$ should be regarded as undefined because system $S^{\ell+1}$ is unknown.

The most rational coordination strategy for such conditions is the *coordination of interactions* when coordinator S_0^{ℓ} allows the systems $\bar{S}^{\ell_\psi}$ to make a free choice of concrete parameters of interactions ${}_{\omega}\bar{\gamma}^{\ell_\psi}$ and, consequently, any dynamic realizations $\bar{\tilde{\omega}}^{\ell_\psi}$ of systems $\bar{S}^{\ell_\psi}$ and its any allowed combinations σ^{ℓ_χ}.

From moment t_{χ}^{ℓ} to t_{φ}^{ℓ} ($t_{\chi}^{\ell}, t_{\varphi}^{\ell} \in T_{\chi}^{\ell} = \bar{T}_{t_\chi, t_\varphi}^{\ell} \subset T^{\ell}$) system S^{ℓ} achieves the state of partial organization. Moment t_{χ}^{ℓ} can be fixed when at least one system S^{ℓ_χ} of level ℓ_{χ}

($\ell_{\chi} = \ell - \xi_{\chi}^{\ell} = (\ell+1)_{\psi}$) reaches the state of complete organization. t_{φ}^{ℓ} is the moment when system S^{ℓ_φ} ($\ell_{\varphi} = \ell - \xi_{\varphi}^{\ell} = (\ell+1)_{\chi}$) has been formed.

For this period of time T_{χ}^{ℓ} coordination strategy in σ^{ℓ} is *evaluation of interactions.* While realizing this strategy coordinator S_0^{ℓ} defines the coordination signals γ^{ℓ} for each new system of level ℓ by bounds of concrete values and standards with the evaluation of belonging to this standard state. Coordination signals $\gamma^{\ell+1}$ and feedback signals $w^{\ell+1}$ are formed in T_{χ}^{ℓ} by the coordination of interactions on level $\ell+1$.

For the moment t_λ^ℓ ($t_\lambda^\ell \in T_\varphi^\ell = \overline{T}_{t_\varphi, t_\lambda}^\ell \subset T^\ell$, $t_\lambda^\ell \in T_\lambda^\ell = T_{t_\lambda}^\ell \subset T^\ell$) structure σ^ℓ of system S^ℓ becomes sufficiently organized and S^ℓ is regarded as system $S^{\ell\lambda}$ of level ℓ ($\ell_\lambda = \ell = (\ell+1)_\varphi = (\ell+2)_\chi = (\ell+3)_\psi$).

For *moment of time* $T_\varphi^\ell \subset T^\ell$:

formal model S^ℓ goes to state $S^{\ell|\ell_\lambda}$ of full certainty with laws of level ℓ of synthesis of structures σ^ℓ;

concrete bearers of $S^{\ell|\ell_\lambda}$ are systems S^ℓ;

the synthesis laws of level ℓ systems become known, and the laws of their functioning in $S^{\ell+1}$ are known with the accuracy of parameters concrete value;

the form of coordination signals γ^ℓ and feedback signals w^ℓ representation is determined system $S^{\ell|\ell_\lambda}\big|T^\ell = {}_\lambda S^\ell$;

it is possible to find the range of systems $\overline{S}^{\ell\varphi}$ sufficiently organized by σ^ℓ ;

coordination systems $\overline{S}_0^\ell$ are in the state of minimal information uncertainty; formal models of systems $S^{\ell+1}$ and $S_0^{\ell+1}$ are realized in states ${}_\varphi S^{\ell+1}$ and ${}_\varphi S_0^{\ell+1}$; models of systems $S^{\ell+2}$ and $S_0^{\ell+2}$ are in states ${}_\chi S^{\ell+2}$ and ${}_\chi S_0^{\ell+2}$ accordingly; systems ${}_\psi S^{\ell+3}$ and ${}_\psi S_0^{\ell+3}$ have been formed.

Coordination strategy in structure σ^ℓ in this case is *interconnections prediction*.

For the *period of time* $T_\lambda^\ell = T_{t_\lambda}^\ell$, which begins from moment t_λ^ℓ, the task of $S^\ell\big|T^\ell$ changing is the task of higher levels.

Summarizing, it should be pointed out that on each level $\ell \in L$ the formal model S^ℓ is consecutively changed from state ${}_\psi S^\ell$ to states ${}_\chi S^\ell$, ${}_\varphi S^\ell$, ${}_\lambda S^\ell$ and the following quality ${}_S\Psi$ is true:

$({}_S\Psi)$

$$\left[\left(S^\ell \middle| T^\ell =_\lambda S^\ell\right) \& (\exists t''') \left(t''' =_\varphi t^{\ell+1} \in_\varphi T^{\ell+1}\right) \left(t''' =_\chi t^{\ell+2} \in_\chi T^{\ell+2}\right) \left(t''' =_\psi t^{\ell+3} \in_\psi T^{\ell+3}\right) \Rightarrow \right. \quad (3.27)$$

$$\left. \Rightarrow \left(S^\ell \middle| T^{\ell+1} =_\varphi S^{\ell+1}\right) \& \left(S^\ell \middle| T^{\ell+2} =_\chi S^{\ell+2}\right) \& \left(S^\ell \middle| T^{\ell+3} =_\psi S^{\ell+3}\right)\right],$$

where symbol ψ means the lowest certainty (highest information uncertainty) of the corresponding value, and symbol λ means the highest certainty.

Abstract system ${}_s\Psi = \{ {}_\psi S^\ell, {}_\chi S^\ell, {}_\varphi S^\ell, {}_\lambda S^\ell \}$ of states of formal model S^ℓ codes the multitude of ways of information representation on each level $\ell \in L$ and is the basis for canonical model of coordinator S_0^ℓ.

3.2.1. Presentation of coordinating element

Similar to *aed* dynamic realization (3.11), which was constructed for mechatronic object, its process and object's environment, the coordinator dynamic realization is constructed for coordinating element – design system analogues, coordination process and environment of coordinator. The process of the creating of canonic presentation of coordinating element is described below.

The canonic representation is constructed on the external interactions ${}_\omega\gamma_0^\ell$ of coordinator. The external connections ${}_\omega\gamma_0^\ell$ of coordinator S_0^ℓ depend on the organization level of mechatronic system S^ℓ and information uncertainty of its coordinator S_0^ℓ.

First, the following mapping is marked by symbol ${}_\Psi R$ as follows:

$${}_\Psi R : {}_\omega\gamma_0^\ell \to \Psi^\ell . \quad (3.28)$$

After that, the introduced equivalence relations $E_\gamma \subset {}_\omega\gamma_0^\ell \times {}_\omega\gamma_0^\ell$:

$$({}_\omega\gamma_{01}^\ell, {}_\omega\gamma_{02}^\ell) \in E_\gamma \Leftrightarrow {}_\Psi R({}_\omega\gamma_{01}^\ell) = {}_\Psi R({}_\omega\gamma_{02}^\ell), \quad (3.29)$$

make it possible to construct the following multitudes:

$$ {}_{\omega}\gamma_0^{\ell}\big|E_{\gamma} = \{ {}_{\psi\omega}\gamma_0^{\ell},\ {}_{\chi\omega}\gamma_0^{\ell},\ {}_{\varphi\omega}\gamma_0^{\ell},\ {}_{\lambda\omega}\gamma_0^{\ell} \}, \tag{3.30} $$

where ${}_{\psi\omega}\gamma_0^{\ell}$, ${}_{\chi\omega}\gamma_0^{\ell}$, ${}_{\varphi\omega}\gamma_0^{\ell}$ and ${}_{\lambda\omega}\gamma_0^{\ell}$ are equivalence classes of coordinator S_0^{ℓ} connections by uncertainty levels of information.

Equivalence classes for coordinator states $c_{0t}^{\ell} \in C_{0t}^{\ell}$ are introduced to construct the state space of the coordinator.

Set $\tilde{C}_0^{\ell} = \bigcup_{t \subset T^{\ell}} C_{0t}^{\ell}$ and equivalence ratio are defined by the following expression $E_c \subset \tilde{C}_0^{\ell} \times \tilde{C}_0^{\ell}$:

$$ (c_{t1}^{\ell}, c_{t2}^{\ell})_0 \in E_c \Leftrightarrow \tag{3.31} $$

$$ \Leftrightarrow [(\forall t \in T^{\ell})(\forall x_{00}^{\ell}) \Rightarrow {}_{\Psi}R(({}_{o}\rho_0^{\ell}(c_{01}^{\ell}, x_0^{\ell})\big|T_t^{\ell})_0) $$

$$ = {}_{\Psi}R(({}_{o}\rho_0^{\ell}(c_{02}^{\ell}, x_0^{\ell})\big|T_t^{\ell})_0)]\ \& $$

$$ \& [(\forall t' > t)(\forall x_{0tt'}^{\ell}) \Rightarrow ({}_{o}\varphi_{0tt'}^{\ell}(c_{0t1}^{\ell}, x_{0tt'}^{\ell}),\ {}_{o}\varphi_{0tt'}^{\ell}(c_{0t2}^{\ell}, x_{0tt'}^{\ell})) \in E_c]. $$

Factor multitude $C_0^{\ell} = \tilde{C}_0^{\ell}\big|E_c = \{ {}_{\psi}C_0^{\ell},\ {}_{\chi}C_0^{\ell},\ {}_{\varphi}C_0^{\ell},\ {}_{\lambda}C_0^{\ell} \}$ is called the *states space* of coordinating element ${}_{o}S_0^{\ell}$, where ${}_{\psi}C_0^{\ell}$, ${}_{\chi}C_0^{\ell}$, ${}_{\varphi}C_0^{\ell}$, ${}_{\lambda}C_0^{\ell}$ are the classes of states equivalence of ${}_{o}S_0^{\ell}$ by information uncertainty levels.

The state transition function in the introduced state space C_0^{ℓ} is defined by the next expression:

$$ {}_{o}\hat{\varphi}_{0tt'}^{\ell} : (\tilde{C}_0^{\ell}\big|E_c) \times X_{0tt'}^{\ell} \to (\tilde{C}_0^{\ell}\big|E_c), \tag{3.32} $$

where ${}_{o}\hat{\varphi}_{0tt'}^{\ell}([c_0^{\ell}], x_{0tt'}^{\ell}) = [{}_{o}\hat{\varphi}_{0tt'}^{\ell}(I_t([c_0^{\ell}]), x_{0tt'}^{\ell})]$, and $I_t : \tilde{C}_0^{\ell}\big|E_c \to C_{0t}^{\ell}$ such as:

$$ I_t([c_0^{\ell}]) = \begin{cases} c_{0t}^{\ell}\ \&\ c_{0t}^{\ell} \in [c_0^{\ell}] \cap C_{0t}^{\ell} \\ c_{0t}^{\ell *} \in C_{0t}^{\ell}\ \&\ [c_0^{\ell}] \cap C_{0t}^{\ell} = \varnothing, \end{cases} \tag{3.33} $$

where $C_{0t}^{\ell *}$ is an arbitrary element from state space C_{0t}^{ℓ}, I_t is mapping.

The composition conditions (i)-(iii) for function ${}_o\hat{\varphi}_{0tt'}^{\ell}$ are presented similarly as for conditions (3.10) for function ${}_o\overline{\varphi}^{\ell}$.

The output function for states space C_0^{ℓ} is presented in the following form:

$$
{}_o\hat{\lambda}_{0t}^{\ell} : (\widetilde{C}_0^{\ell} | E_c) \times \hat{\tilde{X}}_0^{\ell} \to \hat{\tilde{Y}}_0^{\ell}, \tag{3.34}
$$

where: $\hat{\tilde{X}}_0^{\ell}$ is input alphabet; $\hat{\tilde{Y}}_0^{\ell}$ is output alphabet of S_0^{ℓ}, and the following quality is true:

$$
{}_o\hat{\lambda}_{0t}^{\ell}([c_0^{\ell}], \widetilde{x}_0^{\ell}) = [{}_o\lambda_{0t}^{\ell}(I_t([c_0^{\ell}]), x_{0tt'}^{\ell})]. \tag{3.35}
$$

$$
\begin{tikzcd}
C_0^{\ell} \times \overline{X}_{0tt'}^{\ell} \arrow[d, "({}_o\hat{\varphi}_{0tt'}^{\ell}, R\{t'\})"'] & C_{0t}^{\ell} \times \overline{X}_{0tt'}^{\ell} \arrow[l, "{[\ ]\times I}"'] \arrow[d, "({}_o\varphi_{0tt'}^{\ell}, R^*\{t'\})"] \\
C_0^{\ell} \times \hat{\tilde{X}}_0^{\ell} \arrow[dr, "{}_o\hat{\lambda}_{0t'}^{\ell}"'] & C_0^{\ell} \times \overline{X}_{0tt'}^{\ell} \arrow[l, "{[\ ]\times I}"'] \arrow[d, "{}_o\lambda_{0t'}^{\ell}"] \\
& \tilde{Y}_0^{\ell}
\end{tikzcd}
$$

Fig. 3.2. Commutative diagram of output ${}_o\lambda_{0t'}^{\ell}$ and state transition ${}_o\hat{\varphi}_{0tt'}^{\ell}$ functions

From the execution of the last equation (3.35) and conditions (i)-(iii) for function ${}_o\hat{\varphi}_{0tt'}^{\ell}$ it is presumed that the diagram in Figure 3.2 is commutative, and the following system:

$$
{}_o\hat{\varphi}_{0tt'}^{\ell} : C_0^{\ell} \times X_{0tt'}^{\ell} \to C_0^{\ell}, \tag{3.36}
$$

$$
{}_o\hat{\lambda}_{0t}^{\ell} : C_0^{\ell} \times \hat{\tilde{X}}_0^{\ell} \to \hat{\tilde{Y}}_0^{\ell},
$$

is the *canonical presentation* of the dynamic system in the states space [9, 10]. In this particular case ${}_o(\bar{\hat{\varphi}}, \bar{\hat{\lambda}})_0^\ell$ is the canonic representation of coordinating element ${}_oS_0^\ell$ in the space of its states C_0^ℓ .

After that, there appeared a possibility to consider the coordination ways [2,3] as coordinator reaction family ${}_o\bar{\rho}_0^\ell$ or output functions ${}_o\bar{\hat{\lambda}}_0^\ell$ defined on the constructed states space. Coordination strategies are realized on *selection strata*. Therefore, the formal model of selection strata is defined by family of reactions and output functions as follows:

$$
{}_o\bar{\rho}_0^\ell = \{{}_o\rho_{0t}^\ell : C_{0t}^\ell \times X_{0t}^\ell \to Y_{0t}^\ell \;\&\; t \in T^\ell\}, \tag{3.37}
$$

$$
{}_o\bar{\hat{\lambda}}_{0t}^\ell = \{{}_o\hat{\lambda}_{0t}^\ell : C_0^\ell \times \hat{\tilde{X}}_0^\ell \to \hat{\tilde{Y}}_0^\ell \;\&\; t \in T^\ell\}.
$$

The family of output functions ${}_o\bar{\hat{\lambda}}_0^\ell$ is decomposed by coordination strategies in the following way:

$$
{}_{\psi o}\hat{\lambda}_{0t}^\ell : {}_\psi C_0^\ell \times \hat{\tilde{X}}_0^\ell \to {}_\psi\hat{\tilde{Y}}_0^\ell, \tag{3.38}
$$

$$
{}_{\chi o}\hat{\lambda}_{0t}^\ell : {}_\chi C_0^\ell \times \hat{\tilde{X}}_0^\ell \to {}_\chi\hat{\tilde{Y}}_0^\ell,
$$

$$
{}_{\varphi o}\hat{\lambda}_{0t}^\ell : {}_\varphi C_0^\ell \times \hat{\tilde{X}}_0^\ell \to {}_\varphi\hat{\tilde{Y}}_0^\ell,
$$

$$
{}_{\lambda o}\hat{\lambda}_{0t}^\ell : {}_\lambda C_0^\ell \times \hat{\tilde{X}}_0^\ell \to {}_\lambda\hat{\tilde{Y}}_0^\ell,
$$

where ${}_{\psi o}\hat{\lambda}_{0t}^\ell$ corresponds to the strategy of *interactions coordination,* ${}_{\chi o}\hat{\lambda}_{0t}^\ell$ – to the *evaluation of interactions,* ${}_{\varphi o}\hat{\lambda}_{0t}^\ell$ – to the *prediction of interactions* in structure σ^ℓ, and ${}_{\lambda o}\hat{\lambda}_{0t}^\ell$ corresponds to the *prediction of interactions in* ω^ℓ .

The change of coordinator states is executed on the learning and self-organization strata which are described by functions of states transition ${}_o\bar{\varphi}_0^\ell$ and ${}_o\bar{\hat{\varphi}}_0^\ell$ accordingly:

$$
{}_o\bar{\varphi}_0^\ell = \{{}_o\varphi_{0tt'}^\ell : C_{0t}^\ell \times X_{0tt'}^\ell \to C_{0t'}^\ell \;\&\; t, t' \in T^\ell \;\&\; t' > t\}, \tag{3.39}
$$

$$
{}_o\bar{\hat{\varphi}}_0^\ell = \{{}_o\hat{\varphi}_{0tt'}^\ell : C_0^\ell \times X_{0tt'}^\ell \to C_0^\ell \;\&\; t, t' \in T^\ell \;\&\; t' > t\}.
$$

Function ${}_{o}\bar{\hat{\varphi}}_0^{\ell}$ is decomposed into two functions – ${}_{\varphi o}\bar{\hat{\varphi}}_0^{\ell}$ and ${}_{\chi o}\bar{\hat{\varphi}}_0^{\ell}$, where ${}_{\varphi o}\bar{\hat{\varphi}}_0^{\ell}$ performs states transition on the learning stratum, and ${}_{\chi o}\bar{\hat{\varphi}}_0^{\ell}$ – on the self-organization stratum. So, only changes of coordination strategies are possible on the learning stratum. The form of their representation is fixed. On the self-organization stratum the forms of information Ψ^{ℓ} representation and processing ways can be changed.

3.2.2. Process of coordination

Dynamic realization ${}_{\pi}(\bar{\rho},\bar{\varphi})_0^{\ell}$ of coordination process ${}_{o\pi}S_0^{\ell}$ is decomposed into the model of process ${}_{o\pi}(\bar{\rho},\bar{\varphi})_0^{\ell}$ which the coordinating element performs in environment ${}_{\varepsilon}S_0^{\ell}$ and model ${}_{\varepsilon\pi}(\bar{\rho},\bar{\varphi})_0^{\ell}$ of process by which ${}_{\varepsilon}S_0^{\ell}$ constructs ${}_{o}S_0^{\ell}$:

$$ {}_{\pi}(\bar{\rho},\bar{\varphi})_0^{\ell} = \{ {}_{o\pi}(\bar{\rho},\bar{\varphi})_0^{\ell},\ {}_{\varepsilon\pi}(\bar{\rho},\bar{\varphi})_0^{\ell} \}. \tag{3.40} $$

$$ {}_{o\pi}(\bar{\rho},\bar{\varphi})_0^{\ell} :\ {}_{o\pi}\bar{\rho}_0^{\ell} = \left\{ {}_{o\pi}\rho_{0t} : X_{0t} \times C_{0t} \to Y_{0t}\ \&\, t \in T \right\}^{\ell}, $$

$$ {}_{o\pi}\bar{\varphi}_0^{\ell} = \left\{ {}_{o\pi}\varphi_{0tt'} : X_{0t} \times C_{0tt'} \to X_{0t'}\ \&\, t,t' \in T\ \&\, t' > t \right\}^{\ell}. $$

$$ {}_{\varepsilon\pi}(\bar{\rho},\bar{\varphi})_0^{\ell} :\ {}_{\varepsilon\pi}\bar{\rho}_0^{\ell} = \left\{ {}_{\varepsilon\pi}\rho_{0t} : {}_{\varepsilon}X_{0t} \times {}_{\varepsilon}C_{0t} \to {}_{\varepsilon}Y_{0t}\ \&\, t \in T \right\}^{\ell}, $$

$$ {}_{\varepsilon\pi}\bar{\varphi}_0^{\ell} = \left\{ {}_{\varepsilon\pi}\varphi_{0tt'} : {}_{\varepsilon}X_{0t} \times {}_{\varepsilon}C_{0tt'} \to {}_{\varepsilon}X_{0t'}\ \&\, t,t' \in T\ \&\, t' > t \right\}^{\ell}, $$

where ${}_{\varepsilon}X_0^{\ell} = \{ Y_0^{\ell}, {}_{\tau\varepsilon}X_0^{\ell} \}$ and ${}_{\varepsilon}Y_0^{\ell} = \{ X_0^{\ell}, {}_{\tau\varepsilon}Y_0^{\ell} \}$. ${}_{\tau\varepsilon}X_0^{\ell}$ and ${}_{\tau\varepsilon}Y_0^{\ell}$ are components of inputs and outputs of coordinator environment ${}_{\varepsilon}S_0^{\ell}$ which are controlled by coordinator S_0^{ℓ} only in part.

Each of the above models is also decomposed into two ones. Dynamic realization ${}_{o\pi}(\bar{\rho},\bar{\varphi})_0^{\ell}$ is decomposed into the processes in σ^{ℓ} and ω^{ℓ} and ${}_{\varepsilon\pi}(\bar{\rho},\bar{\varphi})_0^{\ell}$ into the processes of coordinator S_0^{ℓ} construction from lower (from $\sigma^{\ell-\tau}, t \ge 0$) and from higher (from $\sigma^{\ell+\tau}, t > 0$) levels:

$$ {}_{o\pi}(\overline{\rho},\overline{\varphi})_0^{\ell} = \{ {}_{o\pi}(\overline{\rho},\overline{\varphi})_0^{\ell/\ell-\tau},\ {}_{o\pi}(\overline{\rho},\overline{\varphi})_0^{\ell/\ell+\tau} \}, \tag{3.41} $$
$$ {}_{\varepsilon\pi}(\overline{\rho},\overline{\varphi})_0^{\ell} = \{ {}_{\varepsilon\pi}(\overline{\rho},\overline{\varphi})_0^{\ell/\ell-\tau},\ {}_{\varepsilon\pi}(\overline{\rho},\overline{\varphi})_0^{\ell/\ell+\tau} \}. $$

The states of process ${}_{o\pi}S_0^{\ell}$ are inputs X_0^{ℓ} of the coordinator and the process inputs are coordinator states C_0^{ℓ}. For the coordinator environment process ${}_{\varepsilon\pi}S_0^{\ell}$ the inputs are states ${}_{\varepsilon}C_0^{\ell}$ and the states are outputs Y_0^{ℓ} of the coordinator.

To construct the canonic presentation of the coordination processes, the classes of equivalence – similarly as for the levels of information uncertainty – are obtained by the restriction of interactions ${}_{\omega}\gamma_0^{\ell}/E_{\gamma}$ on X_0^{ℓ} and Y_0^{ℓ} respectively:

$$ X_0^{\ell}/E_{\gamma x} = ({}_{\omega}\gamma_0^{\ell}/E_{\gamma})/X_0^{\ell} = \{ {}_{\psi}X_0^{\ell},\ {}_{\chi}X_0^{\ell},\ {}_{\varphi}X_0^{\ell},\ {}_{\lambda}X_0^{\ell} \}, \tag{3.42} $$
$$ Y_0^{\ell}/E_{\gamma y} = ({}_{\omega}\gamma_0^{\ell}/E_{\gamma})/Y_0^{\ell} = \{ {}_{\psi}Y_0^{\ell},\ {}_{\chi}Y_0^{\ell},\ {}_{\varphi}Y_0^{\ell},\ {}_{\lambda}Y_0^{\ell} \}. $$

It was found out, that from quality ${}_{s}\Psi$ (3.27) follows, that the states of processes of levels $\ell/\ell+\tau$ and $\ell/\ell-\tau$ for $\tau=1$ are changed with respect to one another and as the result corresponding coordination strategies are also changed. Canonic model for coordination process ${}_{o\pi}S_0^{\ell}$ is described as follows:

$$ {}_{o\pi}(\overline{\hat{\varphi}},\overline{\hat{\lambda}})_0^{\ell/\ell-1}:\ {}_{o\pi}\overline{\hat{\varphi}}_0^{\ell/\ell-1} = \{ {}_{o\pi}\hat{\varphi}_{0tt'}^{\ell/\ell-1} : (\tilde{W}^{\ell}/E_{\gamma x}) \times C_{0tt'}^{\ell} \to (\tilde{W}^{\ell}/E_{\gamma x}) \}, \tag{3.43} $$
$$ {}_{o\pi}\overline{\hat{\lambda}}_0^{\ell/\ell-1} = \{ {}_{o\pi}\hat{\lambda}_{0t}^{\ell/\ell-1} : (\tilde{W}^{\ell}/E_{\gamma x}) \times \hat{\tilde{C}}_0^{\ell} \to \hat{\tilde{G}}^{\ell} \}. $$

Canonic models for processes ${}_{o\pi}S_0^{\ell/\ell+1}$, ${}_{\varepsilon\pi}S_0^{\ell/\ell-1}$ and ${}_{\varepsilon\pi}S_0^{\ell/\ell+1}$ are presented in the same form (3.43). The meanings of states, inputs and outputs of the coordination processes are presented in Table 3.3.

Table 3.3. Connections of states, inputs and outputs in canonic representation of coordination processes of different levels

${}_{\omega\pi}\gamma_0^{\ell}$	C^{ℓ}	X^{ℓ}	Y^{ℓ}
${}_{o\pi}S_0^{\ell/\ell-1}$	$\tilde{W}^{\ell}/E_{\gamma x}$	$\hat{\tilde{C}}_0^{\ell}$	$\hat{\tilde{G}}^{\ell}$
${}_{o\pi}S_0^{\ell/\ell+1}$	$\tilde{G}^{\ell+1}/E_{\gamma x}$	$\hat{\tilde{C}}_0^{\ell}$	$\hat{\tilde{W}}^{\ell+1}$
${}_{\varepsilon\pi}S_0^{\ell/\ell-1}$	$\tilde{G}^{\ell}/E_{\gamma y}$	${}_{\varepsilon}\hat{\tilde{C}}_0^{\ell/\ell-1}$	$\hat{\tilde{W}}^{\ell}$
${}_{\varepsilon\pi}S_0^{\ell/\ell+1}$	$\tilde{W}^{\ell+1}/E_{\gamma y}$	${}_{\varepsilon}\hat{\tilde{C}}_0^{\ell/\ell+1}$	$\tilde{G}^{\ell+1}/E_{\gamma y}$

It was pointed out, that models ${}_{\varepsilon\pi}(\bar{\hat{\varphi}},\bar{\hat{\lambda}})_0^{\ell}$ are controlled by coordinator S_0^{ℓ} only in part, because they contain components not fully defined.

For the description of coordination processes on different strata, the states transition functions of ${}_{o}S_0^{\ell}$ and ${}_{\varepsilon}S_0^{\ell}$ on the learning stratum, which change coordination strategies ${}_{\pi}\bar{\hat{\lambda}}_0^{\ell}$, were marked by index φ :

$$ {}_{\varphi\pi}\bar{\hat{\varphi}}^{\ell} = {}_{\varphi}\{{}_{o\pi}\bar{\hat{\varphi}}^{\ell}, {}_{\varepsilon\pi}\bar{\hat{\varphi}}^{\ell}\}. \tag{3.44} $$

The functions of states transition of process ${}_{\pi}S_0^{\ell}$ caused by self-organization of coordinating element ${}_{o}S_0^{\ell}$ were marked by index χ; functions, caused by self-organization of coordinator environment ${}_{\varepsilon}S_0^{\ell}$ – by index ψ:

$$ {}_{\psi\chi\pi}\bar{\hat{\varphi}}_0^{\ell} = \{{}_{\chi o\pi}\bar{\hat{\varphi}}_0^{\ell}, {}_{\psi\varepsilon\pi}\bar{\hat{\varphi}}_0^{\ell}\}, \tag{3.45} $$

$$ {}_{\chi o\pi}\bar{\hat{\varphi}}_0^{\ell} = \{{}_{\chi o\pi}\bar{\hat{\varphi}}_0^{\ell/\ell-1}, {}_{\chi o\pi}\bar{\hat{\varphi}}_0^{\ell/\ell+1}\}, $$

$$ {}_{\psi\varepsilon\pi}\bar{\hat{\varphi}}_0^{\ell} = \{{}_{\psi\varepsilon\pi}\bar{\hat{\varphi}}_0^{\ell/\ell-1}, {}_{\psi\varepsilon\pi}\bar{\hat{\varphi}}_0^{\ell/\ell+1}\}. $$

Formal models of the processes on selection $({}_{\pi}\bar{\hat{\lambda}}_0^{\ell} = \{{}_{o\pi}\bar{\hat{\lambda}}_0^{\ell}, {}_{\varepsilon\pi}\bar{\hat{\lambda}}_0^{\ell}\})$, learning $({}_{\varphi\pi}\bar{\hat{\varphi}}_0^{\ell})$ and self-organization $({}_{\psi\chi\pi}\bar{\hat{\varphi}}_0^{\ell})$ strata are constructed in the way similar to the corresponding models of the coordinating element (3.37-3.39).

3.2.3. Description of coordinator environment

The development of the model of coordinator environment ${}_{\varepsilon}S_0^{\ell}$ is conducted in a similar way as that of the coordinating element, bearing in mind the following facts. Coordinator S_0^{ℓ} of each concrete systems S^{ℓ} being designed is the connecting unit of lower $\ell-\tau, \tau \geq 0$ and higher $\ell+\tau, \tau > 0$ levels. For $\tau = 1$, the both coordinators $\overline{S}_0^{\ell-1}$ and coordinator $S_0^{\ell+1}$ are in direct contact with coordinator S_0^{ℓ} of level ℓ (trough σ^{ℓ} and $\omega^{\ell} = \sigma^{\ell+1}\Big| S^{\ell}$ respectively) and form its environment ${}_{\varepsilon}S_0^{\ell}$.

Coordinator S_0^{ℓ} connections with other coordinators $S_{0i}^{\ell}, i \in I^{\ell+1}$ of level ℓ also go through its structural elements ω^{ℓ} and σ^{ℓ}, in other words, trough higher and lower levels:

$$
{}_{\omega}\gamma_0^{\ell} = \{{}_{\omega}\gamma_0^{\ell/\ell+1}, \{\omega^{\ell}, \sigma^{\ell}\}, {}_{\omega}\gamma_0^{\ell/\ell-1}\}, \tag{3.46}
$$

$$
{}_{\omega}\gamma_0^{\ell/\ell+1} = \{G^{\ell+1}, \omega^{\ell}, W^{\ell+1}\}, \quad {}_{\omega}\gamma_0^{\ell/\ell-1} = \{W^{\ell}, \sigma^{\ell}, G^{\ell}\} .
$$

Therefore, canonic representation of coordinator environment ${}_{\varepsilon}(\bar{\hat{\varphi}}, \bar{\hat{\lambda}})_0^{\ell}$ is decomposed into two classes of models:

$$
{}_{\varepsilon}(\bar{\hat{\varphi}}, \bar{\hat{\lambda}})_0^{\ell} = \{{}_{\varepsilon}(\bar{\hat{\varphi}}, \bar{\hat{\lambda}})_0^{\ell/\ell+1}, {}_{\varepsilon}(\bar{\hat{\varphi}}, \bar{\hat{\lambda}})_0^{\ell/\ell-1}\}, \tag{3.47}
$$

and their states spaces were marked as ${}_{\varepsilon}C_0^{\ell/\ell+1}$ and ${}_{\varepsilon}C_0^{\ell/\ell-1}$ respectively:

$$
{}_{\varepsilon}C_0^{\ell} = \{{}_{\varepsilon}C_0^{\ell/\ell+1}, {}_{\varepsilon}C_0^{\ell/\ell-1}\}. \tag{3.48}
$$

States space ${}_{\varepsilon}C_0^{\ell}$ of coordinator environment ${}_{\varepsilon}S_0^{\ell}$ is constructed just as the states space of the coordinating element (3.31) in the following form (3.49):

$$
{}_{\varepsilon}C_0^{\ell} = {}_{\varepsilon}\tilde{C}_0^{\ell} / E_{\varepsilon c} = \{{}_{\psi\varepsilon}C_0^{\ell}, {}_{\chi\varepsilon}C_0^{\ell}, {}_{\varphi\varepsilon}C_0^{\ell}, {}_{\lambda\varepsilon}C_0^{\ell}\} \tag{3.49}
$$

The elements of the abstract systems ${}_{\varepsilon}C_0^{\ell} = \{{}_{\varepsilon}C_0^{\ell/\ell+1}, {}_{\varepsilon}C_0^{\ell/\ell-1}\}$ given in (3.48) form represent the following states. ${}_{\varepsilon}C_0^{\ell/\ell-1}$ represents the states of system structure $\sigma^{\ell} \in S^{\ell}$ and ${}_{\varepsilon}C_0^{\ell/\ell+1}$ represents the state of aggregated dynamic realization $\omega^{\ell} \in S^{\ell}$ of the system being coordinated – designed and controlled. Their connection is defined as follows by the law ${}_{\omega\sigma}\Psi$ which arose from ${}_{s}\Psi$ (3.27):

$$({}_{\omega\sigma}\Psi) \qquad \{[(\sigma^{\ell}\big|T_{\lambda}^{\ell} =_{\lambda} \sigma^{\ell}) \Rightarrow (\exists t'')(t'' = t_{\varphi}^{\ell+1}) \,\&\, (t'' = t_{\chi}^{\ell+2}) \,\&\, (t'' = t_{\psi}^{\ell+3})] \Rightarrow$$

$$\Rightarrow (\omega^{\ell}\big|T_{\lambda}^{\ell} =_{\varphi} \omega^{\ell})\} \,\& \tag{3.50}$$

$$\& \,\{[(\sigma^{\ell}\big|T_{\lambda}^{\ell} =_{\lambda} \sigma^{\ell}) \Rightarrow (\exists \hat{t})(\hat{t} = t_{\lambda}^{\ell+1}) \,\&\, (\hat{t} = t_{\varphi}^{\ell+2}) \,\&\, (\hat{t} = t_{\chi}^{\ell+3})] \Rightarrow$$

$$\Rightarrow (\omega^{\ell}\big|T_{\lambda}^{\ell+1} =_{\lambda} \omega^{\ell})\}.$$

Law ${}_{\omega\sigma}\Psi$ represents the connection of structural elements σ^{ℓ} and ω^{ℓ} of coordinator S_0^{ℓ}, in other words ${}_{\omega\sigma}\Psi \subset_{\sigma} \gamma_0^{\ell}$. ${}_{\omega\sigma}\Psi$ means that that level of information uncertainty in ω^{ℓ} is higher than the one in σ^{ℓ} ; ω^{ℓ} becomes sufficiently defined only when system $S^{\ell+1}$of level $\ell+1$ becomes sufficiently defined.

As a result, coordination signal $\gamma^{\ell+1} \in G^{\ell+1}$ and feedback $w^{\ell+1} \in W^{\ell+1}$ become determined for S_0^{ℓ} when the system of a higher level has been formed. This definition is relative because of the synthesis and functioning laws of systems of level ℓ can change the higher levels.

Canonical representation $\hat{\omega}^{\ell}\big|_{\varepsilon} S_0^{\ell}$ of coordinator environment ${}_{\varepsilon}S_0^{\ell}$ is specified as follows:

$$_{\varepsilon}(\bar{\hat{\varphi}}, \bar{\hat{\lambda}})_0^{\ell}: \quad {}_{\varepsilon}\bar{\hat{\varphi}}_0^{\ell} = \{{}_{\varepsilon}\hat{\varphi}_{0tt'}^{\ell} : {}_{\varepsilon}C_0^{\ell} \times Y_{0tt'}^{\ell} \rightarrow {}_{\varepsilon}C_0^{\ell} \,\&\, t, t' \in T^{\ell} \,\&\, t' > t\}, \tag{3.51}$$

$$_{\varepsilon}\bar{\hat{\lambda}}_0^{\ell} = \{{}_{\varepsilon}\hat{\lambda}_{0t}^{\ell} : {}_{\varepsilon}C_0^{\ell} \times \hat{\tilde{Y}}_0^{\ell} \rightarrow \hat{\tilde{X}}_0^{\ell} \,\&\, t \in T^{\ell}\}.$$

Decomposition of states space ${}_{\varepsilon}C_0^{\ell}$ into ${}_{\varepsilon}C_0^{\ell/\ell+\tau}$ and ${}_{\varepsilon}C_0^{\ell/\ell-\tau}$ resulted in the corresponding decomposition of canonical representation ${}_{\varepsilon}(\bar{\hat{\varphi}}, \bar{\hat{\lambda}})_0^{\ell}$. For $\tau = 1$, the decomposition is presented in the following form:

$$_{\varepsilon}(\bar{\hat{\varphi}}, \bar{\hat{\lambda}})_0^{\ell/\ell-1}:$$

$$_{\varepsilon}\bar{\hat{\varphi}}_0^{\ell/\ell-1} = \{{}_{\varepsilon}\hat{\varphi}_{0tt'}^{\ell/\ell-1} : {}_{\varepsilon}C_0^{\ell/\ell-1} \times G_{tt'}^{\ell} \rightarrow {}_{\varepsilon}C_0^{\ell/\ell-1} \,\&\, t, t' \in T^{\ell} \,\&\, t' > t\},$$

$$_{\varepsilon}\bar{\hat{\lambda}}_{0}^{\ell/\ell-1}=\{_{\varepsilon}\hat{\lambda}_{0t}^{\ell/\ell-1}:_{\varepsilon}C_{0}^{\ell/\ell-1}\times\hat{\tilde{G}}^{\ell}\to\hat{\tilde{W}}^{\ell}\ \&\, t\in T^{\ell}\}\ , \qquad (3.52)$$

$$_{\varepsilon}(\bar{\hat{\varphi}},\bar{\hat{\lambda}})_{0}^{\ell/\ell+1}:$$
$$_{\varepsilon}\bar{\hat{\varphi}}_{0}^{\ell/\ell+1}=\{_{\varepsilon}\hat{\varphi}_{0tt'}^{\ell/\ell+1}:_{\varepsilon}C_{0}^{\ell/\ell+1}\times W_{tt'}^{\ell+1}\to_{\varepsilon}C_{0}^{\ell/\ell+1}\ \&\, t,t'\in T^{\ell}\ \&\, t'>t\},$$
$$_{\varepsilon}\bar{\hat{\lambda}}_{0}^{\ell/\ell+1}=\{_{\varepsilon}\hat{\lambda}_{0t}^{\ell/\ell+1}:_{\varepsilon}C_{0}^{\ell/\ell+1}\times\hat{\tilde{W}}^{\ell+1}\to\hat{\tilde{G}}^{\ell+1}\ \&\, t\in T^{\ell}\}\ .$$

Functions $_{\varepsilon}\bar{\hat{\lambda}}_{0}^{\ell/\ell-1}$ and $_{\varepsilon}\bar{\hat{\lambda}}_{0}^{\ell/\ell+1}$ are decomposed in the same way as for co-ordination strategies (3.38) and define the formation ways of feedback W^{ℓ} and coordination signals $G^{\ell+1}$ under conditions of different information uncertainty indexed by $^{\beta}L\leftrightarrow\{\lambda,\varphi,\chi,\psi\}$. The states transition function $_{\varepsilon}\bar{\hat{\varphi}}_{0}^{\ell}$, similar to $_{o}\bar{\hat{\varphi}}_{0}^{\ell}$ and $_{\pi}\bar{\hat{\varphi}}_{0}^{\ell}$ functions, contain two strata – *learning* ($_{\varphi\varepsilon}\bar{\hat{\varphi}}_{0}^{\ell}$) and *self-organization* ($_{\psi\varepsilon}\bar{\hat{\varphi}}_{0}^{\ell}$).

The task of the *self-organization* stratum $_{\psi\varepsilon}\bar{\hat{\varphi}}_{0}^{\ell}$ for coordinator environment $_{\varepsilon}S_{0}^{\ell}$ is to change the level of system S^{ℓ} from ℓ_{ψ} to $\ell_{\lambda}=\ell$. The task of *self-organization* stratum $_{\chi o}\bar{\hat{\varphi}}_{0}^{\ell}$ of coordinating element $_{o}S_{0}^{\ell}$ is to install the structural connections Ψ^{ℓ} $(\Psi^{\ell}=\sigma^{\ell}(S^{\ell}))$ of formal model $S^{\ell}\big|T^{\ell}$ for level ℓ, in other words to change state $S^{\ell}\big|T^{\ell}$ from $_{\psi}S_{0}^{\ell}$ to $_{\lambda}S_{0}^{\ell}$.

The structural elements of *aed* formal model S^{ℓ} are its basic terms S^{ℓ}, S_{0}^{ℓ}, σ^{ℓ}, ω^{ℓ}, $_{\pi}S^{\ell}$, ℓ, etc., which are at the same time connections in S^{ℓ}. For example, system S^{ℓ} connects S_{0}^{ℓ}, ω^{ℓ} and σ^{ℓ}; coordinator S_{0}^{ℓ} connects ω^{ℓ} and σ^{ℓ} in S^{ℓ}; dynamic realization $\tilde{\omega}^{\ell}$ connects S_{0}^{ℓ} , $_{\pi}S^{\ell}$ and $_{\varepsilon}S^{\ell}$; process $_{\pi}S^{\ell}$ connects systems $_{o}S^{\ell}$ and $_{\varepsilon}S^{\ell}$, for which $_{\pi}S^{\ell}$ plays the role of their interconnections; $_{o}S^{\ell}$ and $_{\varepsilon}S^{\ell}$ are the environment for $_{\pi}S^{\ell}$, and processes $_{o\pi}S^{\ell}$ and $_{\varepsilon\pi}S^{\ell}$ are structural elements of $_{\pi}S^{\ell}$, etc.

Each structural element of system $S^{\ell}\big|T^{\ell}$ must be defined on self-organization stratum of coordinator S_0^{ℓ} by its formal model, i.e. by *aed* $S^{\ell}\big|T^{\ell}$. In *aed* model the following components must be indicated: *aed* structural elements (with their dynamics) and their connections, i.e. structure σ^{ℓ} of *aed*, its connections with other elements of S^{ℓ} in general model $S^{\ell}\big|T^{\ell}$, i.e. dynamic realization ω^{ℓ} of *aed*, and the restriction on *aed* coordinator S_0^{ℓ} which defines the ways of concretization of both σ^{ℓ} and ω^{ℓ}.

In other words, on self-organization stratum coordinator S_0^{ℓ} performs the systemic analysis of each element of formal model S^{ℓ} in terms of this model and synthesizes the abstract system $S^{\ell}\big|T^{\ell}$. In this way S_0^{ℓ} controls the conformity of the system model to the laws of synthesis and functioning of an traditional concrete system of level ℓ, which is created by both coordinator S_0^{ℓ} and environment ${}_{\varepsilon}S^{\ell}$.

3.2.4. Performance of the design task of mechatronic systems

Design tasks of mechatronic systems and control process are distributed on coordinator S_0^{ℓ} strata: selection, learning and self- organization (self-coordination). The task is partially described in [49] as well.

On the *selection stratum* at $t \in T$ moment of time, predicting the environment systems states for moment $t' > t$, coordinator defines correspondent inputs and outputs of the system being designed and creates $(\overline{\rho},\overline{\varphi})^{\ell}$ models of the system and its process using the obtained predictions. After that, coordinator selects such states of subsystems and their interactions which correspond to the standard state of system S^{ℓ} required for t', i.e. performs the task of structure σ^{ℓ} synthesis of the object ${}_{o}S^{\ell}$ being designed.

The *synthesis task* is formalized by introducing interlevel relations on information resources of abstract *aed* model $\Psi = \Psi_{\omega} \times \Psi_{\sigma}$ [3, 49-50], where Ψ_{ω} is a relations net on the multitude of dynamic systems $(\overline{\rho},\overline{\varphi})^{\ell}$, Ψ_{σ} is an analogous net of relations $\sigma^{\ell} \times \sigma^{\ell}$ with the types of structures σ^{ℓ}. Ψ_{σ} contains, in particular, equivalence relations on structure σ^{ℓ} types by the connection defects, constructive dimension and way of subprocesses organizations (continuous, parallel connections and feedbacks). Adaptation of structural model σ^{ℓ} to the concrete application area, i.e. mechatronic system design, is executed by the indication of concrete external model $\omega^{\ell} \in \Psi_{\omega}$ and class of structures which contains external models $\overline{\omega}^{\ell-1}$ of elements (subsystems $\overline{S}^{\ell-1}$) of system S^{ℓ} under consideration being designed.

Structure classes – space of mechatronic system constructions – for the defined elements basis correspond to a certain way of designed system functioning. Technical constructions for each way of functioning are selected by changing parameters of the designed object subsystems $\overline{S}^{\ell-1}$ and their interactions ${}_{\sigma,\omega}\gamma^{l}$. The change of the parameters causes the change of external model of the mechatronic object. It allows finding out if the required state of designed object ${}_{o}S^{\ell}$ is obtained (*analysis task*). We return to the *synthesis task* if there is incoherence of the obtained and standard states of the object being designed.

The choice of parameters of the designed object structure depends on the uncertainty level of the coordinator S_0^{ℓ} information about the environment ${}_{\varepsilon}S^{\ell}$. For each level of information uncertainty the equivalence classes of the coordinator states in the design process are introduced. Each class of states of the coordinator has its own coordination strategy: coordination of interconnections, evaluation of interconnections and the prediction of interconnections (most exact) described in Section 3.2.1. Coordination strategies (3.38) are realized on the *selection stratum.*

On the *learning stratum* the coordinator fixes the level of information uncertainty. It compares the data about the environment systems received by the feedback and the same data contained it its informational resources (data bases). In this way the required area Ψ of informational resources is defined. In the frames of this area the selection of required technical construction of mechatronic object will be done. Corresponding state of the design process (coordination strategy) at the selection stratum is also defined.

Information resources Ψ of the coordinator are changed in the design process on its *self-organization (self-coordination) stratum.* The reason of the change could be the absence of coherence with the real mechatronic systems model in information resources Ψ. The criteria for including a new model into Ψ is the exactness of the solutions obtained with the help of the model.

3.3. Metrical characteristics of hierarchical mechatronic systems

Structural-dynamic connections of S^{ℓ}, S_0^{ℓ}, ω^{ℓ}, σ^{ℓ}, $\hat{\omega}_0^{\ell}$, ${}_s\Psi$, ${}_{\omega\sigma}\Psi$ elements of formal model S^{ℓ} of *aed* – the standard formal element of design and control systems – realize the main laws of hierarchical systems. Numerical characteristics necessary for further investigation of the laws and quantitative measurement of particular mechatronic systems being designed are introduced in this Chapter. All numerical characteristics of hierarchical mechatronic systems are determined below by the means of positional numeric system L^s.

3.3.1. Levels of mechatronic systems

The most general numerical characteristic of hierarchical mechatronic systems is level $\ell \in L$, where: $L \in L^s$, L^s is numeric positional system (generally with an unfixed base), specified on multitude $L^s_\beta = \{\ell_\beta : \ell_\beta = (\ell, I^{\ell})\ \&\ \ell \in L\ \&\ I^{\ell} \subset L\}$, L is the multitude of indices of levels, I^{ℓ} is the finite set of indexes on level ℓ, the base on each level is marked by symbol $\mu^{\ell}, \mu^{\ell} \in N^+, N^+$ is the set of natural numbers.

L is determined as $L = \{\ell\} = L^s_\beta \big| E^{\ell}$, E^{ℓ} are relations of equivalence on $L^s_\beta \times L^s_\beta$:

$$(\forall \ell'_\beta, \ell''_\beta)(\ell'_\beta, \ell''_\beta \in L^s_\beta)[(\ell'_\beta, \ell''_\beta) \in E^{\ell} \Leftrightarrow \ell' = \ell'']. \tag{3.53}$$

Abstractive mapping is marked with symbol ${}_\ell R$:

$${}_\ell R : S^{\ell} \to L, \quad {}_\ell R : \{\omega^{\ell}, S_0^{\ell}, \sigma^{\ell}\} \to L, \tag{3.54}$$

which sets correspondence of mechatronic system S^{ℓ} and the index of its level. For dynamic $\omega^{\ell} \in \Omega^{\ell}$ and structural $\sigma^{\ell} \in \Sigma^{\ell}$ representations, the reflections are defined as follows:

$${}_{\omega\ell} R : \Omega^{\ell} \to L, \quad {}_{\sigma\ell} R : \Sigma^{\ell} \to L. \tag{3.55}$$

To determine dynamic Ω_t^ℓ and structural Σ_t^ℓ presentations as the restrictions of Ω_0^ℓ and Σ_0^ℓ at time T_t^ℓ: $\tilde{\Omega}^\ell = \bigcup_{t\in T}\Omega_t^\ell$, $\tilde{\Sigma}^\ell = \bigcup_{t\in T}\Sigma_t^\ell$, the following definition was made.

Definition 3.5. (i) S*pace of dynamic realizations* Ω_ℓ^ℓ of hierarchical mechatronic system on levels $\ell \in L$ is called a factor set of system $\Omega_\ell^\ell = \tilde{\Omega}^\ell \big| E_\omega^\ell$, where E_ω^ℓ are the relations of equivalence on $\tilde{\Omega}^\ell \times \tilde{\Omega}^\ell$:

$$(\forall \omega_1^\ell, \omega_2^\ell)(\omega_1^\ell, \omega_2^\ell \in \tilde{\Omega}^\ell)[(\omega_1^\ell, \omega_2^\ell) \in E_\omega^\ell \Leftrightarrow {}_{\omega\ell}R(\omega_1^\ell) = {}_{\omega\ell}R(\omega_2^\ell)]. \tag{3.56}$$

(ii) The s*pace of structures* of hierarchical mechatronic system on levels $\ell \in L$ is called factor set $\Sigma_\ell^\ell = \tilde{\Sigma}^\ell \big| E_\sigma^\ell$, where:

$$(\forall \sigma_1^\ell, \sigma_2^\ell)(\sigma_1^\ell, \sigma_2^\ell \in \tilde{\Sigma}^\ell)[(\sigma_1^\ell, \sigma_2^\ell) \in E_\sigma^\ell \Leftrightarrow {}_{\sigma\ell}R(\sigma_1^\ell) = {}_{\sigma\ell}R(\sigma_2^\ell)]. \tag{3.57}$$

Systems S_i^ℓ and S_τ^ℓ are considered as equivalent by their level independently of the equality of their bases μ_i^ℓ and μ_τ^ℓ. In other words, the might be systems with different bases on one level. Equivalence $E^{\ell\beta}$ is determined on $L_\beta^s \times L_\beta^s$ for their classification:

$$\forall(\ell'_\beta, \ell''_\beta)(\ell'_\beta, \ell''_\beta \in L_\beta^s)[(\ell'_\beta, \ell''_\beta) \in E^{\ell\beta} \Leftrightarrow (\ell'_\beta = \ell''_\beta)]. \tag{3.58}$$

Spaces $\Omega_{\ell\beta}^\ell$ and $\Sigma_{\ell\beta}^\ell$ are constructed as factor sets $\Omega_{\ell\beta}^\ell = \tilde{\Omega}^\ell \big| E_\omega^{\ell\beta}$ and $\Sigma_{\ell\beta}^\ell = \tilde{\Sigma}^\ell \big| E_\sigma^{\ell\beta}$ by analogy with Ω_ℓ^ℓ and Σ_ℓ^ℓ as follows:

$$(\forall \omega_1^\ell, \omega_2^\ell)(\omega_1^\ell, \omega_2^\ell \in \tilde{\Omega}^\ell)[(\omega_1^\ell, \omega_2^\ell) \in \Omega_{\ell\beta}^\ell \Leftrightarrow {}_{\beta\omega\ell}R(\omega_1^\ell) = {}_{\beta\omega\ell}R(\omega_2^\ell)], \tag{3.59}$$

$$(\forall \sigma_1^\ell, \sigma_2^\ell)(\sigma_1^\ell, \sigma_2^\ell \in \tilde{\Sigma}^\ell)[(\sigma_1^\ell, \sigma_2^\ell) \in \Sigma_{\ell\beta}^\ell \Leftrightarrow {}_{\beta\sigma\ell}R(\sigma_1^\ell) = {}_{\beta\sigma\ell}R(\sigma_2^\ell)], \quad ....$$

$${}_{\beta\omega\ell}R : \Omega^\ell \to L_\beta^\ell, {}_{\beta\sigma\ell}R : \Sigma^\ell \to L_\beta^\ell.$$

Validity of property ${}_\ell\Psi$ for state space C^ℓ follows from ${}_s\Psi$ and ${}_\ell R$:

$$({}_\ell\Psi) \qquad ({}_\ell R(S^\ell) = \ell_\lambda \Leftrightarrow c^\ell \in_\lambda C^\ell)\ \& \tag{3.60}$$

$$\& \ ({}_{\ell}R(S^{\ell}) = \ell_{\varphi} \Leftrightarrow c^{\ell} \in_{\varphi} C^{\ell} \ \& t_{\lambda}^{\ell} =^{0} ?) \ \&$$

$$\& \ ({}_{\ell}R(S^{\ell}) = \ell_{\chi} \Leftrightarrow c^{\ell} \in_{\chi} C^{\ell} \ \& t_{\varphi}^{\ell} =^{0} ?) \ \&$$

$$\& \ ({}_{\ell}R(S^{\ell}) = \ell_{\psi} \Leftrightarrow c^{\ell} \in_{\psi} C^{\ell} \ \& t_{\chi}^{\ell} =^{0} ?).$$

Property ${}_{\ell}\Psi$ connects the values of the level of mechatronic system S^{ℓ} with values ${}_{\Psi}L \leftrightarrow \{\psi, \chi, \varphi, \lambda\}$ of the levels of system organization and coordinator's S_0^{ℓ} information uncertainty. ${}_{\ell}\Psi$ means that each mechatronic system S^{ℓ} spreads at least over three levels up and down in L. The levels' indices in interfaces with natural languages may be replaced by words (machine part, module, machine, etc). Levels on which the reference models introduced are adjustable (by the change of parameters) to any system of this level are called strata.

Levels $\{\psi, \chi, \varphi, \lambda\}$ in $\hat{\omega}_0^{\ell}$ are called:

– the levels of information uncertainty – for signals γ^{ℓ}, $\gamma^{\ell+1}$, w^{ℓ} , $w^{\ell+1}$;

– the levels of system organization – for coordinator's environment ${}_{\varepsilon}S_0^{\ell}$.

Levels $\{\psi, \chi, \varphi, \lambda\}$ also encode output functions $\bar{\hat{\lambda}}_0^{\ell}$ as coordination strategies on the selection stratum of coordinator.

3.3.2. Numeric positional system

Formal models *of aed* and coordinator, as a theoretical basis for the design method proposed in the book, are constructed with the help of their most important particular cases – dynamic system $(\bar{\rho}, \bar{\varphi})$ and numeric positional system L^s. L^s allows determinations of level to be easily introduced.

The dynamic systems are algoristic and sufficiently investigated. But for numeric positional systems there is no formal model in spite of its big importance in numeric information processing.

Numbers in a positional code are usually presented as polynomials, which are not in coordination with most characteristic features of L^s, i.e. with the realization of interlevel transitions in aggregated presentation of numbers. So, addition of numbers in a positional code usually leads to the degree of the result being higher than that of the items. This is impossible for the group of polynomials of degree *n* which are closed with respect to the addition operation. Furthermore, system L^s

can’t be formalized by the means of any other one-level mathematical model either. Being hierarchical by nature L^s is formalized only in terms of hierarchical systems theory.

It is sufficient to examine model L^s for one standard block which is repeatable on each of its level. This block is constructed on the following basic system ${}_{\Psi}\tilde{T}^{\ell}$:

$$
\begin{aligned}
&{}_{\psi}\tilde{T}^{\ell} \leftrightarrow \left\{\tilde{\lambda}^{\ell} = {}_{\lambda}\tilde{\tau}^{\ell}, \tilde{\varphi}^{\ell} = {}_{\varphi}\tilde{\tau}^{\ell}, \tilde{\chi}^{\ell} = {}_{\chi}\tilde{\tau}^{\ell}, \tilde{\psi}^{\ell} = {}_{\psi}\tilde{\tau}^{\ell}\right\}, \qquad (3.61)\\
&\tilde{\lambda}^{\ell} \leftrightarrow \left({}^{-}\tilde{\lambda}^{\ell}, {}^{0}\tilde{\lambda}^{\ell}, {}^{+}\tilde{\lambda}^{\ell}\right) \leftrightarrow \left\{(\lambda,\varphi,\chi,\psi)^{\ell}, \tilde{\varphi}^{\ell}, (\psi,\chi,\varphi,\lambda)^{\ell}\right\},\\
&\tilde{\varphi}^{\ell} \leftrightarrow {}^{0}\tilde{\lambda}^{\ell} \leftrightarrow \left({}^{-}\tilde{\varphi}^{\ell}, {}^{0}\tilde{\varphi}^{\ell}, {}^{+}\tilde{\varphi}^{\ell}\right) \leftrightarrow \left\{(\varphi,\chi,\psi)^{\ell}, \tilde{\chi}^{\ell}, (\psi,\chi,\varphi)^{\ell}\right\},\\
&\tilde{\chi}^{\ell} \leftrightarrow {}^{0}\tilde{\varphi}^{\ell} \leftrightarrow \left({}^{-}\tilde{\chi}^{\ell}, {}^{0}\tilde{\chi}^{\ell}, {}^{+}\tilde{\chi}^{\ell}\right) \leftrightarrow \left\{(\chi,?)^{\ell}, \tilde{?}^{\ell}, (?,\chi)^{\ell}\right\},\\
&\tilde{\psi}^{\ell} \leftrightarrow {}^{0}\tilde{\chi}^{\ell} \leftrightarrow \left({}^{-}\tilde{\psi}^{\ell}, {}^{0}\tilde{\psi}^{\ell}, {}^{+}\tilde{\psi}^{\ell}\right) \leftrightarrow \left\{{}^{-}\tilde{\psi}^{\ell}, {}^{0}\tilde{\psi}^{\ell}, {}^{+}\tilde{\psi}^{\ell}\right\},\\
&(\lambda,\varphi,\chi,\psi)^{\ell} = ({}^{-}\lambda, {}^{-}\varphi, {}^{-}\chi, {}^{-}\psi)^{\ell},\\
&(\psi,\chi,\varphi,\lambda)^{\ell} = ({}^{+}\psi, {}^{+}\chi, {}^{+}\varphi, {}^{+}\lambda)^{\ell}.
\end{aligned}
$$

The meaning of symbols $\{\lambda,\varphi,\chi,\psi\}$ in ${}_{\Psi}\tilde{T}^{\ell}$ is coordinated with the meaning which they have in canonic model $\hat{\omega}_0^{\ell}$ of coordinator, where λ is the index of the most organized system, φ and χ are partly organized ones, ψ is an unorganized system. Property ${}_{\ell\tau}\Psi$ (Fig. 3.3) sets connections between systems ${}_{\Psi}\tilde{T}^{\ell_{\psi}}$, ${}_{\Psi}\tilde{T}^{\ell_{\chi}}$, ${}_{\Psi}\tilde{T}^{\ell_{\varphi}}$ and ${}_{\Psi}\tilde{T}^{\ell_{\lambda}}$ ($\ell_{\lambda} = \ell$, $\ell_{\varphi} = \ell \pm 1$, $\ell_{\chi} = \ell \pm 2$, $\ell_{\psi} = \ell \pm 3$). The whole system ${}_{\Psi}\tilde{T}^{\ell}$ turns into the neutral element ${}^{0}_{\lambda}\tilde{\tau}^{\ell+1} \in {}_{\Psi}\tilde{T}^{\ell+1}$ of the higher level. Element ${}^{0}_{\lambda}\tilde{\tau}^{\ell}$, which is the connection of positively and negatively oriented elements ${}^{+}_{\lambda}\tilde{\tau}^{\ell}$ and ${}^{-}_{\lambda}\tilde{\tau}^{\ell}$ is decomposed into elements of level $\ell - 1$ in ${}_{\Psi}\tilde{T}^{\ell}$.

Bilateral symmetry of elements ${}^{+}_{i}\tilde{\tau}^{\ell}$ and ${}^{-}_{i}\tilde{\tau}^{\ell}$ (where $i \in_{\Psi} L = \{\psi,\chi,\varphi,\lambda\}$) corresponds to the direct and reverse states transitions of system S^{ℓ} in self-organization cycle.

Diagram ${}_{\ell\tau}\Psi$ (Fig.3.3) is in coordination with ${}_{\omega\sigma}\Psi$ and ${}_{\ell}\Psi$. When structure σ^{ℓ} is in state λ, i.e. the system has level ℓ_{λ}, aggregated dynamic representation ω^{ℓ} (for $t_{\lambda}^{\ell+1}={}^{0}?$) is in state φ (level ℓ_{φ}). When the state index of σ^{ℓ} is equal to φ (level ℓ_{φ}) and $t_{\varphi}^{\ell+1}={}^{0}?$, the index of ω^{ℓ} is χ, etc.

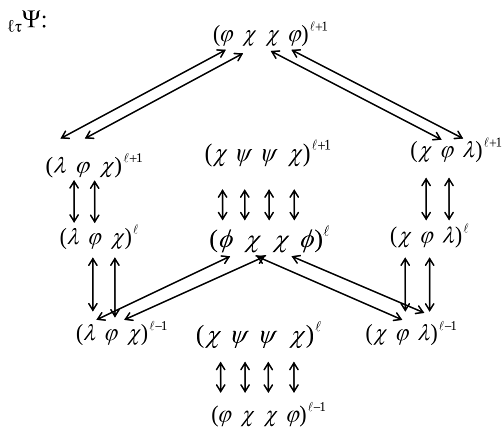


Fig. 3.3. Scheme of inter-level relations ${}_{\Psi}\tilde{T}^{\ell i}$, $i \in_{\Psi} L = \{\psi, \chi, \varphi, \lambda\}$

It is shown that Figure 3.3 is analogous to *aed* S^{ℓ} model in its canonical presentation. The arrows illustrate the elements' inter-level connections which go through coordinator S_0^{ℓ}. System dynamics on each level is specified by the sequence of transitions from ${}^{+}\tilde{\psi}^{\ell}$ to ${}^{+}\tilde{\chi}^{\ell}, {}^{+}\tilde{\varphi}^{\ell}, {}^{+}\tilde{\lambda}^{\ell}$ and vice versa, which is formalized by canonic model $\hat{\omega}_0^{\ell}$ of coordinator.

The change of states $\{\psi, \chi, \varphi, \lambda\}^{\ell}$ of level ℓ of system $L^{s\ell}$ from the state of absolute information uncertainty and the weakest organization ψ^{ℓ} to relative deterministic state λ^{ℓ} is accompanied by alteration of $L^{s\ell}$ characteristics of its higher and lower levels. The relations between the values of numeric characteristics of $L^{s\ell}$ for different levels are determined in the following (3.62) form.

Here 1^{ℓ} is a unit of level ℓ; diverse levels have their own bases $\mu^{\ell-1}$, μ^{ℓ}, $\mu^{\ell+1}$, $\mu^{\ell+2}$, $\mu^{\ell+3}$ are not necessarily equal. Unit of level ℓ is regarded as $0{,}1^{\ell+1}$

on level ℓ and unites μ^{ℓ} =10 units of level ℓ-1. Each system S^{ℓ} is connected through interactions $_{\omega}\gamma^{\ell}$ with the other systems of level ℓ which create environment $_{\varepsilon}S^{\ell}$ of system S^{ℓ}. $_{\omega}\gamma^{\ell}$ is called a prolongation of system S^{ℓ} in its environment $_{\varepsilon}S^{\ell}$ and marked as $\tilde{S}^{\ell} : \tilde{S}^{\ell} = {_{\omega}\gamma^{\ell}}$.

$$\begin{array}{lllllll} (_{\omega\sigma}\Psi) & & & & & & \\ \lambda^{\ell} & & \varphi^{\ell+1} & & \chi^{\ell+2} & & \psi^{\ell+3} \\ \Updownarrow & & \Updownarrow & & \Updownarrow & & \Updownarrow \\ 1^{\ell} & = & 0,1^{\ell+1} & = & 0,01^{\ell+2} & = & 0,001^{\ell+3} \\ \| & & \searrow & & & & \\ 10^{\ell-1} & & \omega^{\ell}(L^{S\ell}) & & & & \\ \nwarrow\!\searrow & & & & & & \\ \sigma^{\ell}(\overline{L}^{S(\ell-1)}) & & & & & & \end{array} \tag{3.62}$$

System $L^{s\ell}$ of level ℓ with its prolongation $\tilde{L}^{s\ell}$ in environment $_{\varepsilon}S^{\ell}\big|L^{s\ell}$ is described and presented in Figure 3.4.

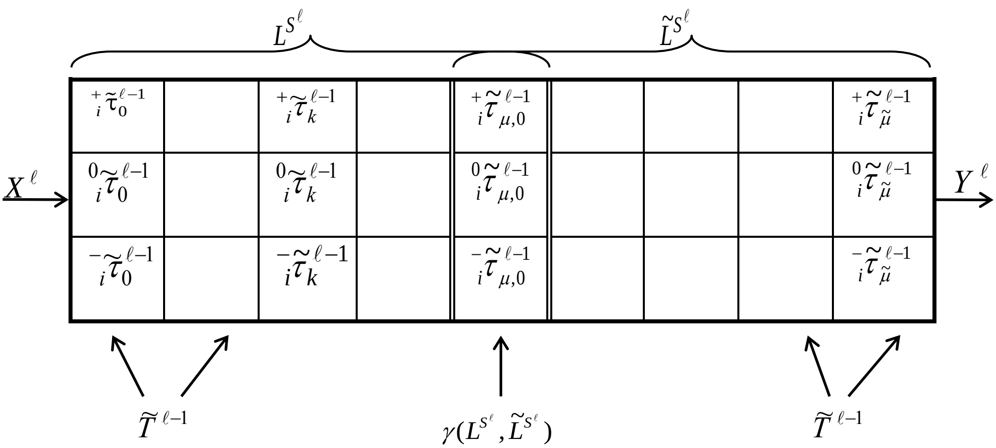


Fig. 3.4. System $L^{S^{\ell}}$ and its prolongation $\tilde{L}^{S^{\ell}}$ in environment $_{\varepsilon}S^{\ell}\big|L^{s\ell}$, where: $_{i}\tilde{\tau}_{k}^{\ell-1}$ – element of *k* number of basic system $\tilde{T}^{\ell-1}$, *i* – index $i \in_{\Psi} L = \{\psi, \chi, \varphi, \lambda\}$; $\gamma(L^{S^{\ell}}, \tilde{L}^{S^{\ell}})$ – interaction of systems $L^{S^{\ell}}$ and $\tilde{L}^{S^{\ell}}$

Theoretical construction of *aed*-processor which is oriented at the processing of numeric and geometric information of mechatronic system being designed is defined as a structure of three connected levels of system $L^{s\ell}$. Construction of *aed*-processor is presented in Figure 3.6.

For system $L^{s\ell}$ in Figure 3.4, μ^{ℓ} elements of the lower level are determined on multitude ${}_{\Psi}\widetilde{T}^{\ell-1}$, the base of system $\widetilde{L}^{s\ell}$ is equal to $\widetilde{\mu}^{\ell}$ and sometimes may not coincide with μ^{ℓ} ($\widetilde{\mu}^{\ell}$ characterizes the quantity of $L^{s\ell}$ connections in ${}_{\varepsilon}S^{\ell}\big|L^{s\ell}$). Elements of $\widetilde{L}^{s\ell}$ are transferred by system $L^{s\ell}$ to the other systems of its level. In particular, the whole system $\widetilde{L}^{s\ell}$ can be transferred to only one system $L_i^{s\ell}, i \in I^{\ell+1}$. The relation of $L^{s\ell}$ and $\widetilde{L}^{s\ell}$ systems goes through element $L_{\mu}^{s(\ell-1)}$ $=\widetilde{L}_0^{s(\ell-1)}$. Number μ^{ℓ} defines the address of interactions $\widetilde{\gamma}(L^{s\ell}, \widetilde{L}^{s\ell})$ in $L^{s\ell}$, which is coordination signal $\gamma^{\ell}\big|L^{s}$ for system $L^{s\ell}$.

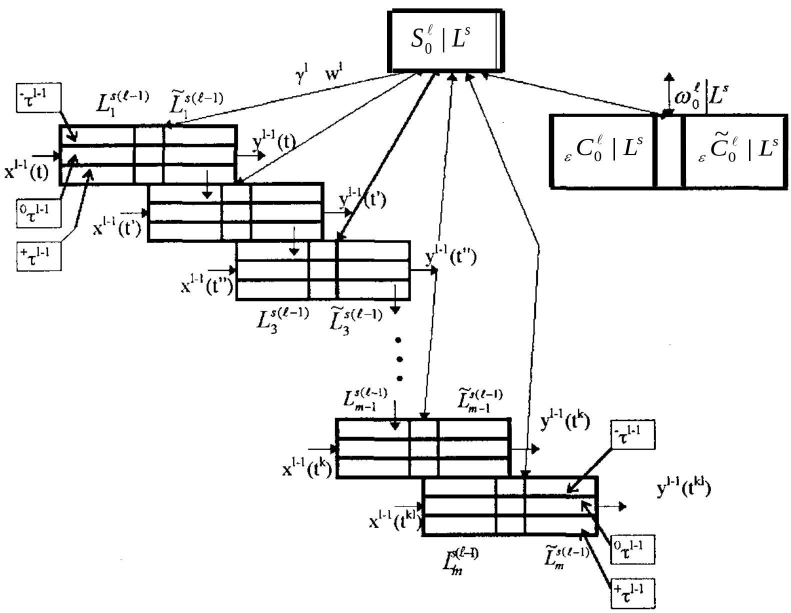


Fig. 3.5. Connection of discrete elements of system L^s continuous in time, where: $S_0^{\ell} \mid L^s$ – coordinator of L^s; ${}_{\varepsilon}C_0^{\ell} \mid L^s$ – states of coordinator environment [140]

System $\{ L^{s\ell}, \widetilde{L}^{s\ell} \}$ in its initial state contains non-oriented elements only. The uniting of elements with different orientation which come to the input of $L^{s\ell}$ gives a non-oriented element again. When the quantity of elements with the same orientation is equal to μ^{ℓ}, signal $w^{\ell+1}\big| L^{s}$ (unit *1* of level ℓ, i.e. *0.1* of level $\ell+1$) goes to higher level $\ell+1$ informing that system $L^{s\ell}$ has passed into its organization state λ, see (3.61).

The further process in L^{s} is illustrated in Figure 3.5 and described as follows. Being in its organization state λ (3.61), each system $L_i^{s\ell}$ gives its prolongation $\widetilde{L}_i^{s\ell}$ to system $L_{i+1}^{s\ell}$ and after that $L_{i+1}^{s\ell}$ begins to receive the inputs of level ℓ (in the moment of prolongation transfer it may occur that $\widetilde{L}_i^{s\ell}$ contains a certain number of elements with the same orientation). In this way, discrete systems $L_i^{s\ell}$ and $L_\tau^{s\ell}(i, \tau \in I^{\ell+1})$ become continuously connected.

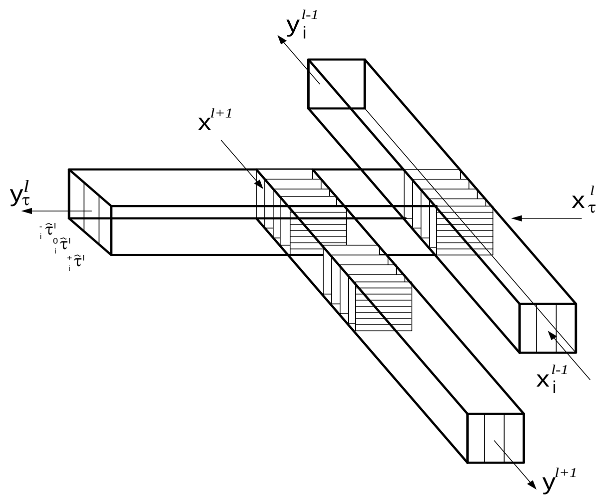


Fig. 3.6. *Aed*-processor construction, i.e. three connected levels of $L^{S^{\ell}}$, where: x^{ℓ}, y^{ℓ} – inputs and outputs of level ℓ; ${}_i\widetilde{\tau}^{\,\ell}$ – element of basic system $\widetilde{T}^{\,\ell}$ [140]

It is necessary to point out that information for higher levels is aggregated and the sizes of units of different levels with equal bases are equal.

Each level in L^s has its own inputs and outputs ${}_{\omega}\gamma^{\ell}\big|L^{s\ell}$. Interconnections ${}_{\omega}\gamma^{\ell}\big|L^{s\ell}$ of system $L^{s\ell}$ of level ℓ go through structural connections ${}_{\sigma}\gamma^{\ell}\big|L^{s\ell}$ which connects elements $\overline{L}^{s(\ell-1)}$ of system $L^{s\ell}$. The connections in L^s have all the features of the process of addition in column. When one unit of the higher level goes to the left, part of the sum stays in its position in $\widetilde{L}^{s\ell}$ for the following addition operation.

The change of states $(\lambda,\varphi,\chi,\psi)^{\ell}$ of system $L^{s\ell}$ is executed by its coordinator $S_0^{\ell}\big|L^{s\ell}$ through states transition function $\bar{\hat{\varphi}}_0^{\ell}\big|L^s \in \hat{\omega}_0^{\ell}\big|L^s$ (Fig. 3.5). For the connection of L^s with the fields of real ($\mathbb{R}^{\ell}$), complex ($\mathbb{C}^{\ell}$) and with hyper-complex ($\mathbb{Z}^{\ell}$) numbers the multiplying operations on corresponding contractions of ${}_{\psi}\widetilde{T}^{\ell}$ are considered below.

The multiplication in the field of real numbers is determined by states transition function ${}_{\mathrm{R}}(\hat{\varphi}_0^{\ell}\big|L^S)^* \leftrightarrow {}_{\mathrm{R}}(*)$ (Tab. 3.4) on contraction ${}_{\mathrm{R}\psi}\widetilde{T}^{\ell}$ of system ${}_{\psi}\widetilde{T}^{\ell}$:

$${}_{\mathrm{R}\psi}\widetilde{T}^{\ell} \leftrightarrow \{{}^{-}\widetilde{\lambda}, \widetilde{\varphi}, {}^{+}\widetilde{\lambda}\}^{\ell} \,\&\, \widetilde{\varphi}^{\ell} \in {}^{\pm}\widetilde{\lambda}^{\ell} \qquad (3.63)$$

where $\widetilde{\varphi}^{\ell}$ plays the role of zero, and ${}^{\pm}\widetilde{\lambda}^{\ell}$ are the units.

Table 3.4. Multiplication in field of real numbers $\mathbf{R}^{(*)}$

X^{ℓ} \ C^{ℓ}	${}^{-}\lambda$	φ	${}^{+}\lambda$
${}^{-}\lambda$	${}^{+}\lambda$	φ	${}^{-}\lambda$
φ	φ	φ	φ
${}^{+}\lambda$	${}^{-}\lambda$	φ	${}^{+}\lambda$

Table 3.5 of function of states transition ${}_{\mathrm{C}}(*) \leftrightarrow {}_{\mathrm{C}}(\hat{\varphi}_0^{\ell}\big|L^S)^*$ defines the multiplication of complex numbers on the contraction ${}_{\mathrm{C}\psi}\widetilde{T}^{\ell} \leftrightarrow \{{}^{-}\widetilde{\lambda}, {}^{-}\widetilde{\varphi}, \widetilde{\chi}, {}^{+}\widetilde{\varphi}, {}^{+}\widetilde{\lambda}\}^{\ell}$

of system ${}_{\psi}\widetilde{T}^{\ell}$, where ±Re(ω^{ℓ})= $^{\pm}\widetilde{\lambda}^{\ell}$ is real and ±Im(ω^{ℓ})= $^{\pm}\widetilde{\varphi}^{\ell}$ is imaginary part of the complex number, $\widetilde{\chi}^{\ell} \leftrightarrow 0^{\ell}$ is zero, and $\omega^{\ell} \in \mathrm{C}^{\ell}$.

Table 3.5. Multiplication of complex numbers $\mathbf{C}^{(*)}$

X^{ℓ} \ C^{ℓ}	$^{-}\lambda$	$^{-}\varphi$	χ	$^{+}\varphi$	$^{+}\lambda$
$^{-}\lambda$	$^{+}\lambda$	$^{+}\varphi$	χ	$^{-}\varphi$	$^{-}\lambda$
$^{-}\varphi$	$^{+}\varphi$	$^{-}\lambda$	χ	$^{+}\lambda$	$^{-}\varphi$
χ	χ	χ	χ	χ	χ
$^{+}\varphi$	$^{-}\varphi$	$^{+}\lambda$	χ	$^{-}\lambda$	$^{+}\varphi$
$^{+}\lambda$	$^{-}\lambda$	$^{-}\varphi$	χ	$^{+}\varphi$	$^{+}\lambda$

Table $_{\mathrm{Z}}(*)$ of hypercomplex number multiplication (Table 3.6) is plotted for the example of quaternions (their contraction on the positive area). The table is defined on multitude ${}_{\mathrm{Z}\psi}\widetilde{T}^{\ell} \leftrightarrow \{^{-}\widetilde{\lambda},^{-}\widetilde{\varphi},^{-}\widetilde{\chi},^{-}\widetilde{\psi},?,^{+}\widetilde{\psi},^{+}\widetilde{\chi}^{+}\widetilde{\varphi},^{+}\widetilde{\lambda}\}^{\ell}$, where $\widetilde{?}^{\ell} \leftrightarrow 0^{\ell}$ is zero; $^{\pm}\widetilde{\lambda}^{\ell}$ are positive and negative real units, and $^{\pm}\widetilde{\varphi}^{\ell}$, $^{\pm}\widetilde{\chi}^{\ell}$, $^{\pm}\widetilde{\psi}^{\ell}$ are imaginary units.

Table 3.6. Multiplication of hypercomplex numbers $\mathbf{Z}^{(*)}$

X^{ℓ} \ C^{ℓ}	?	$^{+}\psi$	$^{+}\chi$	$^{+}\varphi$	$^{+}\lambda$
?	?	?	?	?	?
$^{+}\psi$	?	$^{-}\lambda$	$^{-}\varphi$	$^{+}\chi$	$^{+}\psi$
$^{+}\chi$	?	$^{+}\varphi$	$^{-}\lambda$	$^{-}\psi$	$^{+}\chi$
$^{+}\varphi$	?	$^{-}\chi$	$^{+}\psi$	$^{+}\lambda$	$^{+}\varphi$
$^{+}\lambda$	?	$^{+}\psi$	$^{+}\chi$	$^{+}\varphi$	$^{+}\lambda$

Tables of any numbers multiplication are particular cases of ${}_{\ell\tau}\Psi$ diagram. Its fragment is reflected in Figure 3.3.

The multiplication law of any elements of system ${}_{Z\psi}\tilde{T}^{\ell}$ (Table 3.6) is implemented as follows:

– index $i_c \in \{\psi, \chi, \varphi, \lambda\}$ of state element ${}_{i}\tilde{\tau}_{c}^{\ell}$ defines ${}_{i}\tilde{\tau}_{c}^{\ell}$ in the initial state of system ${}_{\psi}\tilde{T}_0^{\ell}$, where (conditional) the origin of coordinates is ${}_{\lambda}\tilde{\tau}^{\ell}$;

– index i_x of input element ${}_{i}\tilde{\tau}_{x}^{\ell}$ (coordinating signal) identifies level $i_x \in \{\psi, \chi, \varphi, \lambda\}$ from which system ${}_{\psi}\tilde{T}_t^{\ell}$ will be considered, i.e. identifies a new origin of coordinates;

– ${}_{i}\tilde{\tau}_{c}^{\ell}$ elements in new coordinates are transformed into other elements ${}_{\psi}\tilde{T}^{\ell}$, i.e. their indices change, which is illustrated by ${}_{\ell\tau}\Psi$ diagram and multiplication tables ${}_{R}(*)$ ${}_{C}(*)$, ${}_{Z}(*)$; the process of indices transformation runs in such a way that the results of any table line cannot be the same for any other line.

For instance, when system ${}_{R\psi}\tilde{T}^{\ell} \leftrightarrow \{{}_{\lambda}^{-}\tilde{\tau}^{\ell}, {}_{\varphi}\tilde{\tau}^{\ell}, {}_{\lambda}^{+}\tilde{\tau}^{\ell}\} \leftrightarrow \{{}_{\lambda}^{-}\tilde{\tau}^{\ell}, {}_{\lambda}^{0}\tilde{\tau}^{\ell}, {}_{\lambda}^{+}\tilde{\tau}^{\ell}\}$ is multiplied by ${}_{\lambda}^{+}\tilde{\tau}^{\ell}$, its state does not change and both elements ${}_{\lambda}^{\pm}\tilde{\tau}^{\ell}$ (unit) and ${}_{\varphi}\tilde{\tau}^{\ell}$ (zero) stay in their places. When ${}_{R\psi}\tilde{T}^{\ell}$ is multiplied by ${}_{\varphi}\tilde{\tau}^{\ell}$, index λ of the origin of coordinates goes into φ, i.e. φ is replaced by λ, and vice versa λ is changed by φ (element ${}_{\varphi}\tilde{\tau}^{\ell}$ on level ℓ_{φ} comes into ${}_{\lambda}\tilde{\tau}^{\ell}$ and ${}_{\lambda}\tilde{\tau}^{\ell}$ comes into ${}_{\varphi}\tilde{\tau}^{\ell}$).

It should be pointed out that for the multitude of real numbers in L^s code, ${}_{R}(*)$ table is insufficient and a diagram similar to ${}_{Z}(*)$ is required. The diagram is also a particular case of ${}_{\ell\tau}\Psi$. For instance, the real and complex numbers multiplication in L^s positional code (which may be also reduced to addition in L^s) is obtained from ${}_{\ell\tau}\Psi$ by the following way. The addition of $\mu^{\ell}(\mu^{\ell}=10)$ real units ${}_{\lambda}^{+}\tilde{\tau}^{\ell-1}$ of level $\ell-1$ is equivalent to multiplication ${}_{\lambda}^{+}\tilde{\tau}^{\ell-1} * \mu^{\ell}$, i.e. 0.1*10 or multiplication ${}_{\alpha}^{+}\tilde{\tau}^{\ell-1} * {}_{\alpha}^{+}\tilde{\tau}^{\ell+1}$ of units of levels $\ell-1$ and $\ell+1$.

Both indices $\ell-1$ and $\ell+1$ of units of level λ are equal to φ and these units for $\ell=\lambda$ can be recognized as imaginary ones, i.e. ${}_{\lambda}^{+}\tilde{\tau}^{\ell-1} = {}_{\varphi}^{+}\tilde{\tau}^{\ell}$, ${}_{\lambda}^{+}\tilde{\tau}^{\ell+1} = {}_{\varphi}^{+}\tilde{\tau}^{\ell}$ and imaginary numbers ${}_{\varphi}^{+}\tilde{\tau}^{\ell} * {}_{\varphi}^{+}\tilde{\tau}^{\ell}$ are multiplied.

The result of multiplication in both cases is unit ${}^{+}_{\lambda}\tilde{\tau}^{\ell}$ of level ℓ. Its sign – the symbol of direction in ${}_{\psi}\tilde{T}^{\ell}$ – is plus for the case of real numbers, and minus for imaginary ones. The connections between the real numbers and imaginary units ψ and χ are explained by analogy.

Therefore, in numerical positional system L^s the fundamental difference not only between integer and real numbers but also between the real and imaginary ones disappears. The difference between adding and multiplication is also lost. A real number could be obtained by the addition of imaginary numbers if levels in ${}_{\psi}\tilde{T}^{\ell}$ are taken into account.

The example of arithmetical operations in L^s is reflected in Figure 3.4. When on level $\ell - 1$ there are elements with the opposite orientation with respect to the unit of level ℓ, the unit is decomposed into μ^{ℓ} units of level $\ell - 1$, which are later united with the oppositely oriented elements, as a result of which the neutral ones are obtained. That number of units of level $\ell - 1$ for which there are no oppositely oriented units on level $\ell - 1$ and lower levels becomes a numerical characteristic of level $\ell - 1$ with its sign.

To summarize the presented description of numeric positional system L^s, the following characteristic features of the system were underlined and it was pointed out that properties ${}_{\ell\tau}\Psi$, ${}_{\ell\mu}\Psi$ which reflect both the structure and dynamics of L^s on its different levels and interlevel relations do not have analogies in the known mathematical models used for mechatronic system design. For instance:

– discrete systems of each level ℓ in L^s are connected in such a way, that the system of level $\ell + 1$ becomes continuous;

– the interlevel transitions are also continuous;

– information for higher levels is aggregated;

– systems of any level ℓ in L^s can be divided;

– relations of real and imaginary numbers in L^s were assigned not only by multiplication but also by structural and dynamic uniting of canonic states;

– block of coordinator $S_0^{\ell}\big|L^s$ sets values μ^{ℓ} of the basis on each level of L^s in the process of its adjustment to concrete systems with a natural basis, e.g. measuring systems of time, length, etc.

Therefore, the formal model of standard block $L^{s\ell}$ of system L^s can be constructed as *aed* formal model contraction $S^{\ell}\big|L^s = L^{s\ell}$ only. Models of numerical systems in terms of algebra and automata theory are particular cases of $L^{s\ell}$, which is caused by the ways of its formalization [3,8-10]. The system for geometrical and physical objects measuring in H. Lebesque theory [48] is also a particular case of L^s.

The possibility of physical realization of L^s arises from its coordination with natural processes of calculation performed in positional codes. That possibility is partly illustrated by Figures 3.4-3.6 in which L^s is reflected by analogues of modern calculating machines.

3.3.3. Measuring units of mechatronic objects

To introduce the measuring units of hierarchical mechatronic objects, the following definitions are formulated. Previously, it was pointed out, that states of mechatronic system S^ℓ are indicated by symbols of L^s, i.e. they could be of different levels, orientation or neutral on each level. Their interlevel dynamics is defined by scheme ${}_{\ell\tau}\Psi$ (Fig. 3.3).

Definition 3.6 (i). Any neutral state ${}_i c^\ell_{t_i} \in_i C^\ell$, where $t^\ell_i \in T^\ell_i$ is the minimal element of T^ℓ_i, is called *initial state* ${}_i\eta^\ell$ (the unit from ${}_i\sigma^\ell$ for measuring S^ℓ) for any period of time $T^\ell_i \in_\psi T^\ell = \{T^\ell_\psi, T^\ell_\chi, T^\ell_\varphi, T^\ell_\lambda\}$ of system S^ℓ. Multitude ${}_i\tilde{H}^\ell = \{{}_i\eta^\ell\} \subset_i C^\ell$ is called the *object of initial states* for T^ℓ_i.

(ii) For any period of time $T^\ell_i \in_\psi T^\ell$ any neutral state ${}_i c^\ell_{t_i} \in_i C^\ell$ is called the standard state, or *measuring unit* ω^ℓ, where $\tilde{t}^\ell_i$ is the maximal element of T^ℓ_i. Multitude ${}_i\tilde{H}^\ell = \{{}_i\eta^\ell\} \subset_i C^\ell$ is called the *object of standard states* for T^ℓ_i, $i \in_\psi L$.

It follows from ${}_s\Psi$ (3.27) that the standard state of each period $T^\ell_i \in_\psi T^\ell$ is the initial state for period $T^\ell_{i+1} \in_\psi T^\ell$: ${}_{i+1}\mathrm{H}^\ell = {}_i\tilde{H}^\ell = {}_i\mathrm{H}^{\ell+1}$.

Definition 3.7. Set $\{{}_\psi\chi^\ell(\tilde{\eta}^\ell_s, S^\ell), \tilde{\eta}^\ell_s\}$ is called *metrical characteristic* $\mu^\ell_{\tilde{\eta}^\ell_s}$, $S^\ell = \mu^\ell(\tilde{\eta}^\ell_s, S^\ell)$ of mechatronic system S^ℓ in relation to standard $\tilde{\eta}^\ell_s$, where ${}_\psi\chi^\ell(\tilde{\eta}^\ell_s, S^\ell)$ is the measure of S^ℓ in units $\tilde{\eta}^\ell_s$.

Metrical characteristic concerning the initial state is identified by analogy.

The representation of metrical characteristics:

$$_{\sigma}\mu^{\ell} = \mu^{\ell}(\tilde{\eta}^{\ell}_{\sigma}, \sigma^{\ell}),\ \ _{\omega}\mu^{\ell} = \mu^{\ell}(\tilde{\eta}^{\ell}_{\omega}, \omega^{\ell}), \tag{3.64}$$

$$_{c}\mu^{\ell} = \mu^{\ell}(\tilde{\eta}^{\ell}_{c}, c^{\ell}),\ _{x}\mu^{\ell} = \mu^{\ell}(\tilde{\eta}^{\ell}_{x}, x^{\ell}),\ _{y}\mu^{\ell} = \mu^{\ell}(\tilde{\eta}^{\ell}_{y}, y^{\ell}),$$

$$_{\omega c}\mu^{\ell} = \mu^{\ell}(\tilde{\eta}^{\ell}_{\omega x}, _{\omega}c^{\ell}),$$

for ω^{ℓ} or σ^{ℓ}, states of object $_{o}S^{\ell}$, processes $_{\pi o}S^{\ell}$, $_{\pi\varepsilon}S^{\ell}$ and for environment $_{\varepsilon}S^{\ell}$, respectively, allows a range of state spaces to be constructed in addition.

Metrical characteristics $\mu^{\ell} \in \mathrm{M}^{\ell}$ are dimensional qualities and are constructed from two elements, one of which – measure – is the number in L^{S} code and the other one is any unit of measurement.

It should be pointed out, that the metrical characteristics of object $_{o}S^{\ell}$, process $_{\pi}S^{\ell}$ and environment $_{\varepsilon}S^{\ell}$ are connected with one another by relations determined by aggregated dynamic realization ω^{ℓ}. Metrical characteristics $_{\omega}\mu^{\ell}$ and $_{\sigma}\mu^{\ell}$ are connected by interlevel relations realized by coordinator S^{ℓ}_{0}. The connections for each particular mechatronic system are defined by its formal model S^{ℓ}.

Preliminary conclusions

In Chapter 3, the element of theoretical basis of HS design method – the *aed* formal model of coordinator – was constructed with the help of the dynamic system. Coordinator's state spaces concerning the level of its information uncertainty were introduced. Canonical presentation of the coordinating element, the process of coordination and the environment of coordinator were built in the state spaces obtained.

The design task of mechatronic systems is performed by coordinator on its strata by one of its coordination strategies. Strategies of selection strata were described by the output functions of canonical model. Change of states of coordinator was described by states transition functions, which correspond to the functions of leaning strata (change of coordinator's strategies) and functions of self-organization strata (change of the state of the formal model). Coordinator actions while performing the main design tasks of mechatronic objects – synthesis and analysis – were described.

So, the model developed meets the requirements of the design systems formulated in Introduction and Section 3.1 of this Chapter.

With the help of numeric positional system, connections of levels and metrical characteristics of the hierarchical mechatronic objects being designed were introduced. The known numeric systems were subsequently described with the help of hierarchical systems. All this allows quantitative calculation on the space of levels of the designed mechatronic objects to be executed. Such a description causes realization of coordinator in the form of the theoretical construction of *aed*-processor, i.e. cybernetic system oriented to numeric and geometric information processing.

Theoretical basis of the conceptual HS design method, i.e. *aed* formal model presented in this Chapter 3, was improved in comparison with its previous version described in [140], and has got the following additional characteristic features: general scheme of *aed* (Fig. 3.1) has improved so that system S^{ℓ} block became included in its environment ${}_{\varepsilon}S^{\ell}$; all elements of *aed* model were presented in the form of systems with correspondent indexes from the set ${}_{k}L \leftrightarrow \{o, o\pi, \pi\varepsilon, \varepsilon\}$, which indicate object (o), environment (ε), and their processes $(o\pi)$ and $(\pi\varepsilon)$; symbol presentation of systems connections was changed from ${}_{\sigma}U^{\ell}$ to ${}_{\sigma}\gamma^{\ell}$ what made it agreed with presentation of coordination signals γ_0^{ℓ} generated by coordinator while performing systems structures synthesis in design process by arranging subsystems structural connection; fundamental *aed* law of systems levels grows was modified by the law of systems outlook increasing in the levels space, i.e. increasing of number τ of system levels $\ell \pm \tau$ being coordinated 'up' and 'down' in regard with current level ℓ of system under consideration; symbols set of levels of system organization ${}_{\Psi}L$ and correspondent basic system ${}_{\Psi}\tilde{T}^{\ell}$ were changed to ${}_{\Psi}L \leftrightarrow \{\psi, \chi, \varphi, \lambda\}$ and improved in such a way, that they become coordinated with coordinator canonic model $(\hat{\varphi}, \hat{\lambda})$ and its reactions λ and functions φ performed on selection and learning strata respectively. Furthermore, modified *aed* model presented in this Chapter was further improved. New *aed* formal model for systems design and control was presented and described by Novikava and Miatluk in [8], and used after that by Miatluk and Kim in [80].

4. DESCRIPTION OF THE DESIGN METHOD

The formal models of hierarchical mechatronic objects and coordinator given above in Chapter 3 are used in this Chapter for the description of both mechtronic system (MS) being designed and the design process itself. Each mechatronic system can be presented at the conceptual design phase according to (3.1) in the following *aed* form (4.1) of HS:

$$ {}_{m}S^{\ell} \leftrightarrow {}_{m}\{\omega, S_0, \sigma\}^{\ell}, \tag{4.1}$$

where: ${}_{m}\omega^{\ell}$ is the dynamic representation of mechatronic system ${}_{m}S^{\ell}$ as a unit in its environment ${}_{m\varepsilon}S^{\ell}$, ${}_{m}\sigma^{\ell}$ is the structure of the system ${}_{m}S^{\ell}$, ${}_{m}S_0^{\ell}$ is coordinator, which connects ${}_{m}\sigma^{\ell}$ with ${}_{m}\omega^{\ell}$, *m* is the index which indicates the mechatronic nature of the object being designed.

The states of a mechatronic system in its ω^{ℓ} and σ^{ℓ} forms are detected by its metrical characteristics. Aggregated dynamic (functional) representation of mechatronic system (MS) is described in accordance with (3.11):

$$ {}_{m}\omega^{\ell} = {}_{m}\{ {}_{o}(\bar{\rho},\bar{\varphi})^{\ell}, {}_{\pi}(\bar{\rho},\bar{\varphi})^{\ell}, {}_{\varepsilon}(\bar{\rho},\bar{\varphi})^{\ell} \}, \tag{4.2}$$

where: ${}_{o}(\bar{\rho},\bar{\varphi})^{\ell}$ – dynamic description of mechatronic object (MO) itself,

${}_{\pi}(\bar{\rho},\bar{\varphi})^{\ell}$ – description of the mechatronic process (see Subsection 4.4.2),

${}_{\varepsilon}(\bar{\rho},\bar{\varphi})^{\ell}$ – dynamics of the MO environment (external representation).

An examples of the description of MO dynamics by ${}_{o}(\bar{\rho},\bar{\varphi})^{\ell}$ is presented below in Subsection 4.4.2.

Structure ${}_{m}\sigma^{\ell}$ of mechatronic object is presented according to (3.14) in the following form (4.3):

$$ {}_{m}\sigma^{\ell} = {}_{m}\{S_0^{\ell}, \{\bar{\omega}^{\ell-1}, \gamma_{\sigma}^{\ell}\}\}, \tag{4.3}$$

where $\overline{\omega}^{\ell-1}$ is a set of mechatronic subsystems of lower level ℓ-1 and their structural connections γ_{σ}^{ℓ}.

Coordinator ${}_mS_0^{\ell}$ is the system which sets the characteristics of mechatronic object elements and coordinates their connections by the sequence of operations and the decision-making actions, such as a choice of the decision and the constructing of semantic net on the MO elements set. These actions include the operations of MO metric characteristics changing, operations with MO geometric elements structures and their connection.

So, to construct an mechatronic object it is necessary to define its structure, its function – aggregated dynamic representation as the unit in its environment, its environment construction and technology, MO coordinator (design and control unit), and their metrical characteristics.

The *main design tasks – analysis and synthesis – of mechatronic objects* are performed by coordinator ${}_mS_0^{\ell}$ on its strata (see Chapter 3) under conditions of different information uncertainty. ${}_mS_0^{\ell}$ creates the desired construction of mechatronic object in accordance with its required dynamic functional representation in the environment (*synthesis task*). In its turn, it partly changes the structure parameters of the resultant mechatronic object and analyses its behavior in the environment (*analysis task*). All the tasks are performed at both Conceptual Design (CD) and Detailed Design (DD) phases of mechatronic object life cycle using HS design method, i.e. coordination technology of HS.

So, **the main steps of the HS design method** realization are the following.

1) Conceptual model creation – description of the mechatronic object being designed in *aed* form of HS; *aed* is a standard block of hierarchical systems – formal analogue of two-level system with dynamic units.

2) Representation of basic geometric elements (point, line, surface, 3D object) of the mechatronic object being designed in the hierarchical form – conditional point, line, surface, 3D object.

3) Definition of metric and constructive characteristics – constructive dimension and connections defect – of the mechatronic object being designed in the codes of numeric positional system.

4) Realization of the design process of mechatronic objects – synthesis and analysis tasks performing – as the coordination process of *aed*.

At the *first step* of the method realization – conceptual model creation – the connected formal models of the mechatronic object structure, its dynamic functional representation as a unit in its environment, the object environment and the environment processes are developed (see Chapter 3). Formal descriptions of object-environment interactions, mechatronic object coordinator and its coordination (design and control) processes, processes executed by subsystems and general process are presented as well.

At the *second step*, the representation task of basic geometric elements of the mechatronic object being designed in the hierarchical form is performed. The representations of MO geometric elements in HS forms of conditional point, line, surface and 3D object are described in details below in Subsection 4.3.

At the *third step*, definition of metric and constructive characteristics of the mechatronic object being designed is performed with the help of numeric positional system L^s. Main numeric characteristics of MO – constructive dimension and connections defect – are introduced in L^s basis. Formal definitions of these characteristics are presented below in Section 4.2.

At the *fourth step* – synthesis and analysis tasks performing – the conceptual descriptions of the tasks are presented first. The task of MO structure *synthesis* is given in the following form of the coordinator mapping:

$$S_0^{\ell} : \{\overline{\omega}^{\ell-1}, {}_{\sigma}\gamma^{\ell}\} \to \sigma^{\ell} \tag{4.4}$$

where MO structure σ^{ℓ} which is synthesized from set of sub-subsystems $\overline{S}^{\,\ell-1}$ presented in aggregated dynamic forms $\overline{\omega}^{\ell-1}$ by establishing of the structural connections ${}_{\sigma}\gamma^{\ell}$ of these elements $\overline{S}^{\,\ell-1}$.

The steps of the synthesis process ${}_{\pi}S_1^{\ell}$ are formally described by reaction of dynamic system $(\rho,\varphi)^{\ell}$:

$$\{\rho_i : C_i \times X_i \to Y_i \,\&\, i \in T\}^{\ell}, \tag{4.5}$$

where *i* is a time moment of the synthesis process, initial element (or composition of elements previously connected in the synthesis process) is recognized as state C_i, at the step *i*, element to be added to initial one is recognized as input X_i, and the composition of the both connected initial and input elements is recognized as output Y_i. The output Y_i becomes the state C_{i+1} at the next step of the synthesis process.

The *analysis* task of MO is performed by coordinator and presented at CD phase in the following form of the coordinator mapping:

$$S_0^{\ell} : \{\overline{\omega}^{\ell-1}, {}_{\sigma}\gamma^{\ell}\} \to \omega^{\ell} \tag{4.6}$$

In this task, coordinator partially changes the structural connections ${}_{\sigma}\gamma^{\ell}$ of MO by sending its coordination signals to MO structure and observe the variations of MO behavior – change of MO functioning in its environment caused by MO structural variations.

While performing analysis tasks, e.g. kinematics and dynamics tasks, conceptual models ω^{ℓ} of MO are presented at CD phase in $(\rho,\varphi)^{\ell}$ form. At the DD phase the conceptual models are concretized and presented in forms of DE, State Space Equations or algebraic form. The exemplary scheme of MO conceptual model concretization at the design phase of the object life cycle (Fig. 1.2, Chapter I) is presented in Fig. 4.1.

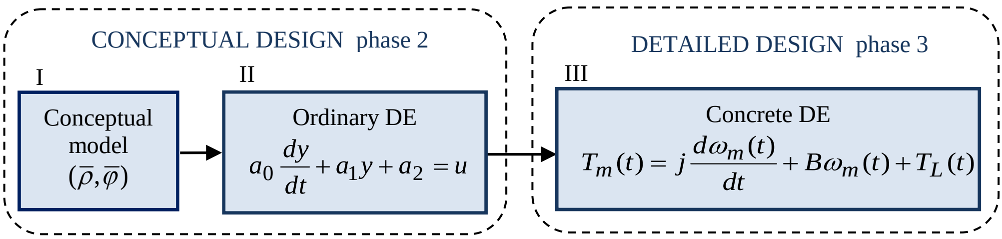


Fig. 4.1. Steps of conceptual model $(\overline{\rho},\overline{\varphi})$ concretization at the design phases of MO life cycle; conceptual model is transformed to ordinary DE at CD phase and concretized at DD phase by the exemplary dynamics equation of object being in rotational motion, where *ω(t)* is the angular velocity and is *T(t)* is a torque

Synthesis and analysis tasks of geometric constructions of mechatronic objects being designed are partially described in Chapters 6-8 as well. The steps of the above HS design method are described below in Chapters 5-9 for the cases of the exemplary mechatronic objects design tasks. The method is also partially regarded in [49, 51-54].

4.1. General geometric characteristics of mechatronic objects

For the description of geometric elements of mechatronic object construction at the *second step* of the HS design method realization the geometric information is presented according to (4.1) as a set of variables of three classes:

$$\overline{Z} = \{Z_1, Z_2, Z_3\} = \{\Omega, \Gamma, \Sigma\}, \tag{4.7}$$

where: $\Omega=\{\omega_i : i\in I_\Omega\}=Z_1$ is the description of the state of geometric object (with the help of metrical characteristics μ_i^ℓ which are the coordinates, length, squares, volumes, angles) presented in aggregated dynamic form ω^ℓ, $\Gamma=\{\gamma_i : i\in I_\Gamma\}=Z_2$ – is the description of geometric object interconnections ${}_\omega\gamma^\ell$ with other objects (boundary of the object defined by the coordinator),

$\Sigma=\{\sigma_i : i\in I_\Sigma\}=Z_3$ – is the description of object structure σ^ℓ.

Then, any geometric representation of mechatronic object is presented in form of relations $S\subset\Omega\times\Gamma\times\Sigma$ on the Cartesian product of variables Ω,Γ,Σ. Each class of variables has its own structure, i.e. there are the sets:

$$\overline{\psi}_1=\{{}_\Omega\psi, {}_\Gamma\psi, {}_\Sigma\psi\}, \tag{4.8}$$

where: ${}_\Omega\psi$ is the structure of class Ω, ${}_\Gamma\psi$ is structure of Γ class, and ${}_\Sigma\psi$ is structure of class Σ.

Besides, there are gluing relations $\overline{\psi}_2$ between Ω,Γ,Σ. The multitude of all the relations on $\overline{Z}$ is marked as $\overline{\psi}$: $\overline{\psi}=\{\overline{\psi}_1,\overline{\psi}_2\}$.

One of the relations from $\overline{\psi}_2$ is $\Gamma\subset\Sigma$. That means that the boundary Γ of each geometric object ${}_gS^\ell$ is included in its structure Σ. The boundary is the area of geometric construction ${}_{gm}S^\ell$ of mechatronic object ${}_mS^\ell$ interconnections with its environment ${}_\varepsilon S^\ell$ (i.e. Γ defines the location of ${}_{gm}S^\ell$ in ${}_\varepsilon S^\ell$). Each $\gamma\in\Gamma$ corresponds to the coordination signal from coordinator ${}_{gm}S_0^\ell$ to the object ${}_{gm}S^\ell$.

Ω^ℓ is defined through the interactions of the geometric mechatronic object ${}_{gm}S^\ell$ with the higher and lower levels, i.e. measured with the help of its structural units to be evaluated in its environment, and, inversely, being a unit of the environment, it is measured with the help of the other environment units. So, the higher and the lower level units for the object are its measuring units.

To construct geometric construction of mechatronic object ${}_{gm}S^\ell$ it is necessary to define all its variables μ, γ, σ. In design task any subset of parameters from $S\subset\Omega\times\Gamma\times\Sigma$ must be initially defined. Other parameters are defined in the synthesis process (Tab. 4.1).

The unknown variable (which is not presented in $\bar{Z}$) is marked in Table 4.1 by symbol *?*. Definition of *?* symbol is a task of self-organization strata of the coordinator.

Taking into account the type of the design task and the knowledge level of the coordinator, ${}_{gm}S_0^{\ell}$ chooses the strategy of knowledge settlement, i.e. the algorithm of the synthesis process.

Therefore, the structure of the design process is defined by:

– the dividing of set $\bar{Z}$ into known and unknown parameters (inputs and outputs), i.e. the type of constructing task,
– by the geometric representation ${}_{gm}S^{\ell}$ of the mechatronic object being designed (concrete meanings of variables),
– by the uncertainty level of semantic net relations $\bar{\psi}$ (former design experience).

Coordinator ${}_{gm}S_0^{\ell}$ in this case plays the role of a system which defines the characteristics of the elements set and coordinates their interactions with the help of a sequence of actions and procedures of decision making.

These action classes include the procedures of the changing of metrical characteristics as well as actions with boundaries and structures of geometric elements of MO.

Table 4.1. Subsets of parameters $\bar{Z} = \{\Omega, \Gamma, \Sigma\}$ in design tasks, processes and semantic relations

Design process	Known parameters (input)	Unknown parameters (output)	Semantic net relations
$P_{\Omega\Gamma\Sigma,S}$	μ, γ, σ	S	$\bar{\psi}\|_S$
$P_{\Gamma\Sigma,\Omega S}$	γ, σ	Ω, S	$\psi_{\Gamma\Sigma,\Omega}\|_S$
$P_{\Omega\Sigma,\Gamma S}$	μ, σ	Γ, S	$\psi_{\Omega\Sigma,\Gamma}\|_S$
$P_{\Sigma,\Omega\Gamma S}$	σ	Ω, Γ, S	$\psi_{\Sigma,\Omega\Gamma}\|_S$
$P_{\Omega\Gamma,\Sigma S}$	μ, γ	Σ, S	$\psi_{\Omega\Gamma,\Sigma}\|_S$
$P_{\Gamma,\Omega\Sigma S}$	γ	Ω, Σ, S	$\psi_{\Gamma,\Omega\Sigma}\|_S$
$P_{\Omega,\Gamma\Sigma S}$	μ	Γ, Σ, S	$\psi_{\Omega,\Gamma\Sigma}\|_S$
$P_{?,\Omega\Gamma\Sigma S}$	?	Ω, Γ, Σ, S	$\varnothing$, S^{ℓ}

4.2. Numerical characteristics of mechatronic objects

To realize the *third step* of proposed method, all numerical characteristics of geometric elements of MO (mechatronic object) S^{ℓ} are described in the work and connected with the help of numeric positional system L^S presented in Section 3.3.

So, metrical characteristic $\mu^{\ell} \in M^{\ell}$ (measure of length, squares, volumes, angles) is obtained in units ${}_{\tau}\eta^{\ell} \in H^{\ell} = \{{}_{\psi}\eta, {}_{\chi}\eta, {}_{\varphi}\eta, {}_{\lambda}\eta\}$, presented in Subsection 3.3.3, with the coefficients from L^S:

$$ {}_{\tau}\tilde{\mu}^{\ell} = {}_{\tau}\left({}^{-}\tilde{\mu}^{\ell}, {}^{0}\tilde{\mu}^{\ell}, {}^{+}\tilde{\mu}^{\ell}\right), \qquad {}^{\tau}_{0}\tilde{\mu}^{\ell} = {}_{\tau}\left({}^{-}\tilde{\mu}^{\ell-1}, {}^{0}\tilde{\mu}^{\ell-1}, {}^{+}\tilde{\mu}^{\ell-1}\right) = {}_{\tau}\tilde{\mu}^{\ell-1}, \tag{4.9} $$

where: $\tau \in_{\psi} L$, ${}^{-}\tilde{\mu}^{\ell}$ – negative, ${}^{0}\tilde{\mu}^{\ell}$ – neutral, ${}^{+}\tilde{\mu}^{\ell}$ – positive parts of $\tilde{\mu}^{\ell}$; unit ${}^{0}\tilde{\mu}^{\ell}$ is at the same time a unit of lower level ℓ-1, etc. Numerical characteristics of system S^{ℓ} organisation – connection defect $\tilde{\xi}^{\ell}$ and constructive dimension $\tilde{\delta}^{\ell}$ were defined in the following way.

Let the object Ξ^{ℓ} and the reflection ${}_{\xi}\mathrm{R}:{}_{\sigma}U^{\ell} \to \Xi^{\ell}$ be as follows:

$$ \Xi^{\ell} = \left\{\xi^{\ell}_{\sigma} = (\xi,n)^{\ell}_{\sigma} : (\xi,n)^{\ell}_{\sigma} \in L \times N \& \xi \in L \& n \in N \& N = \mathrm{N}^{+} \cup \{0\}\right\}, $$

$$ \left[{}_{\xi}\mathrm{R}({}_{\sigma}\gamma^{\ell}) = (\xi,n)^{\ell}_{\sigma}, \xi = \ell - \hat{\ell}\right] \Leftrightarrow \tag{4.10} $$

$$ \Leftrightarrow \left[(\exists_{\omega}\bar{\gamma}^{\hat{\ell}} \subset {}_{\sigma}\gamma^{\ell})({}_{\omega}\bar{\gamma}^{\hat{\ell}} = \{{}_{\omega}\gamma^{\hat{\ell}}_{i} : {}_{\ell}\mathrm{R}({}_{\omega}\gamma^{\hat{\ell}}_{i}) = \hat{\ell}\} \& {}_{n}\mathrm{R}({}_{\omega}\bar{\gamma}^{\hat{\ell}}) = n\right], $$

${}_{\ell}\mathrm{R}:{}_{\omega}\gamma^{\hat{\ell}}_{i} \to L$, ${}_{n}\mathrm{R}:{}_{\omega}\bar{\gamma}^{\hat{\ell}} \to N$, n – cardinality of ${}_{\omega}\bar{\gamma}^{\hat{\ell}}$, ${}_{\omega}\gamma^{\hat{\ell}}_{i}$ – interconnections of $S^{\ell-1}_{i}$ in σ^{ℓ}; N^{+} – natural numbers multitude, $L \in L^S$.

Then ξ^{ℓ}_{σ} is called a *connection defect* of σ^{ℓ} of order ξ and cardinality n, and

$$ (\forall \ell \in L) \Rightarrow (\xi_{\sigma} \in I_{\xi}, I_{\xi} = \{0,1,2,3\}). \tag{4.11} $$

The locations of the defects in MO structure σ^{ℓ} are defined by parameter $\xi^{\ell}_{\sigma,\gamma}$:

$$\xi_{\sigma,\gamma}^{\ell} = (i,\tau)_{\xi}...(i,\tau)_0 : \tag{4.12}$$

$$(\forall \xi^{\ell} \geq 0) \Rightarrow \left[(i,\tau)_{\xi} = \left\{ (i,\tau) : (i,\tau) \in I^{\ell} \times I^{\ell} \& \tau \neq i \& {}_{\xi}\mathrm{R}(\gamma_{i,\tau}^{\ell} = \ell - \hat{\ell} = \xi^{\ell} \right\} \right].$$

For each system S^{ℓ} of level $\ell \in L$ the connection defect ξ_{ω}^{ℓ} in environment ${}_{\varepsilon}S^{\ell}$ is called the contraction of connection defect $\xi_{\sigma,\gamma}^{\ell+\tau}$ of system $S^{\ell+t}$ of higher level $\ell + \tau$, $\tau \in L^{s}$ on ${}_{\omega}\gamma^{\ell}$.

In the code of numeric positional system L^{S} the connection defect is defined as follows:

$$\tilde{\xi}^{\ell} = (n_3...n_0)_{\xi}, \tilde{\xi}^{\ell} \in \{\xi_{\sigma}^{\ell}, \xi_{\omega}^{\ell}\}. \tag{4.13}$$

The formal definition of constructive dimension and the ways of its calculation are obtained from the definition of connection defect $\tilde{\xi}^{\ell}$.

Constructive dimension $\delta^{\ell} \in \Delta^{\ell}$ of system S^{ℓ} is called its numeric characteristic, which is presented in L^{S} code:

$$\tilde{\delta}^{\ell} = (n_3...n_0)_{\delta}, \tilde{\delta}^{\ell} \in \{\delta_{\sigma}^{\ell}, \delta_{\omega}^{\ell}\}, (n_i)_{\delta} = (n_{3-i})_{\xi}, \tag{4.14}$$

where: $(n_i)_{\delta} \in N$, i=0,1,2,3; δ_{ω}^{ℓ} and δ_{σ}^{ℓ} are constructive dimensions of σ^{ℓ} and ω^{ℓ}, respectively. The instance of constructive dimension $\tilde{\delta}^{\ell}$ connection with Euclidean dimension is presented by Table 4.2.

Table 4.2. Connection of constructive dimension $\tilde{\delta}^{\ell}$ with Euclidean dimension

Constructive dimension $\tilde{\delta}^{\ell}$	Geometrical object S^{ℓ}
0 0 0 1	Point
0 0 1 0	Line
0 1 0 0	Surface
1 0 0 0	3D object

Therefore, the described representation of geometric information allows all operations with graphic images in *aed*-processor (subsection 3.3.2) to be executed in the design process of MO as operations with numeric codes. It allows increasing the efficiency of computational processes while performing the design procedures.

L^s may be recognized as new coordinate space (hierarchic coordinates) which has not only habitual transformations of coordinates, but also dimension changes (in line with interlevel connections of real, complex and hyper complex numbers).

The records of $\tilde{\xi}^{\ell}$ and $\tilde{\delta}^{\ell}$ in L^s code allow carry out all L^s tasks with $\tilde{\xi}^{\ell}$ and $\tilde{\delta}^{\ell}$, and change the dimensions and connections of the units with changing their scales in $\tilde{\mathrm{T}}^{\ell}$. The actions with $\xi^{\ell}_{\sigma,\gamma}$ and $\delta^{\ell}_{\sigma,\gamma}$ are executed in the space of L^s laws.

All geometric characteristics in hierarchical space are changeable. The changes of construction connections in $\sigma^{\ell-\tau}$ (changes in $\xi^{\ell-\tau}$ and $\delta^{\ell-\tau}$) cause alteration of coordinates ω^{ℓ} (the movements of Γ^{ℓ} in structure σ^{ℓ}) and thereby changes in the construction $\sigma^{\ell+\tau}$. Thanks to that a possibility appeared to design the connected motions and deformations of mechatronic systems on their different levels.

4.3. Hierarchical geometric representation of mechatronic objects

In accordance with the *second step* of HS design method, the following geometric elements of mechatronic objects (MOs) are presented in hierarchical form:

– points S^0,

– parts of lines S^1,

– surface S^2 patches,

– 3D objects S^3.

The structure of hierarchical systems is introduced on the multitude of the structures of these elements $\Sigma = \left\{\sigma^0, \sigma^1, \sigma^2, \sigma^3\right\}$. Thus, it became possible to operate in design process not with ideal mathematical objects but with their equivalents – *conditional* S^0, S^1, S^2 objects, which have all metrical characteristics of 3D objects of S^3 type.

4.3.1. Conditional point

The conditional point is described as follows. Conditional point S^{ℓ}, where level ℓ =0, is a minimal element of geometric representation. In comparison with the usual point it has all the characteristics of hierarchical systems:

$$S^{0}=\{\sigma,\ \gamma,\ \omega\}^{0}. \tag{4.15}$$

Conditional point S^{0} can be graphically presented by a cube, a sphere, a pyramid, etc.

Metrical characteristics of ω^{0} (in case of cube) are: μ_0^0 – tops coordinates, μ_1^0 – length of edges, μ_2^0 – square of sides, μ_3^0 – cube volume.

The conditional point has the following qualities.

It may be constructed from lower level units (also points):

$$S^{\ell}=\{S_1, S_2, ..., S_n, n \in I^{\ell-1}\}^{\ell}, \tag{4.16}$$

but graphically its structure can't be observed. Points $S_i^{\ell-1}$ of lower level ℓ *-1* have the same type of structure but their metrical characteristics are smaller.

Each conditional point $S_i^{\ell-1}$ can be in its turn decomposed into $\overline{S}^{\ell-2}$. The sequence of such decomposition operations leads to the ideal point with metrical characteristics $\mu_1^0=\mu_2^0=\mu_3^0$ =0.

In comparison with the ideal point, the conditional one has more operations of coordinator: scaling (which does not take conditional points beyond their class) and level lowering. In addition, the points can be united into a higher level system. Thanks to that, it is easy to introduce the definitions of conditional line and surface, which have all the characteristic features of ideal ones, and in addition, have their own qualities.

As in the case of numbers, the point representation contains its positive (+), negative (-), i.e. prolongation, and neutral (0) parts. In the coordinates it was presented as shown in Figure 4.2. In the case of conditional point interconnections in the structure of higher level objects, the coordinates (directions of interconnections in Fig. 4.2b) are defined by the structural addresses of the other elements of the objects.

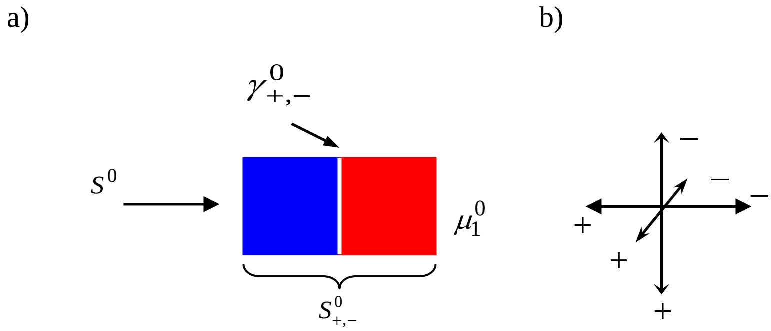


Fig. 4.2. a) conditional point S^0 with its positive (+), negative (-) and neutral $\gamma^0_{+,-}$ parts and metrical characteristic μ^0_1 – length of the edge; b) directions of point interactions in the structure of a higher level

4.3.2. Conditional line

The part of continuous conditional line S^1 is defined in hierarchical form as a two-level system (*aed*) which contains the finite number of conditional points $\{S^0_1,...,S^0_n\}=\bar{S}^0$. The points are ordered in a linear sequence in the structure of line S^1, i.e. each point S^0_i, i=1, ..., n has one preceding and one following points.

The main elements of *aed* definition for conditional lines $S^1=\{\sigma,\gamma,\omega\}^1$ are:

– structure σ^1 as the finite set of conditional points;

– connections $\gamma^1=\{{}_\sigma\gamma,{}_\omega\gamma\}^1$ which contain structural connections ${}_\sigma\gamma^1$ – the sides and angles between conditional points, and external interactions ${}_\omega\gamma^1$ – any point (or some points) from line structure, and/or boundary points $\{S^0_1,S^0_n\}\in S^1$ included into ${}_\omega\gamma^1$;

– aggregated representation ω^1 – the description of line as a unit with its metrical characteristics (for example, μ^1_0 are the coordinates of the points of the left and right ends of the line, μ^1_1 is the length, etc.).

Coordinator S^1_0 of line S^1 can execute all operations with conditional points which allow the points interactions to be coordinated in order to meet the requirements to system S^1, i.e. the line equation. So, coordinator S_0^1 can define all elements in accordance with the requirements to the system. When the boundary points $\{S_1^0, S_n^0\}$ coordinates μ_0^1, line length μ_1^1 and the distance between the centres of two neighbouring points μ_1^0 are given, then the number *n* of elements S_i^0 is precisely defined by the coordinator S_0^1.

The subset of the line structural points is called the basis of the line points.

The following *quality* is formulated as follows: i*n comparison with the ideal line, the coordinator of conditional one realises operations of decomposition and level rise*.

The part of conditional line S^1 in its ω^1 form is described by the same equation as the ideal one. So, the coordinator can execute the same operations with the conditional line as with the ideal one:

– parallel transfer;

– rotation;

– continuous deformation (the elements S_i^0 and their number *n* are constant, but elements interconnections ${}_\sigma\gamma^l$ and locations are changed);

– the division into lines of smaller length (in comparison with the similar operation with the ideal line, this operation, after the execution of finite *n* number of steps, leads out of S^1 class to class of points S^0);

– the continuous connecting of line S_i^1 with systems of the same class of lines (this action leaves a line in class S^l in case of saving linear order in the whole system structure).

The set of relations $\overline{\psi}$ on $\Omega\times\Gamma\times\Sigma$ contracted on S^1, ($\overline{\psi}\big|S^1$) is a set of two-level systems (*aed*) relations. Thanks to that, the standard way of constructing any system S_i^1 from class S^1 was defined and it made possible to introduce new variables in $\{\Omega, \Gamma, \Sigma\}$.

The part of conditional line S^1, which is constructed from conditional points $\overline{S}^0$ (cubes) is shown in Figure 4.3. The figure is a screen-shot of the computer where geometric construction of the conditional line was synthesised form conditional points, i.e. cubes.

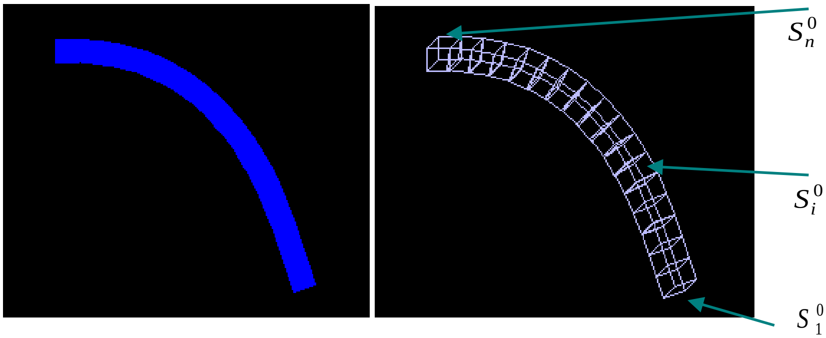


Fig. 4.3. Computer screenshot with conditional line S^1 presented in its: a) aggregated dynamic form ω^ℓ, and b) structural form σ^ℓ, where: S_1^0 – the first structural element; S_i^0 – any i structural element; S_n^0 – the last structural element, i.e. conditional point

One of the important manifestations of conditional line is its part. The part of conditional line, as in the case of the ideal one, contains all point S_i^0. The coordinates of the points are defined by the following equation:

$$S_i^0 = \alpha \cdot S_1^0 + (1-\alpha) \cdot S_n^0, \ \alpha \geq 0, \ \alpha \leq 1, \tag{4.17}$$

where S_1^0 and S_n^0 are the boundary points of S^1. Since the number of points S_i^0 of line part S^1 is finite, parameter α has a discrete set of values (0, 1/*n*, 2/*n*, ..., 1).

Coefficients α and (1-α) are called a bury-centric coordinates or weight coefficients of the points in $\{S_1^0, S_n^0\}$ basis.

4.3.3. Conditional surface

Conditional surface S^2, or patch, is a two-level system which is composed of the finite number of conditional lines which have a strictly linear order, i.e. each line has only one preceding and one following relative line.

Conditional patch is also presented in form (4.1) as object S^2. Its characteristics in general basis $\{\Omega, \Gamma, \Sigma\}$, $\overline{\psi}$ are defined as follows:

$\Sigma = \sigma^2$ – is a two-level system's (*aed*) structure defined above (which is composed from the finite number of parts of connected conditional lines) and criteria of lines selection;

$\Gamma = \gamma^2$ – a closed segment of conditional line and elements, i.e. conditional points, which create the segment;

$\Omega = \left\{\omega_i^2\right\}$ with their metrical characteristics μ_i^2, i=1, ..., n connected by the interlevel relations with the metrical characteristics μ_i^1 of the set of conditional lines $\overline{S}^1$ which create surface S^2.

Coordinator S_0^2 of system S^2 can execute all possible actions with conditional patches. These actions allow lines S^1 to be coordinated in the way meeting the requirements to the whole system S^2.

The following *qualities* of the part of conditional surface S^2 are defined as follows.

1. Since each conditional line from $\overline{S}^1$ has a strictly linear order T, it is necessary to have two parameters for the ordering of conditional points $\overline{S}^0$ of patch S^2, i.e. τ and t, t for points $\overline{S}^0$ of lines $\overline{S}^1$ ordering and τ for the ordering of conditional lines of the patch S^2.

2. Any subset of conditional lines from $\overline{S}^1$ containing initial S_1^1 and final S_n^1 lines is called a linear framework of the patch S^2. Interconnection points of any two linear frameworks are called the framework of the points. Any framework is simply defined by the boundaries and choice criteria of conditional points and lines.

3. The contraction $\overline{\psi}\big|S^2$ of the relations set $\overline{\psi}$ defined on $\Omega\times\Gamma\times\Sigma$ for S^2 class is the set of two-level system relations. Thanks to that the standard way of any system constructing from S^2 class is defined.

4. Boundary γ^2 of conditional patch S^2 contains not only conditional lines but also elements S^{-1} of conditional points which do not take part in these points interconections.

5. The interconnections of two patches, or a patch and line, or a patch and point and defined similar to the case of line in accordance with the general laws of hierachical systems.

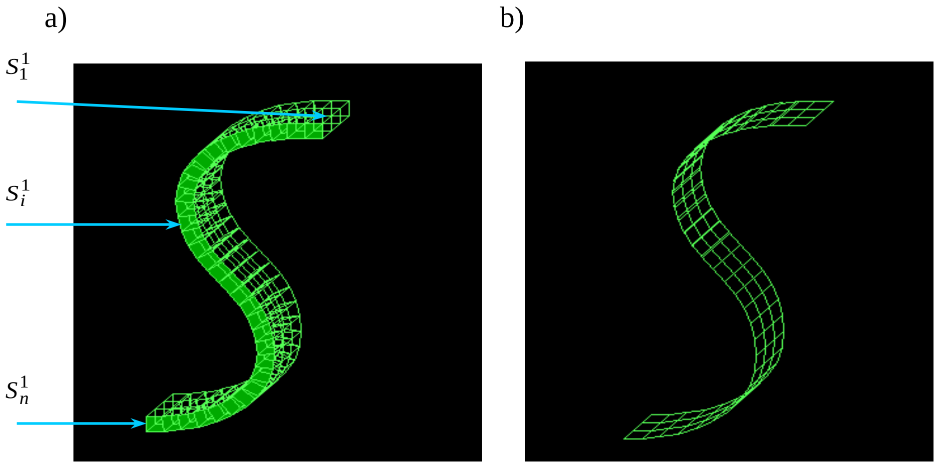


Fig. 4.4. Computer screenshot with exemplary conditional surface patch S^2 of *s*-similar form presented in its: a) structural form σ^{ℓ}, where S_1^1 is the first structural conditional line, S_i^1 is any *i* one, S_3^1 is the last *n* structural conditional line, and b) aggregated form ω^{ℓ}

The exemplary image of conditional surface patch of *s*-similar form constructed from three conditional lines S_n^1 , $n=3$, was obtained from computer monitor (Fig. 4.4). It is the result of the MO geometric element, i.e. surface synthesis using computer program implementation of HS design method proposed in the book.

4.3.4. 3D Object

3D object S^3 is a real construction (in comparison with mathematical notions of ideal point and surface) and therefore it is not necessary to introduce the term "conditional" for it.

At the same time the boundary γ^3 of object S^3 can be ideal (γ^{i3}) or conditional (γ^3) surface. For instance, when S^3 is a polyhedron it can have both the ideal and conditional sides, edges and tops (objects from S^2, S^1, S^0 classes). Each conditional top S^0 is defined by interconnections of a finite number of conditional sides S^2 and edges S^1. Each edge S^1 is defined by the interconnections of two sides S^2.

3D object S^3 is defined on the basis of geometric variables $\{\Omega, \Gamma, \Sigma\}$ and their relations $\overline{\psi}$ in the following way:

$\Sigma = \sigma^3$ is a two-level system (*aed*) composed of the finite number of connected surface patches $\overline{S}^2$ and containing the criteria of patches selection;

$\Gamma = \gamma^3$ is a closed conditional surface;

$\Omega = \{\omega_i^3\}$ with their metrical characteristics μ_i^3 which are connected by interlevel relations with characteristics μ_i^2, i=1,...,n of conditional patches $\overline{S}^2$; the patches create object S^3.

Coordinator S_0^3 of system S^3 can execute all the actions necessary for coordinating the interconnections of conditional patches $\overline{S}^2$ so as to meet the requirements to the whole system S^3. S_0^3 also executes all the actions necessary to construct S^3 from the elements of class S^3 but of smaller sizes, e.g. from conditional points S^0 which are in reality primitives from the same class S^3 but of smaller sizes.

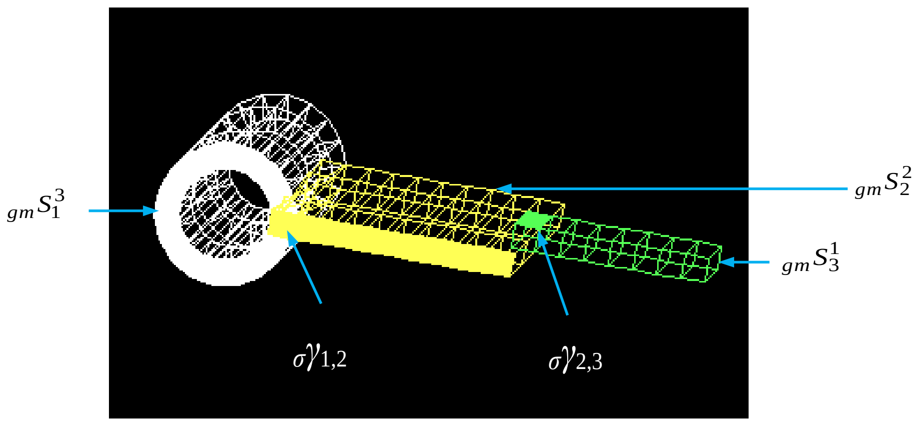


Fig. 4.5. Computer screenshot – structural interconnections of geometric elements of MO, where: $_{gm}S_1^3$ is 3D object - cylinder, $_{gm}S_2^2$ is conditional surface, $_{gm}S_3^1$ is conditional line, connected by $_\sigma\gamma_{1,2}$ (conditional **line**) and $_\sigma\gamma_{2,3}$ (conditional **point**)

The following qualities of S^3 object were defined.

1. For ordering conditional patches $\overline{S}^2$ of object S^3 it is sufficient to have one parameter *t*; for conditional points S^3 ordering it is necessary to have 3 parameters (t, τ, θ).

2. The construction $\overline{\psi}\big|S^3$ of the relations set $\overline{\psi}$ of object S^3 parameters $\Omega \times \Gamma \times \Sigma$ is defined by standard relations of *aed* – two-level system.

Exemplary 3D objects synthesized in computer are presented in Figs. 4.5 and 4.7.

4.3.5. Interconnections coordination of geometric elements of mechatronic objects

The interconnections γ of geometric elements of MO were defined by:

– indication of numbers or signs of interconnected element – geometric objects (first interaction);

– indication of quantity and set of numbers or signs of connected systems of a lower level (second interaction), etc.

The accuracy of interconnections γ grows with the consecutive, step by step indication of interacting lower level systems.

If the measure of conditional point S^0 is defined as equal to 1, and if *n* equals to 10, then measure of S^{-1} is equal to $0.1=10^{-1}$, measure of S^{-2} is equal $0.01=10^{-2}$, etc. Measure of ideal point S^{i0} ($S^{-\infty}$) is equal to $10^{-\infty}=0$.

When a part of line S^1 contains *n* (10) elements $S_1^0,...,S_{10}^0$, then its measure equals to $10^1=10$. If the quantity of the elements on each level is the same, then after the decomposition of the systems into lower level subsystems, the sizes of the subsystems are defined by the number of the level – by *log* with *n* base.

Metrical characteristics of MO geometric element of ${}_{gm}S$ of level ℓ (object ${}_{gm}S^{\ell}$) are defined by metrical characteristics of its elements ${}_{gm}\overline{S}^{\,\ell-1}$ of level ℓ-1 taking into account the number of the elements and the metrical characteristics of their interconnections $\overline{\gamma}$, as presented in Figure 4.6 (subsection 4.4.2).

An example of interconnections of geometric elements of MO is presented in Figure 4.5. Constructive dimensions for object $\delta_{\omega}^{\ell}=$1_1_1_0, and its structural connections ${}_{\sigma}\gamma^{\ell}$: $\delta_{\gamma_{\sigma}}^{\ell}=$0_0_1_1.

4.4. Procedures of geometric design of mechatronic objects

Actions with geometric elements of mechatronic objects (MO) in the design process executed by the coordinator, its *synthesis* and *analysis* algorithms are defined in this book to realise the final, *fourth step* of proposed HS design method.

4.4.1. Coordinator actions with geometric elements of mechatronic objects

The following classes of coordinator actions in design process for each type of above introduced structures $\Sigma = \left\{ \sigma^0, \sigma^1, \sigma^2, \sigma^3 \right\}$ = {point, line, surface, 3D object} are defined as follows.

1) Actions which change the object but do not lead it out of the present structure type. Due to these actions each type of structures may be regarded as a dynamic system. Operations of linear movement, rotation, changing of size μ_1 and some kinds of deformations belong to this class of operations.

2) Actions which cause limited changes of the structure. All the elements of the object are changed in this process.

This class of actions includes the above-mentioned (in 1) operations and, in addition the following ones:

– adding, erasing, changing of elements (such operations have not to change the structure type, i.e. constructive dimension and connections of elements, for instance the uniting of several linear elements into a long one);

– decomposition of structural elements into elements of lower levels;

– changing of elements connections.

3) Actions which cause a change of object level.

Such actions contain all the above-mentioned procedures (1 and 2) and also the following ones:

– size changing which causes changes of the present structure type of object:

$$\sigma^0 \leftrightarrow \sigma^3; \sigma^1 \rightarrow \sigma^0; \sigma^2 \rightarrow \sigma^0;$$

– erasing and decomposition of elements into the elements of a lower level:

$$\sigma^3 \rightarrow \sigma^2 \rightarrow \sigma^1 \rightarrow \sigma^0;$$

– synthesis of higher level system from lower level elements:

$$\bar{S}^0 \to S^1, \bar{S}^0 \to S^2, \bar{S}^0 \to S^3,$$
$$\bar{S}^1 \to S^2, \bar{S}^1 \to S^3,$$
$$\bar{S}^2 \to S^3.$$

These operations are performed by coordinator S_0^3 of the most complex geometric object S^3, which contains a set of systems from every level. Coordinators of systems S^2, S^1 and S^0 have less abilities in executing the above-mentioned actions.

For instance, the coordinator S_0^0 of conditional point S^0 performs the operations of linear motion only, which are limited by the changes of size and decomposition into points from lower levels S^{-1}, S^{-2}, etc.

Geometric elements of mechatronic objects defined on $\{\Omega, \Gamma, \Sigma\}$ and $\bar{\psi}$ go to the input of the given operations. Object parameters Ω, Γ, Σ may be changed at the output of operations with the changes of relations $\bar{\psi}$ (where the transition between the structures classes takes place) or without it. The scheme of the action *R* of changing point $S^{0'}$ coordinates $\mu_1^{0'}$ by adding coordinates $\mu_1^{0''}$ of another point $S^{0''}$ is presented in Table 4.3.

Table 4.3. Scheme of adding of two points coordinates

R	State	Input	Output
+	$\mu_1^{0'}$	$\mu_1^{0''}$	$\mu_1^{0'''}$

General representation of any action is similar to that in Table 4.3 but the inputs, outputs and states have more complicated descriptions. The description (4.15) of coordinator process of exemplary synthesis of simple GO is presented if the following subsection 4.4.2.

4.4.2. Geometric synthesis

Synthesis is the basic design task performed at both conceptual and detailed design phases. The actions realized by coordinator in the process of geometric synthesis of mechatronic object are as follows:

1. defines the constructive dimension of the resultant geometric structure of mechatronic object (chooses the required structure class $\sigma^0, \sigma^1, \sigma^2, \sigma^3$ from Σ);

2. defines kinds of changes of initial standard element of the defined structure which are necessary to perform the constructing of the required object ${}_{gm}S$;

3. sets the sequence of the changing actions from the above determined classes (subsection 4.4.1);

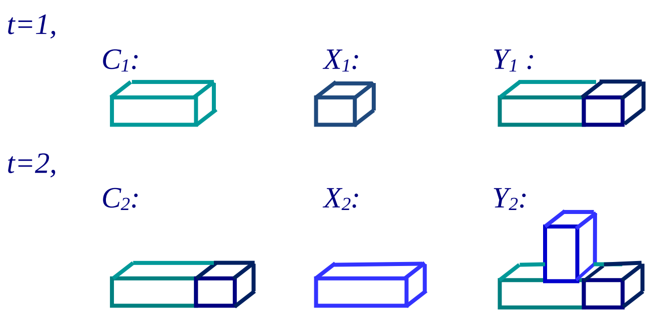
4. performs these actions (this process corresponds to the calculations of the resultant object ${}_{gm}S$ characteristics $\{\Omega, \Gamma, \Sigma\}$).

Relations between the basic geometrical structures are presented in the form of procedures from the given classes. So, it became possible to choose a class of procedures for the synthesis of a required output object from the given input object. The dynamics of states of input object ${}_{gm}S^{\ell}$ is described with the help of dynamic system ${}_{o}(\bar{\rho}, \bar{\varphi})^{\ell}$.

An example of the dynamics of the object states in the synthesis process is given in the following Scheme 4.1. Both the structure and form of GO are continuously changed. The dynamics of the process (two steps) is described by the following reactions:

$$\{{}_{o}\rho_1 : C_1 \times X_1 \to Y_1 \;\&\, 1 \in T\}^{\ell}, \tag{4.18}$$

$$\{{}_{o}\rho_2 : C_2 \times X_2 \to Y_2 \;\&\, 2 \in T\}^{\ell}.$$



Scheme 4.1. Two states of GO synthesis process

That is, at the moment of time $t=1$ the basic GO is the conditional line (C_1), the input (X_1) is the conditional point and the output (Y_1) is a composition of these objects. Reaction ρ_1 corresponds to the coordinator action of type 2 (subsection 4.4.1). At the next moment $t=2$, the basic element $C_2=Y_1$ is the conditional line constructed in the previous moment $t=1$, input object (X_2) is another conditional line, and the output object (Y_2) is a composition of two conditional lines which is the initial state of a conditional surface. Reaction ρ_2 corresponds to the coordinator action of type 3 (subsection 4.4.1).

The set of primitives, which are used in the synthesis process is presented in Table 4.4.

Table 4.4. Set of primitives of different level systems

Levels	S^0	S^1	S^2	S^3
Primitives	Points	parts of lines, curves	circles, cubes, triangles, rings	spheres, cubes, cones, cylinders, pyramids (3D simplex)

For instance, coordinator actions $R_0{:}C{\times}X{\to}Y$ (where $C=\left\{S_1^0,\ldots,S_n^0\right\}$, $X=\left\{\sigma^1,\omega\right\}$, $Y=S^1$) make it possible to synthesise the part of line S^1 from frame C of points (cubes $S_1^0,\ldots,S_n^0$), when the structure type and additional information are received from designer (Fig. 4.6). All the parameters $\left\{\Omega,\Gamma,\Sigma\right\}$ of the part of line S^1 are defined in this process as follows:

- GO being synthesised S^ℓ is the conditional line S^1;
- structure type $\sigma^\ell \leftrightarrow$ line; structural connections ${}_\sigma\gamma^\ell$ – sides of cubes;
- structural elements are conditional point $\omega^{\ell-1}=S_1^0$ – cubes;
- metrical characteristic of the elements $\mu_1^0 = w$;
- another metrical characteristic of the line $l = \mu_1^1 = n\cdot \mu_1^0$, is defined by the number n of structural elements and their sizes $w = \mu_1^0$;
- constructive dimension of the line in its aggregated ω^ℓ form is δ_ω^ℓ = 0 0 1 0; constructive dimension of the line in its structural σ^ℓ form is δ_σ^ℓ = 0 0 0 n, etc.

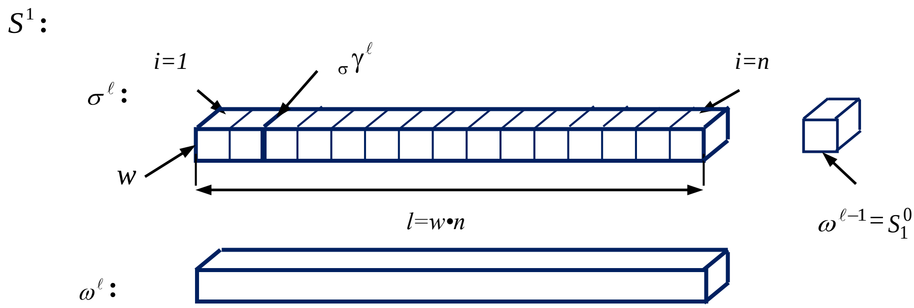


Fig. 4.6. Geometric object (conditional line) S^1 synthesis from the conditional points (cubes)

The result of the computer synthesis of the exemplary 3D geometric objects realized using HS design method is presented in Fig. 4.7.

a) b)

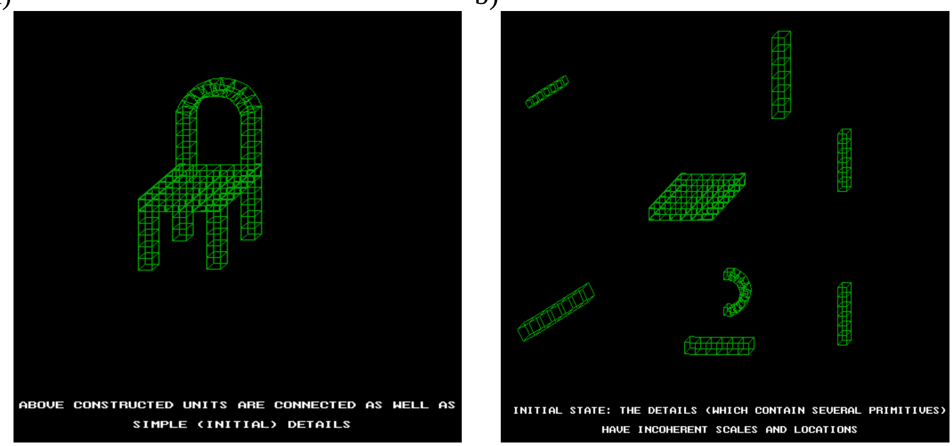


Fig. 4.7. Computer screenshots with: a) 3D object (chair), synthesized from: b) its structural elements

The constructive dimension of 3D object – the chair – in its initial state (Fig. 4,7b) is:

$$\delta_{\omega}^{\ell} = 0\_1\_7\_0, \tag{4.19}$$

and for its structural connections ${}_{\sigma}\gamma^{\ell}$ is:

$$\delta_{\lambda_\sigma}^{\ell} = 0\_0\_0\_0, \tag{4.20}$$

i.e. there are *1* conditional surface and *7* lines which are not connected.

In the final state of the chair synthesis (Fig. 4.7a) there are *1* conditional surface and *4* lines connected by conditional *6* points, and constructive dimensions are as follows:

$$\delta_{\omega}^{\ell} = 0\_1\_4\_0 \text{ and } \delta_{\gamma_\sigma}^{\ell} = 0\_0\_0\_6. \tag{4.21}$$

One conditional line is constructed form three initial ones. Therefore the *7* turns into *4* in the second position of δ_{ω}^{ℓ} numeric code which is the position of lines in the constructive dimension δ^{ℓ} code. All constructive elements of the chair in its final state are connected by conditional points. Figs. 4.7a and 4.7b are computer screenshots.

4.4.3. Motion design and deformation as analysis tasks

The tasks of mechatronic object (MO) motion design and deformations - *analysis tasks* - are defined as the coordination tasks of hierarchical system standard block S^{ℓ} (*aed*).

Object ${}_{m}S^{\ell}$ *movement* in its environment ${}_{\varepsilon}S^{\ell}$ is called a change of its interconnections ${}_{\omega}\gamma^{\ell}$ with the elements S_i^{ℓ} ($i \in I^{\ell+1}$) of environment system ${}_{\varepsilon}S^{\ell}$.

The change of interconnections ${}_{\sigma}\gamma^{\ell}$ of the elements of MO structure σ^{ℓ} is identified as *deformation*.

Adding of negative and positive units μ^{ℓ} (elements) to the connections ${}_{\sigma}\gamma^{\ell}$ with structure σ^{ℓ} remaining it in the same structures class causes the effect of structure σ^{ℓ} deformation. The change of ${}_{\sigma}\gamma^{\ell}$ causes the external interconnections ${}_{\omega}\gamma^{\ell}$ changes which are equivalent to object ${}_{o}S^{\ell}$ movement in its environment. The accordingly organized deformations - for instance running waves of deformations [55] - correspond to the classes of ways of object motion coordination.

The objects mass measure is defined as the metrical characteristics $\tilde{\mu}^{\ell} \in \Omega^{\ell}$ of system ${}_{m}S^{\ell}$ state $c \in C^{\ell}$, its inputs X^{ℓ} and outputs Y^{ℓ}.

Neutral element ${}^{0}\tilde{\mu}^{\ell} = \left({}^{-}\tilde{\mu}^{\ell-1}, {}^{0}\tilde{\mu}^{\ell-1}, {}^{+}\tilde{\mu}^{\ell-1}\right) = \tilde{\mu}^{\ell-1}$ is the length, square or volume measure – depending on which object state (i.e. which constructive dimension $\xi_{\omega,\sigma}^{\ell}$ and connection defect $\delta_{\omega,\sigma}^{\ell}$) is considered as ${}_{\tau}C^{\ell}$ or ${}_{\tau\omega}\gamma^{\ell}$, $\tau \in_{\psi} L$.

Movement measures (velocity, impulse, etc.) are presented as the measures of state changing of dynamic systems during the time unit ${}_{\tau}\eta_{T}^{\ell}$, ${}_{\tau}\eta_{T}^{\ell} \in {}_{\tau}T^{\ell} = \{{}_{\psi}T^{\ell}, {}_{\chi}T^{\ell}, {}_{\varphi}T^{\ell}, {}_{\lambda}T^{\ell}\}$ in spaces M^{ℓ}, Ξ^{ℓ}, Δ^{ℓ}. Movement measures have their directions in accordance with environment ${}_{\varepsilon}S^{\ell}$ element addresses with regard to which the movement is considered. The general law of movement and deformation was also presented in form of *aed* .

Geometric and physical values were defined in state spaces M^{ℓ}, Ξ^{ℓ}, Δ^{ℓ}, T^{ℓ}. All calculations are carried out in these spaces with the help of numeric positional system L^{S}. The ways of motion design and deformation calculations depend on the units $\tilde{\mu}^{\ell}$, $\tilde{\xi}^{\ell}$, $\tilde{\delta}^{\ell}$ of both structure σ^{ℓ} and aggregated dynamic realization ω^{ℓ}. The ways correspond to coordination strategies ${}_{\tau}\lambda_{o}^{\ell} \in \Lambda_{o}^{\ell} = \left\{{}_{\psi}\lambda_{o}^{\ell}, {}_{\chi}\lambda_{o}^{\ell}, {}_{\varphi}\lambda_{o}^{\ell}, {}_{\lambda}\lambda_{o}^{\ell}\right\}$ for different levels of information uncertainty of coordinator.

It is possible to define different kinds of movements and deformations by introducing corresponding conditions. For instance, the following relation corresponds to MO physical movement (object's coordinates changing) at time interval $\overline{T}_{tt'}^{\ell}$:

$$
{}_{\sigma}\gamma^{\ell}\Big|\overline{T}_{tt'}^{\ell} = const, \tag{4.22}
$$

where structural connections ${}_{\sigma}\gamma^{\ell}$ are not changed (at this time period) and in this way structure σ^{ℓ} remains in its current state. This movement is conditioned by environment process ${}_{\pi\varepsilon}S^{\ell}$, which was presented in the following way:

$$
\begin{aligned}
{}_{\pi\varepsilon}(\overline{\rho},\overline{\varphi})^{\ell}:\ \ & {}_{\pi\varepsilon}\overline{\rho}^{\ell} = \left\{{}_{\pi\varepsilon}\rho_{t}^{\ell} : {}_{\varepsilon}X_{t}^{\ell} \times {}_{\varepsilon}C_{t}^{\ell} \rightarrow {}_{\varepsilon}Y_{t}^{\ell}\ \&\, t \in T^{\ell}\right\}, \\
& {}_{\pi\varepsilon}\overline{\varphi}^{\ell} = \left\{{}_{\pi\varepsilon}\varphi_{tt'}^{\ell} : {}_{\varepsilon}X_{t}^{\ell} \times {}_{\varepsilon}C_{tt'}^{\ell} \rightarrow {}_{\varepsilon}X_{t'}^{\ell}\ \&\, t, t' \in T^{\ell}\ \&\, t' > t\right\}, \\
& {}_{\varepsilon}X^{\ell} = {}_{\tau\varepsilon}X^{\ell} \times Y^{\ell}, \qquad {}_{\varepsilon}Y^{\ell} = {}_{\tau\varepsilon}Y^{\ell} \times X^{\ell}.
\end{aligned} \tag{4.23}
$$

The following relation corresponds to the bio-mechanical type of movements:

$$
({}_{\tau\varepsilon}X^{\ell} \times {}_{\tau\varepsilon}Y^{\ell})\Big|\overline{T}_{tt'}^{\ell} = {}_{\tau\omega}\gamma^{\ell}\Big|\overline{T}_{tt'}^{\ell} = const, \tag{4.24}
$$

in other words the connections ${}_{\omega}\gamma^{\ell}$ at time interval $\left|\overline{T}_{tt'}^{\ell}\right.$ depend on the structural interconnection changes ${}_{\sigma}\gamma^{\ell}\left|\overline{T}_{tt'}^{\ell}\right.$ only. This movement is conditioned by process ${}_{o\pi}S^{\ell}$, which was described in form (3.3) as follows:

$$
{}_{o\pi}\left(\overline{\rho},\overline{\varphi}\right)^{\ell} : {}_{o\pi}\overline{\rho}^{\ell} = \left\{{}_{o\pi}\rho_{t}^{\ell} : X_{t}^{\ell} \times C_{t}^{\ell} \rightarrow Y_{t}^{\ell} \ \& t \in T^{\ell}\right\}, \tag{4.25}
$$
$$
{}_{o\pi}\overline{\varphi}^{\ell} = \left\{{}_{o\pi}\varphi_{tt'}^{\ell} : X_{t}^{\ell} \times C_{tt'}^{\ell} \rightarrow X_{t'}^{\ell} \ \& t,t' \in T^{\ell} \ \& t' > t\right\}.
$$

The other classes of movements occupy the positions between the above mentioned ones and presented in the basis of S^{ℓ} symbol construction.

The examples of bio-mechanical movements coordination – analysis tasks – are given below for snake and caterpillar like robots motion design, as well as for other biologically inspired robots [56,57] and human motion design (Chapter 7).

The following *analysis algorithm* is defined and realised in these and other tasks of the work. The coordinator performs the following steps:

1. synthesis of geometrical structure of mechatronic object being designed;
2. definition of the way of interconnection changing of object's elements;
3. realization of the interconnections changing process at the definite time interval by one of the coordinator operations described in subsection 4.4.1.
4. checking and evaluation of MO position (address) in its environment, or MO external representation improved, i.e. geometric form of the mechatronic object being designed.

The analysis tasks performing for caterpillar and snake like robots bio-mechanical movements coordination (motion design) by the suggested method are partially described in [55] and presented below. The tasks are performed in accordance with the analysis algorithm presented above.

For the case of caterpillar-like robot at the *first step*, the geometric construction σ_{τ}^{ℓ} of the caterpillar ${}_{gm}S^{\ell}$ which is the conditional line, and structural representations $\sigma^{\ell+\tau}$ $\tau \in L^{s}, \tau \neq 0$ for the environment ${}_{\varepsilon}S^{\ell} \leftrightarrow S^{\ell+\tau}$, which is the pathway of robot are synthesized. Structural interconnections ${}_{\sigma}\gamma_{\tau}^{\ell}$ are defined as the common points and angles between the structural elements (primitives) of caterpillar robot.

At the *second step*, definition of the way of the structural interconnections ${}_{\sigma}\gamma^{\ell}$ changing to perform the required robot motion ${}_{o\pi}S^{\ell}$ is its environment ${}_{\varepsilon}S^{\ell}$ was done. The caterpillar motion mode was selected as linear with constant velocity. The direction *M* of robot motion in its environment was also defined. Another motion modes can be defined and selected as well.

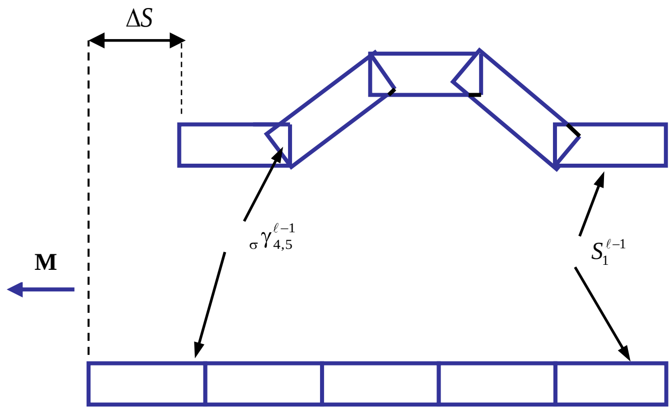


Fig. 4.8. Caterpillar robot motion, where: ΔS – robot body displacement during one motion cycle; ${}_{\sigma}\gamma_{4,5}^{\ell}$ – structural connections of 4 and 5 elements of the body, **M** – direction of the motion

At the *third step*, the consecutive change of structural interconnections (angles) in the way defined by coordinator at the second algorithm step leads to the whole robot movements in its environment ${}_{\varepsilon}S^{\ell} \leftrightarrow S^{\ell+\tau}$ (Fig. 4.8). The change of angles ${}_{\sigma}\gamma_{i,\tau}^{\ell}$, $i,\tau \in L^{s}$ from their maximum value to *0* during one motion circle causes displacement equal to ΔS of the robot body.

At the *fourth step*, the checking and evaluation of the changes of the both position and form, i.e. external representation of caterpillar robot ${}_{gm}S^{\ell}$ in its environent ${}_{\varepsilon}S^{\ell}$ are executed in parallel with the actions of the third algorithm step.

a) b)

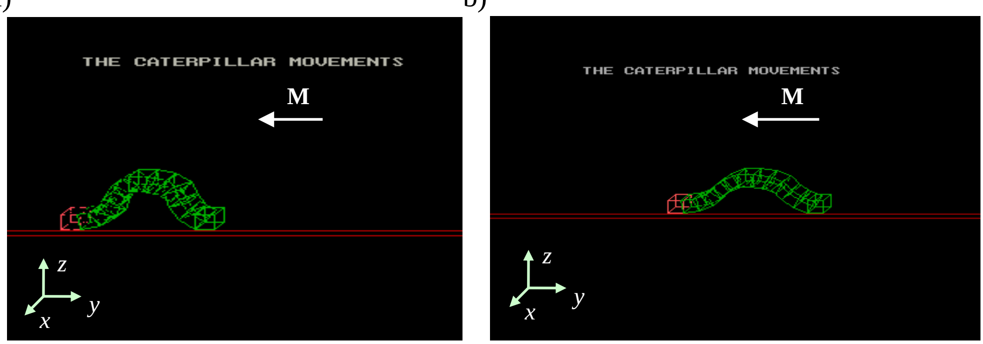


Fig. 4.9. Two states of motion design of caterpillar like robot movements, M – motion direction

The result of computer performing of motion design task for the caterpillar robot is presented in Figure 4.9. The figure is a computer screen-short obtained.

Motion design of *snake like robot* is the another exemplary analysis task. The *mechanism* of snake like robot motion is a combined mechanism of transverse and lengthwise waves of structural interconnection change spreading in the snake body. For lengthwise waves of deformation the interconnections ${}_{\sigma}\gamma^{\ell}_{i,\tau}$ volume $\mu^3(\gamma_{i\tau})(t)$ changes for relative elements i, τ and continuous interconnections transfer (i=i+1 or i=i-1) depending on the direction of the mass being transferred take place (Fig. 4.10). The strength of the structural connections is graphically illustrated by their volumes. Robot body displacement during one motion cycle is equal to ΔS. Motion design of *snake like robot* is performed according to the analysis algorithm given above.

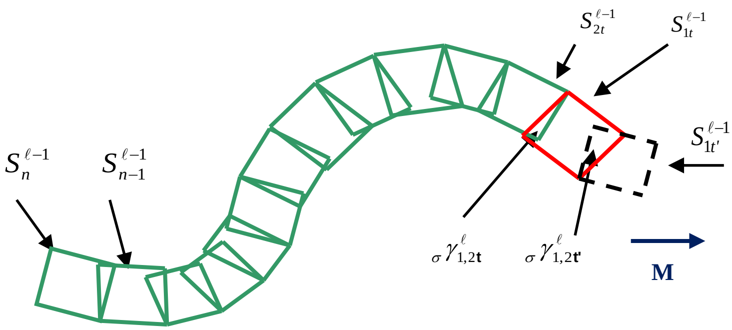


Fig. 4.10. Construction of a body of snake-like robot being in motion, where $S^{\ell-1}_{1t'}$ is the new position of the first element (head) in the moment of time *t'*, *M* – direction of the motion

At the first step of the analysis task, geometric construction ${}_{m}\sigma^{\ell}$ of biomechatronic object ${}_{m}S^{\ell}$ – snake like robot – is synthesized in the form of conditional line built from conditional points – cubes (Fig. 4.10). Interconnections ${}_{\sigma}\gamma^{\ell}_{i,j}$ of conditional points *i* and *j* are their common edges and angles. The snake robot construction ${}_{m}\sigma^{\ell}_{t}$ at initial *t* moment is in the state presented in Figure 4.10.

At the second and the third algorithm steps, definition of the way of the snake robot structural interconnections ${}_{\sigma}\gamma^{\ell}$ changing to perform the required robot motion ${}_{o\pi}S^{\ell}$ is its environment ${}_{\varepsilon}S^{\ell}$ was done. The direction *M* of robot motion in its environment was also defined. After that, the coordination of the snake motion process was realized in the following way. The character of snake motion is based on the principle that each part of the snake body goes the same way in the motion process as all the others. To initiate the motion process the head of the snake $S_1^{\ell-1}$ makes one step – moves forward – obtaining in this way its new position $S_{1t'}^{\ell-1}$. In this case, connection ${}_{\sigma}\gamma_{1,2}^{\ell}$ between the first structural element $S_1^{\ell-1}$ (the head) and the second structural element $S_2^{\ell-1}$ of snake body becomes weaker. To remove the weakness of the connection, the second element is built on to the head (reconstructed in a new position) taking the place of the previous head position. In this way the interconnection is changed by robot coordinator S_0^{ℓ} from state ${}_{\sigma}\gamma_{1,2\mathbf{t}}^{\ell}$ to the new state ${}_{\sigma}\gamma_{1,2\mathbf{t}'}^{\ell}$. The third element $S_3^{\ell-1}$ is built on to the second one $S_2^{\ell-1}$ taking its previous position, the fourth is built on to the third one, etc. One step of the process was finished when the last *n* element $S_n^{\ell-1}$ was built to the last but one *n-1* element $S_{n-1}^{\ell-1}$ being in its new position.

Formally, the coordination task of robot motion is presented as reaction ρ_0^{ℓ} of coordinator, see (4.5), and as function $\varphi_{tt'}^{\ell}$ of states transition for the system, i.e. robot:

$$\rho_0^{\ell} : C_0^{\ell} \times \sigma_t^{\ell} \to \sigma_{t'}^{\ell} \,\&\, \sigma_{t'}^{\ell} = Y_0, \sigma_t^{\ell} = X_0, \tag{4.26}$$

$$\varphi_{tt'}^{\ell} : \sigma_t^{\ell} \times {}_{\sigma}\gamma_{tt'}^{\ell} \to \sigma_{t'}^{\ell},$$

where σ_t^{ℓ} and $\sigma_{t'}^{\ell}$ are the states of the snake-like robot structures at moments of time *t* and *t'* respectively, *t'>t;* ${}_{\sigma}\gamma_{tt'}^{\ell}$ is the change of structural connections during the period of time *tt'*; C_0^{ℓ} is coordinator state (the state of control program).

At the *fourth step,* the evaluation of the position and form changes – external representation of snake robot ${}_{gm}S^{\ell}$ in its environment ${}_{\varepsilon}S^{\ell}$ – is executed in parallel with the actions of the third algorithm step.

a) b)

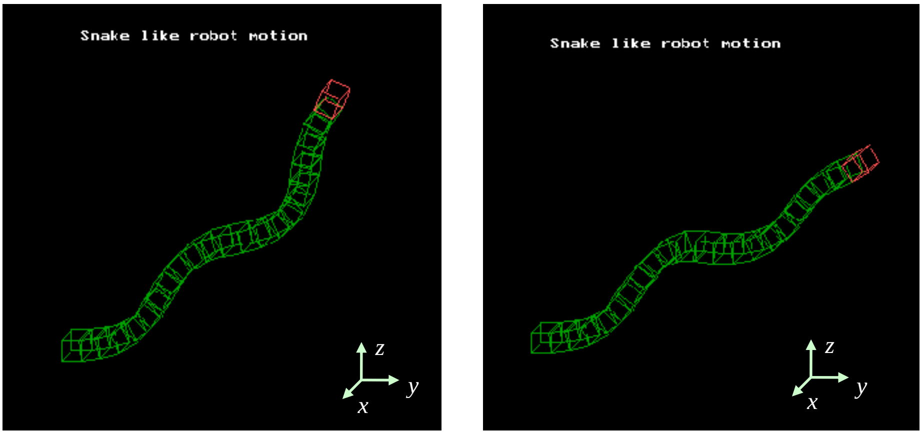


Fig. 4.11. Computer screen shorts obtained in the process of the snake-like robot motion design

In the program realization of snake robot motion design, the coordinator S_0^ℓ, which is realized as the main control program, controls the interconnections ${}_\sigma\gamma_{i,j}^\ell$ change of the body elements $\bar{S}^{\ell-1}$ and reconstructs the elements consecutively in their new positions. The wave of the interconnections change runs from the head $S_1^{\ell-1}$ to the last structural element $S_n^{\ell-1}$. This process of ${}_\sigma\gamma_{i,j}^\ell$ change causes the shake robot motion in the opposite direction $\boldsymbol{M}$. Two states of the body construction of the snake robot being in motion are given in Fig. 4.11.

So, in the case of motion design of caterpillar and snake like robots we deal with two kinds of geometric design (analysis) processes – deformation (in the lower level structure of the biomechatronic object) and motion, i.e. its dynamics in the structure of the environment.

The results obtained in performing the given tasks have a general character and may be used in an abstract motion design of hierarchical systems. The conducted researches show that the HS design method proposed in this book can be used both in the conceptual and detailed design tasks of mechatronic objects motion, i.e. coordination of biomechanical movements, and deformation analysis of the objects constructed.

Preliminary conclusions

The HS design method of mechatronic objects (MO) has been described in this Chapter 4.

First, the steps of the method realization were defined.

After that, in accordance with the method steps, the basic geometric elements of MO were presented in the hierarchical form. The results of this presentation are conditional point, line, surface and 3D object.

Later, the geometrical and numeric characteristics of MO were introduced. The main numeric characteristics, i.e. constructive dimension and connections defect, are presented in the numeric positional code.

Next, coordinator procedures of mechatronic systems design were defined. Algorithms of hierarchical objects synthesis and analysis were presented.

Finally, the exemplary computer realizations of the method for solving the basic design tasks were described: synthesis of 3D objects and analysis – design of caterpillar and snake like robots movements. Program modules have been written in Pascal language and work in the frames of standard operation systems. The role of coordinator is performed by the main control program unit, which realizes the functions of coordinator selection stratum, i.e. solves synthesis and analysis design tasks.

The computer realization shows, that the method has the main advantages of the wide spread technologies of geometric design. At the same time hierarchical representation of geometric elements of MO is free from certain drawbacks of the known methods.

5. CONCEPTUAL DESIGN OF BIOMECHATRONIC SURGICAL ROBOT SYSTEM

The Chapter presents the application of Hierarchical System (HS) technology in the task of conceptual design of biomechatronic Surgical Robot System (SRS). This task is topical for the systems of Computer Aided Design (CAD) and processes of conceptual mechatronic design in particular [1,58]. In design process of biomechatronic systems we deal with objects which contain connected mechanical, electromechanical, biological, electronic, computer and human-computer subsystems. It is important to define the common conceptual model which will describe all the above mentioned systems of biomechatronic SRS being designed in common formal basis.

The model must also be coordinated with numerical and geometrical systems, i.e. the traditional forms of information representation in mechatronics. The theoretical basis of design process in agreement with these requirements must be a hierarchical construction connecting any level unit with its lower and higher levels. Mathematical and cybernetic theories based on the set theory are incoherent with the above design requirements since the set theory describes one-level outlook.

So, the coordination technology of Hierarchical System with its standard block *aed* described in Chapters 3 and 4 is applied in this Chapter as a theoretical basis for the performing of conceptual design task of biomechtronic SR system. According to the definitions – given above in the Introduction and Chapter 1 – Conceptual Design (CD) is recognized in this book as a process of creation of the systemic model of the object being designed on the early phase of its life cycle which is before the Detailed design (DD) phase of object's concrete mathematical model creation and numeric calculations realization. In the frames of the HS method proposed, to create conceptual model for design of biomechatronic object, e.g. Surgical Robot System (SRS) in our case, means to define SR system structure; its dynamic representation as the unit in its environment; SRS environment, its process and SRS-Environment interactions; SR system coordinator and its design and control processes; processes executed by SRS subsystems and general SRS process.

Therefore, the conceptual model of biomechatronic surgical robot system (SRS) presented in *aed* formal basis of HS is given first in this Chapter. The exemplary tasks performed by SRS subsystems are described after that in frames of HS technology as well.

5.1. Conceptual formal model of surgical robot system

SR systems, such as commercial ROBODOC system [59] (Integrated Surgical Systems, CA, USA), have been widely used recently in TKA (Total Knee Arthoplasty) surgery [60]. Total knee arthroplasty is one of the major type of surgery for people with severe knee damage to relieve the pain and disability of cartilage injury, osteoarthritis, rheumatoid arthritis, or psoriatic arthritis [61, 62]. Due to the increasing aging population recently, the TKA surgery has been grown rapidly in the world. The main surgical procedure of the TKA is removing damaged cartilage and bones from the surface of the knee joint and replacing them with femoral and tibia components of TKA implant (Fig. 5.4). In order for the surgery to be successful and the longevity of the implant to be achieved, the accurate bone cutting and implant alignment along the preoperative planning are required. Conventional surgical planning is performed basing on 2D plane x-rays and bone cutting is performed manually using electric saw, which strongly relies on a surgeon's experience. Even for a skilled surgeon, the implant alignment with respect to the bone in frontal plane is a very complicated task. So it is necessary to achieve excellent implant alignment and bone cutting by utilizing Computer Tomography (CT) based 3D planning and robot assisted accurate bone cutting. The minimal invasive surgery (MIS) in the knee arthroplasty has been increased since MIS can improve the surgical outcomes such as the recovery time and hospital stay by reducing the incision in surgery.

Recently, commercially available computer-aided surgical robot systems have been introduced to enable surgeons to improve the accuracy of cutting and alignment in knee and hip arthroplasty. The surgical robot systems could be classified into semi-active type and active type. The semi-active robot systems such as the MAKOPLASTY® system (MAKO Surgical Co, Fort Luderdale, FL, USA [63]) or ACROBOT Sculptor (The Acrobot Co., Elstree, UK [64]) provide a surgeon active control over the robot by hands.

These robot systems also protect the cutting tool from moving out of the planned cutting space by actively blocking the robot arm attached to the cutting tool when the surgeon executes the bone cutting process. The active robot systems such as ROBODOC® system (Curexo Tech. Co., Anyang, Korea [65]) and CASPAR (URS Ortho GmbH, Rastatt, Germany [60]) move the cutting tool autonomously to cut the patient's bone basing on the results of the pre-operative planning.

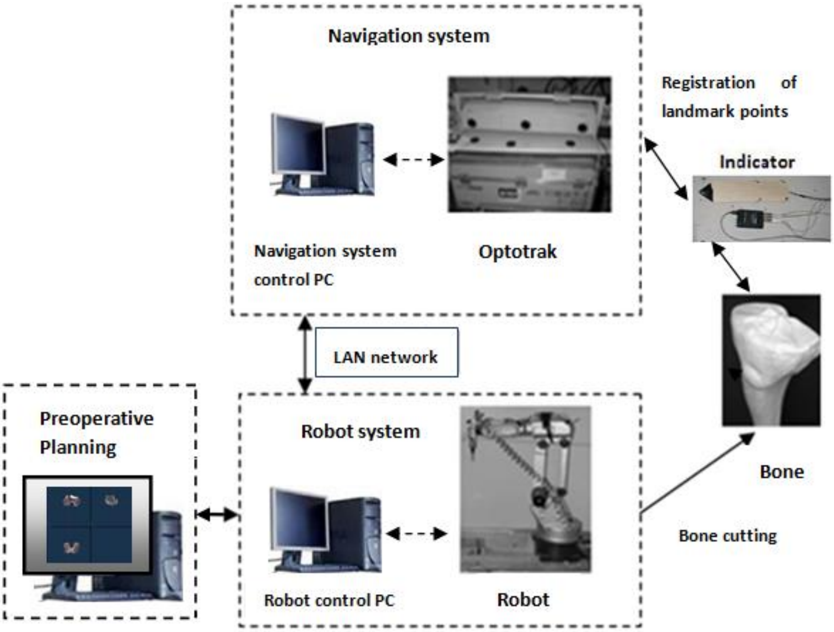


Fig. 5.1. Structure of laboratory SRS for TKA

In this Chapter the main attention is paid to laboratory-level SRS with industrial robot and navigation system for TKA (Fig. 5.1) developed in BioMech Lab., School of Engineering, Kyung Hee University, South Korea [58, 61]. Conceptual systemic model of the SR system is presented in *aed* formal form as follows:

$$S^{\ell} \leftrightarrow \{\omega, S_0, \sigma\}^{\ell} \tag{5.1}$$

where ω^{ℓ} is an aggregated dynamic representation of SR system S^{ℓ}, σ^{ℓ} is the SR system structure, S_0^{ℓ} is SRS coordinator, i.e. design and control system, ℓ is the index of level.

The *SR system structure* σ^{ℓ} contains the set of sub-systems $\overline{\omega}^{\ell-1}$ and their structural connections ${}_{\sigma}\gamma^{\ell}$. Thus, according to *aed* model described in Chapter 3, SRS structure is presented in the following form:

$$\sigma^{\ell} = \{S_0^{\ell}, \{\overline{\omega}^{\ell-1}, {}_{\sigma}\gamma^{\ell}\}\}, \tag{5.2}$$

where the SRS subsystems presented in aggregated dynamic form $\overline{\omega}^{\ell-1}$ are

$\omega_1^{\ell-1}$: robot system (RS);

$\omega_2^{\ell-1}$: pre-operative planning system (PP);

$\omega_3^{\ell-1}$: navigation system (NS);

$\omega_4^{\ell-1}$: computer control system (CCS).

CCS is presented in the form of SRS coordinator (Fig. 5.7) and contains control PC of robot, control units of pre-operative planning and navigation subsystems, and communication program developed to integrate the SRS subsystems. Therefore, CCS $\omega_4^{\ell-1}$ partially plays the role of SRS coordinator S_0^{ℓ} which performs design and control functions of SRS.

In their turn, each subsystem has its own structural elements – lower level $\ell-1$ subsystems. For the robot mechatronic subsystem RS $\omega_1^{\ell-1}$ they are manipulator $\omega_{1,1}^{\ell-2}$ (mechanical), servomotors $\omega_{1,2}^{\ell-2}$ (electromechanical), cutting machine $\omega_{1,3}^{\ell-2}$ (pneumatic) and its own control system $\omega_{1,4}^{\ell-2}$ (computer). All the subsystems are connected by their structural connections ${}_{\sigma}\gamma^{\ell-2}$. For instance, cutting machine $\omega_{1,3}^{\ell-2}$ and manipulator $\omega_{1,1}^{\ell-2}$ are connected by end effector ${}_{\sigma}\gamma_{1,3}^{\ell-2}$ of robot. By analogy, the higher level subsystems $\overline{\omega}^{\ell-1}$ are connected by their common parts – structural connections ${}_{\sigma}\gamma^{\ell-1}$ – that are the elements of lower levels. For instance, navigation system $\omega_3^{\ell-1}$ and robot $\omega_1^{\ell-1}$ are connected by their common element – communication program of computer control system ${}_{\sigma}\gamma_{1,3}^{\ell-1} = \omega_{1,4}^{\ell-2} = \omega_{3,4}^{\ell-2}$, where $\omega_{1,4}^{\ell-2}$ is dynamic representation of the control program being the subsystem of robot $\omega_1^{\ell-1}$, and $\omega_{3,4}^{\ell-2}$ the one of the control program being the subsystem of the navigation system $\omega_3^{\ell-1}$.

Aggregated dynamic realizations $\overline{\omega}^{\ell-1}$, i.e. dynamic models ${}_i(\overline{\rho},\overline{\varphi})^{\ell-1}$ of SRS subsystems $\overline{S}^{\ell-1}$, are formed after the definition of subsystems' inputs-outputs concerning each particular sub-process they execute. Thus, for the navigation system $\omega_3^{\ell-1}$ concerning its registration process of bone and robot, the input

$X_3^{\ell-1}$ is optic signal received by Optotrak 3020 cameras system (Northern Digital Inc., Canada) and the output $Y_3^{\ell-1}$ is the robot instrument and real bone coordinates written on control PC. State $C_3^{\ell-1}$ in this case is the stage of registration process completeness. This process is of informational nature. As for the robot subsystem $\omega_1^{\ell-1}$, concerning its mechanical process of cutting instrument motion, the aggregated realization $\omega_{1,1}^{\ell-2}$ presented in $(\overline{\rho},\overline{\varphi})^{\ell}$ form at the conceptual design (CD) phase (Fig. 1.2, Chapter 1) can be transformed to differential equations [58] of inverse kinematics at the detailed design (DD) phase:

$$\dot{q} = J^{+}(q)\dot{x} \tag{5.3}$$

Equation (5.3) connects robot joints velocities $\dot{q}$ as the output parameter $Y_1^{\ell-1}$ with velocities $\dot{x}$ of robot cutting instrument as input $X_1^{\ell-1}$, where J^{+} is the pseudo inverse of the Jacobian matrix.

Object ${}_{o}S^{\ell}$ (SR system), its environment ${}_{\varepsilon}S^{\ell}$, their processes ${}_{o\pi}S^{\ell}$ and ${}_{\varepsilon\pi}S^{\ell}$ (TKA and SRS control) and coordinator S_0^{ℓ} (design and control system) create a general biomechatronic SR system for TKA.

Environment ${}_{\varepsilon}S^{\ell}$ of the SR system has its own structure and contains:

ω_1^{ℓ} : bone (biological system),

ω_2^{ℓ} : surgeon, who communicates with SRS via video information, registration and cutting motion planning system (human-computer system),

ω_3^{ℓ} : other biomechatronic systems being in interaction with SRS (e.g. computer tomography (CT) system, which supply PP subsystem of SRS with the CT images of bone),

ω_4^{ℓ} : implant producing system (bioengineering system),

ω_5^{ℓ} : higher level coordinator (higher level design and control system).

Thus the control interactions between surgeon (operator) ω_2^{ℓ} and SR system are realized through obtaining video information of bone, checking the cutting path and robot cutting motion planning. In their turn, the interactions between robot cutting instrument and bone ω_1^{ℓ} are the executive interactions between SR system and its environment.

The immediate input X^{ℓ} for the SR system (which is at the same time the output ${}_{\varepsilon}Y^{\ell} = X^{\ell}$ of the environment of SR system) are control actions produced by surgeon and the inputs generated by other environment systems, e.g. inserted implant presented by ω_4^{ℓ}. The output Y^{ℓ} of the SR system is the cutting bone. The output of the SR system S^{ℓ} is at the same time the input ${}_{\varepsilon}X^{\ell} = Y^{\ell}$ of the environment. According to the relations established for the elements of $(\overline{\rho},\overline{\varphi})$ model – see Table 3.1, Chapter 3 – the states C_i^{ℓ} of SR system S^{ℓ} are the inputs of TKA surgical process.

Dynamic representation ω^{ℓ} of SR system and its subsystems are constructed in $(\overline{\rho},\overline{\varphi})$ form (3.2) by the inputs X^{ℓ}, states C^{ℓ} and outputs Y^{ℓ} mentioned above. For example, in the case of description of robot end effector motion, the dynamic representation which is given in $(\overline{\rho},\overline{\varphi})$ form at CD phase can be transformed at DD phase to the states space equations as follows:

$$\dot{x} = Ax + Bu \tag{5.4}$$

$$y = Cx\,.$$

First state equation in system (5.4) corresponds to the state transition function $\overline{\varphi}$ in (3.2) and the second output equation corresponds to the reaction $\overline{\rho}$. Vectors *x*, *y*, *u* and matrices *A*, *B*, *C* must be predefined. In the case of robot end effecter motion the elements of states vector $x=[x_1\ x_2\ x_3]^T$ are the displacement x_1, velocity x_2 and acceleration x_3.

SR TKA system process ${}_{o\pi}S^{\ell}$ is part of higher-level process ${}_{\pi}S^{\ell+1}$ in SRS environment ${}_{\varepsilon}S^{\ell}$ which contains biomechatronic system of higher level – general design&control system including surgeon, person being operated, other biomechatronic systems being in interaction with SRS (e.g. CT system and implant production system).

This process contains:

${}_{\pi}S_1^{\ell}$: VM of bone and implant graphic image forming (by PP system),

${}_{\pi}S_2^{\ell}$: real bone image and robot instrument location registration (by NS),

${}_{\pi}S_3^{\ell}$: VM matching to real bone model – bone image,

${}_{\pi}S_4^{\ell}$: cutting path forming in PP and transforming to coordinates system of RS,

${}_{\pi}S_5^{\ell}$: allocation of robot instrument in the registration points by surgeon using control haptic device, and definition of robot joints angels control law by solving inverse kinematics problem,

${}_{\pi}S_6^{\ell}$: sending control signal to AS3 robot system,

${}_{\pi}S_7^{\ell}$: AS3 robot motion and end effector (pneumatic milling instrument) displacement,

${}_{\pi}S_8^{\ell}$: bone cutting (or milling) and implant inserting.

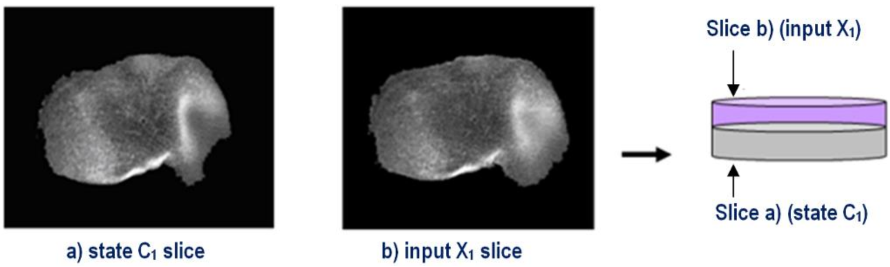


Fig. 5.2. Stacking process of the bone model creation from CT bone images a) and b), i.e. slices.

For instance, ${}_{\pi}S_1^{\ell}$ process of the bone virtual model – bone image – creation is described at conceptual design (CD) phase as follows. The model of a bone biological subsystem of general biomechatronic SR system is constructed in virtual space from computer tomography (CT) images, i.e. bone slices (Fig. 5.2). The process of the bone model creation is called *stacking*.

The stacking process ${}_{\pi}S_1^{\ell}$ (Fig. 5.2) is formally described at the phase of CD in the form of reaction of dynamic system $(\rho,\varphi)^{\ell}$:

$$\{\rho_1 : C_1 \times X_1 \rightarrow Y_1 \,\&\, 1 \in T\}^{\ell} \tag{5.5}$$

where initial slice (or composition of previously connected slices) is recognized as state C_1, at step *1*, slice to be added to the initial one is recognized as input X_1, and the composition of both the connected initial and input slices is recognized as output Y_1. Output Y_1 becomes state C_2 at the next step of stacking process.

Virtual bone structure is presented at CD phase according to *aed* model in the form:

$$\sigma^{\ell} = \{\overline{\varpi}^{\ell-1}, {}_{\sigma}\gamma^{\ell}\} \tag{5.6}$$

where $\overline{\varpi}^{\ell-1}$ is a set of bone CT slices, and ${}_{\sigma}\gamma^{\ell}$ are their structural interconnections, i.e. common surface of neighboring slices.

The general synthesis process of the virtual bone construction is formally described as coordinator task according to the HS design method step 4 (See Chapter 4) in form of (4.4) as follows:

$$S_0^{\ell} : \{\overline{\varpi}^{\ell-1}, {}_{\sigma}\gamma^{\ell}\} \rightarrow \sigma^{\ell} \tag{5.7}$$

a) b)

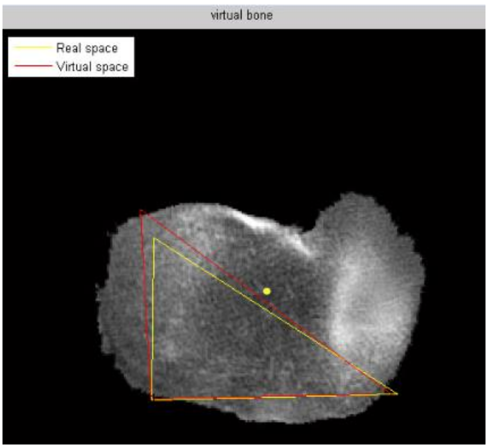


Fig. 5.3. a) top view of tibia bone in matching ${}_{\pi}S_3^{\ell}$ process, where red and yellow triangles represent three corresponding points from PP and robot coordinate systems respectively, b) AS3 Samsung robot system, man-operator and fixed bone being operated

Because of the fact that pre-operative planning (PP) system and the robot system (RS) have their own coordinate systems, registration process ${}_{\pi}S_2^{\ell}$ is required to transfer information to RS coordinates about bone cutting trajectory represented in coordinates of PP system. A transformation matrix between two coordinate systems is calculated with three corresponding points – represented by red and yellow triangles – from each coordinate system where each point has the same location in both coordinates (see Fig. 5.3). After that, the cutting trajectory calculated in the pre-operative planning system is transformed into the cutting trajectory in the coordinates of the robot system. This process ${}_{\pi}S_3^{\ell}$ is called *matching*.

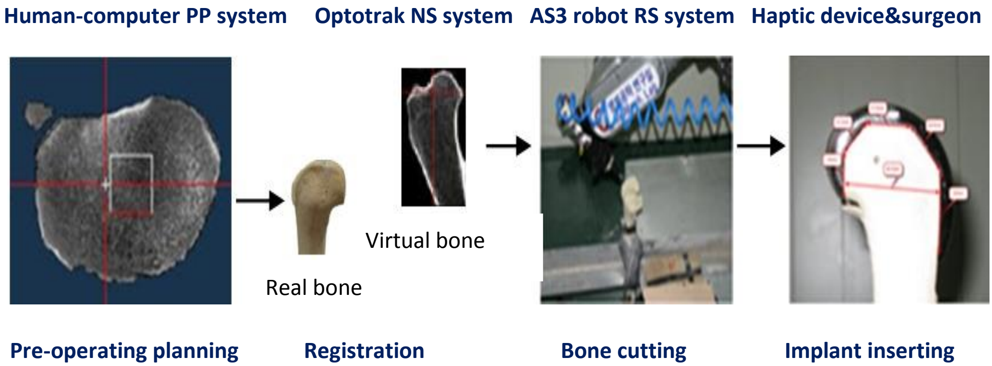


Fig. 5.4. Several steps of *SRS TKA process* ${}_{o\pi}S^{\ell}$ – bone cutting and implant inserting – executed by SRS subsystems $\overline{\omega}^{\ell-1}$ and $\overline{\omega}^{\ell}$

After performing a matching process a bone cutting and implant inserting processes ${}_{\pi}S_3^{\ell} \ldots {}_{\pi}S_8^{\ell}$ are realized (see Fig. 5.4). To move the milling tool along the cutting trajectory, the values of the robot joint angles at each time step are calculated for AS3 robot system (Rockwell Samsung Automation Inc., Korea) by performing the inverse kinematics task. For the accuracy of the inverse kinematics analysis, first, each length of the robot link from the manufacture specification is analyzed and calibrated to match the pre-defined known positions and orientations of the end effecter of the robot system by trial-end errors. Then, the exact transformation of the robot links from the origin coordinate to the robot end effecter is reconstructed for the calculation of the joint angle profiles for given bone cutting trajectory input.

Pre-operative planning (PP) system has to reconstruct the 3D model of a bone, choose an implant model, align the bone and the implant models, and calculate the cutting trajectory of the tools to allow the appropriate location and orientation of the implant with respect to the tibia bone. PP system connects the robot and virtual bone spaces to calculate a bone cutting trajectory. Processes ${}_{\pi}S_1^{\ell}$ and ${}_{\pi}S_3^{\ell}$ are associated with PP human-computer system, ${}_{\pi}S_2^{\ell}$ and ${}_{\pi}S_5^{\ell}$ – with navigation system (NS), and ${}_{\pi}S_6^{\ell} \ldots {}_{\pi}S_8^{\ell}$ processes are associated with robot system (RS) (Figs. 5.1, 5.3b and 5.4). NS registers 3D positions of real bone and AS3 robot instrument by Optotrak 3020 cameras system (Northern Digital Inc., Canada) to transfer the cutting trajectory being planned to robot system. NS also transforms the bone coordinates system in real space to the coordinate system in virtual space in PP system. The trajectory calculated in PP is transformed into coordinates of robot system. The control signals to robot joints are generated by Robot Control System and Multi Motion Controller (Rockwell Samsung Automation Inc., Korea) [58] to realize the cutting trajectory. The inverse kinematic task is performed before it. NS and RS are coordinated by different control PC computers connected by LAN network (Fig. 5.1) with correspondent communication program [61] developed to integrate the systems.

${}_{\pi}S_7^{\ell}$ is realized by electromechanical subsystems (AS3 robot servomotors) of the general biomechatronics SR system, ${}_{\pi}S_1^{\ell} - {}_{\pi}S_4^{\ell}$ are realized by video-information and computer subsystems, and ${}_{\pi}S_8^{\ell}$ is realized by pneumatic and mechanical ones. The general process is composed by sub-processes ${}_{\pi}\bar{S}^{\ell}$, executed by the general biomechatronic system, which includes the SR system ${}_{o}S^{\ell}$ and its environment ${}_{\varepsilon}S^{\ell}$. All the sub-processes ${}_{\pi}S_i^{\ell}$ are realized by different nature subsystems of the general SR biomechatronic system.

So, all the subsystems of general biomechatronic SR system, i.e. mechanical (AS3 manipulator $S_{11}^{\ell-2}$, haptic system), electromechanical (servomotor $S_{12}^{\ell-2}$), pneumatic (cutting-milling instrument $S_{13}^{\ell-2}$), computer-electronic (navigation $S_3^{\ell-1}$ and control system $S_4^{\ell-1}$), human-computer (surgeon and preregistration system) have their aggregated dynamic ω^{ℓ} and structural σ^{ℓ} descriptions. The sub-processes ${}_{\pi}S_i^{\ell}$ are also presented at CD phase in $(\bar{\rho},\bar{\varphi})^{\ell}$ form (3.5), see Chapter 3. All the connected descriptions of the subsystems $\bar{S}^{\ell}$ and processes ${}_{\pi}\bar{S}^{\ell}$ are presented in the informational resources (data bases) of the coordinator which performs the design and control processes, connecting in this way structure σ^{ℓ} and the functional dynamic realization ω^{ℓ} of the SRS being coordinated.

Coordinator S_0^{ℓ} in our case is realized in the form of human-computer design&control system of the SRS, which maintains its functional modes by surgeon and control system and realizes the design process by higher level computer aided design (CAD) system if necessary. All metrical characteristics of SR subsystems and processes are presented at CD phase in the form of numeric positional systems L^s (see Chapters 3,4).

5.2. Realization of coordination – design and control – tasks

Coordinator S_0^ℓ tasks in control process ${}_\pi S_0^\ell$ of SRS are performed by human-computer system, i.e. surgeon communicating with servers and robot. The main control tasks, i.e. functions, are indicated in the graphic user interface (GUI) window of control program (Fig. 5.5) developed in Biomechanics Lab., School of Engineering, Kyung Hee University, Yongin, Korea [61]. The main control processes are: 1) bone fixation and its position checking ${}_{1\pi} S_0^\ell$, 2) registration of robot position ${}_{2\pi} S_0^\ell$, 3) registration of real-virtual bone relations ${}_{3\pi} S_0^\ell$, 4) calculation of the cutting path and realization of a bone cutting ${}_{4\pi} S_0^\ell$.

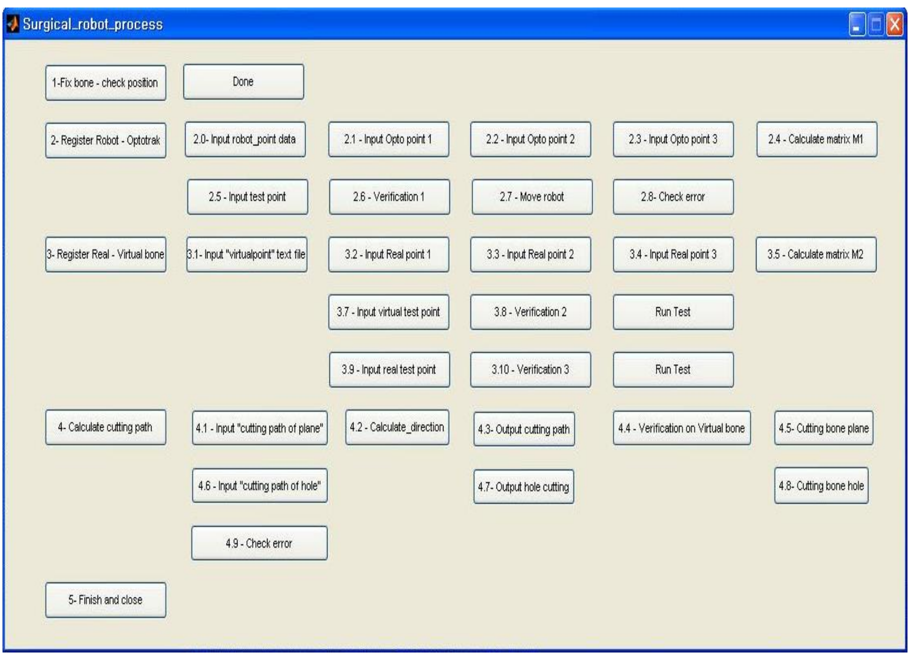


Fig. 5.5. GUI window of control program of SRS which performs TKA process ${}_\pi S^\ell$

All the control processes have their sub-processes indicated by correspondent buttons on the GUI control panel.

${}_{1\pi} S_0^\ell$ control process of bone fixation and its position checking contains the sub-processes performed by surgeon:

1.1. bone fixation and allocation according to the device position;

1.2. moving robot to the working area and checking the position of the robot end effecter;

${}_{2\pi}S_0^{\ell}$ process of robot position registration contains the sub-processes of:

2.1. robot displacements using haptic device and getting 3 registration points coordinates and joint angles by Multi Motion Controller (MMC) – data collecting,

2.2. registration of 3 registration points by Optotrak – collecting coordinates in another coordinate system,

2.3. loading of the data collected to SRS control program,

2.4. calculating the coordinates matrix transform – setting the relation of coordinate systems of Optotrak and AS3 robot,

2.5. checking the matrix transform – choosing any control point and comparison of the coordinates values;

${}_{3\pi}S_0^{\ell}$ control process of registration of real-virtual bone relations contains the sub-processes of:

3.1. loading data of 3 registration points of virtual bone from GUI to program,

3.2. collecting and loading data of real bone using Optotrak cameras,

3.3. performing matching algorithm by calculating matrix transformation and setting relation between real and virtual bones,

3.4. checking of matrix transformation;

${}_{4\pi}S_0^{\ell}$ control process of calculation of cutting path and bone cutting realization contains the sub-processes of:

4.1 cutting path loading from PP system – planning program,

4.2. calculation of robot end effecter cutting path by performing matrix transformations and inverse kinematics task,

4.3. realization of cutting – loading of manipulator angular change data to control MMC unit and execution of the event.

Formally, SRS control processes are described at CD phase in $(\overline{\rho},\overline{\varphi})_0^{\ell}$ form of coordinator functions (see Chapter 3) after definition of the states C_{0t}^{ℓ}, inputs X_{0t}^{ℓ} and outputs Y_{0t}^{ℓ} of control subsystems:

$$(\overline{\rho},\overline{\varphi})_0^{\ell} : \overline{\rho}_0^{\ell} = \{\rho_{0t} : C_{0t} \times X_{0t} \to Y_{0t} \,\&\, t \in T\}^{\ell}, \tag{5.8}$$

$$\overline{\varphi}_0^{\ell} = \{{}_o\varphi_{0tt'} : C_{0t} \times X_{0tt'} \to C_{0t'} \,\&\, t,t' \in T \,\&\, t' > t\}^{\ell}.$$

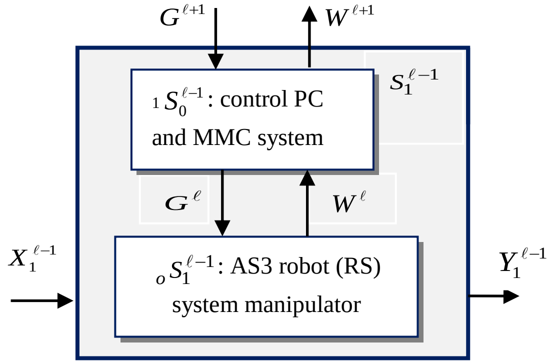


Fig. 5.6. MMC and control PC system presented in the form of coordinator ${}_1S_0^{\ell-1}$ of manipulator ${}_oS_1^{\ell-1}$ of robot system $S_1^{\ell-1}$

As for the control system of robot itself (Fig. 5.6), it is presented by Multi Motion Control (MMC) system and control PC in the form of coordinator according to the scheme presented in Fig. 3.1 (Chapter 3). The inputs and outputs of coordinator are defined on the sets of its coordination signals G^{ℓ} and feedbacks W^{ℓ} as follows:

$$X_0^{\ell}=\{\,G^{\ell+1},W^{\ell}\,\},\qquad\qquad Y_0^{\ell}=\{\,G^{\ell},W^{\ell+1}\},\tag{5.9}$$

where: G^{ℓ} are coordination signals for systems $\bar{S}^{\ell-1}$, W^{ℓ} is feedback from $\bar{S}^{\ell-1}$, $W^{\ell+1}$ is feedback from S_0^{ℓ} to coordinator $S_0^{\ell+1}$ of higher level, $G^{\ell+1}$ are coordination signals from $S_0^{\ell+1}$ to S_0^{ℓ} [58].

Concerning Multi Motion Control (MMC) system [58] of AS3 robot the input is the preplanning cutting trajectory of robot's arm (end effecter) and the output is the electronic control signals (manipulator joints angular values) which go to manipulator servomotors to realize predefined motion of its joints after solving the inverse kinematic problem. State of general control system is the state of control process in the current moment of time.

For MMC control system ${}_1S_0^{\ell-1}$ (see Fig. 5.6) input X_{0t}^{ℓ} are $G^{\ell+1}$ coordinates of cutting trajectory [*x(t), y(t), z(t)*] and feedback signals W^{ℓ}. W^{ℓ} signals bring the actual values of robot joints current positions from servomotors. Output Y_{0t}^{ℓ} contains both electronic signals G^{ℓ} which define manipulator joint angular values θ_i, *i=6* (new position), and feedback signals $W^{\ell+1}$ to PP system, i.e. position of robot and cutting instrument, registered by Optotrak system at step 2 of surgical control process (button 2 – register robot and robot point data, Fig. 5.5). C_{0t}^{ℓ} are the actual positions of the manipulator joints and cutting instrument at t moment of time. Concerning positioning and bone cutting process controlled by MMC system the input $X_1^{\ell-1}$ of AS3 robot (Rockwell Samsung Automation Inc., Korea) are input loads of servomotors. The robot output $Y_1^{\ell-1}$ are the positioning and cutting operations performed, which are the immediate input of the environment ${}_{\varepsilon}S^{\ell}$ element ${}_{\varepsilon}S_i^{\ell}$, i.e. bone being operated.

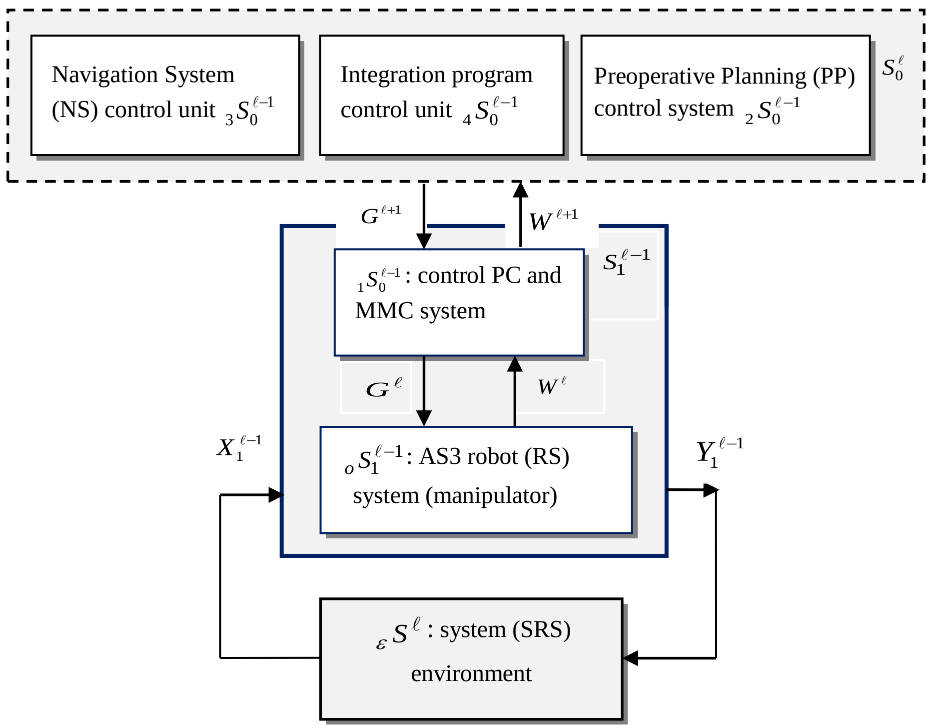


Fig. 5.7. Computer control system (CCS) subsystems $S_0^{\ell-1}$ interaction in control process of manipulator ${}_oS_1^{\ell-1}$ of robot system $S_1^{\ell-1}$ (RS) which realizes a cutting path

The bone cutting process is executed by surgeon using the control program by activating step 4 “Calculate cutting path” of GUI (see Fig. 5.5). The processes of PP system are activated at steps 4.1-4.3 by pressing the correspondent GUI buttons. The control processes of robot MMC system are activated by steps 4.4 and 4.5 (Fig. 5.5). Internal control loop flowchart of MMC while executing the motion process of robot end effecter along the cutting trajectory is given in Fig.5.8. The control loop flowchart is presented in the form of coordination process ${}_{1\pi}S_0^{\ell-1}$ of coordinator ${}_1S_0^{\ell-1}$ (MMC connected with control PC, see. Figs. 5.6 and 5.7) and contains the sub-processes ${}_{1\pi}S_{0i}^{\ell-1}$ (Fig. 5.8).

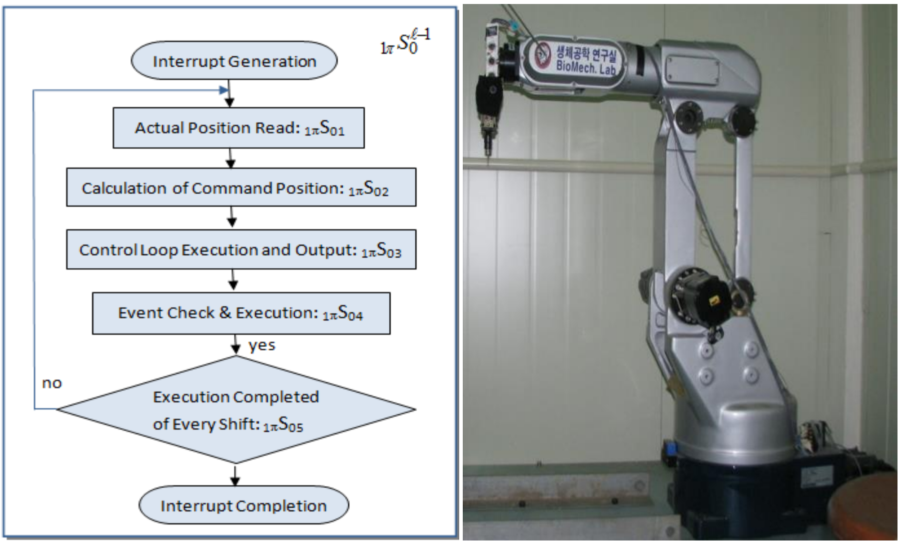


Fig. 5.8. a) internal control loop flowchart of coordinator ${}_1S_0^{\ell-1}$ – sub-processes ${}_{1\pi}S_{0i}^{\ell-1}$ of Multi Motion Controller (MMC), and b) manipulator – AS3 robot being controlled

Control signals of AS3 robot manipulator (Figs. 5.7 and 5.8) are formed as a result of inverse kinematic task performing and calculation of robot's joints angular change after consequent placing of end effecter of robot to the positions of 3 registration points. The placement is realizes by operator (surgeon) using haptic device connected with manipulator via robot's control system – man-machine mechatronic subsystem.

One of the functions of control PCs of coordination system is the commands forming and control of each subsystem while executing general surgical process, e.g. for Navigation System (NS), it is control of bone registration process, for Robot System (RS) – control of manipulator motion along the predefined cutting path while executing cutting-milling process. Other functions include creation of virtual bone and real bone images, obtaining registration points coordinates, matching, cutting path forming according to the shape of implant, etc.

Coordinator S_0^ℓ tasks in the *design process*, realized by higher level SRS design system, are distributed on the coordinator's strata: selection, learning and self-organization. The design process realized by SRS coordinator correspond to the general HS design method performance described in Chapter 4 and Section 3.2 of Chapter 3.

On the *selection stratum* at $t \in T$ moment of time, predicting the environment systems states for moment $t' > t$, coordinator defines correspondent inputs and outputs of SRS system being designed and creates $(\bar{\rho}, \bar{\varphi})^\ell$ models of SRS and its process using the obtained predictions. After that, coordinator selects such states of subsystems and their interactions which correspond to the standard state (required for t' moment of time) of SRS system S^ℓ, i.e. performs the task of structure σ^ℓ *synthesis* of the SRS being designed. Synthesis task – described in Chapter 4 – is performed by SRS coordinator realizing its coordination strategies at both conceptual (CD) and detailed (DD) design phases. Coordination strategies realized on the *selection stratum* of SRS coordinator are described in *aed* general form (3.38) in Section 3.2.1.

The change of coordination strategies and SRS coordinator states is executed on the learning and self-organization strata which are described by functions of states transition ${}_o\bar{\varphi}_0^\ell$ and ${}_o\bar{\hat{\varphi}}_0^\ell$ given in (3.39) form in Section 3.2.1. All metric characteristics μ of SRS sub-systems being coordinated – designed and controlled – and the most significant geometrical signs are defined in the frames of *aed* formal model of HS in codes of numeric positional system L^S, and presented in (3.64) and (4.9-4.14) forms in Sections 3.2 and 4.2 respectively.

Preliminary conclusion

The conceptual model of SRS biomechatronic system presented in *aed* theoretical basis of hierarchical systems for the design and control technology realization is briefly presented in the Chapter. In comparison with models of mathematics and artificial intelligence the proposed SRS formal model contains connected descriptions of the biomechtronic system structure, its aggregated dynamic representation as a unit in its environment and SRS environment model. All the descriptions are connected by the coordinator which performs the design and control tasks on its strata. Exemplary control loop of robot system (RS) coordinator is presented in the Chapter in the form of Multi Motion Controller (MMC) which realizes coordination of AS3 robot notion. Computer control system (CCS) is presented in the form of SRS coordinator, and CCS subsystems interaction in control process is revealed. Model of SRS design system is given in the form of coordinator of higher level which performs the design tasks on its strata: selection, learning and self- organization. The conceptual SRS model presented in this chapter is coordinated with traditional systems of information presentation in biomechatronics – numeric and geometrical forms (see Chapter 4). *Aed* technology of HS is also coordinated with general requirements of design and control systems (see Chapter 1), considers the elements of SRS conceptual model as well as connected SRS subsystems of different nature – mechanical, electromechanical, electronic, human-computer, biological – in common *aed* theoretical basis. It brings new possibilities in creating of a formal language for conceptual design of surgical robots and other biomechatronic systems.

Besides, the proposed conceptual model facilitates considerably the transfer from the conceptual design phase to the detailed design phase in the mechatronic object's life cycle. The presentation of the conceptual model elements in the form of the dynamic systems $(\overline{\rho},\overline{\varphi})^{\ell}$ – which are the generalization of mathematical models (DE, automata, algebra systems [9.10]) – turns the transfer algorithm to the detailed design phase into the concretization process of the conceptual $(\overline{\rho},\overline{\varphi})^{\ell}$ models. It allows the efficiency of the design processes of the biomechatronic systems to be enhanced.

6. CONCEPTUAL MODEL OF BIOLOGICALLY INSPIRED ROBOT

In this Chapter, a conceptual model for design of biologically inspired robot – Bioloid Dinosaur robot [66] – is presented. Design process of robots – industrial, universal, biologically inspired, etc. [56, 67–71, 141] – in general product life circle (see Fig. 1.2) contains several phases [1], similar to other engineering objects. The first conceptual design phase is very important, because the main design concept is generated and evaluated at this phase. According to the Hierarchical Systems (HS) technology described above (see Chapters 3,4), to create a conceptual model of robot being designed it is necessary to take into account its several levels: structure of robot (lower level), its aggregated dynamic representation as a unit in environment (current level), the environment construction and technology (higher level) and robot coordinator, i.e. robot design and control system. Therefore, a conceptual model of Dinosaur robot in formal basis of HS technology is given first in the Chapter. Examples of conceptual descriptions of assembly design and robot motion design are presented after that.

6.1. Conceptual model of dinosaur robot

Conceptual model of Bioloid Dinosaur robot is presented in the frames of HS theoretical basis as follows:

$$S^{\ell} \leftrightarrow \{\omega, S_0, \sigma\}^{\ell} \ , \qquad (6.1)$$

where ω^{ℓ} is aggregated dynamic realisation, which represents the robot as a unit in its environment, σ^{ℓ} is structure of Dinosaur robot, S_0^{ℓ} is coordinator which connects σ with ω and in this way performs the design tasks, ℓ is index of level.

Structure σ^{ℓ} of Dinosaur Bioloid robot is defined in the form of (3.14) – see Chapter 3 – and contains aggregated representations of lower level elements (head $\omega_6^{\ell-1}$, arm $\omega_5^{\ell-1}$, body $\omega_4^{\ell-1}$, leg $\omega_3^{\ell-1}$, tail $\omega_2^{\ell-1}$, foot $\omega_1^{\ell-1}$) and their connections ${}_{\sigma}\gamma^{\ell}$ – the joints, see Figure 6.1.

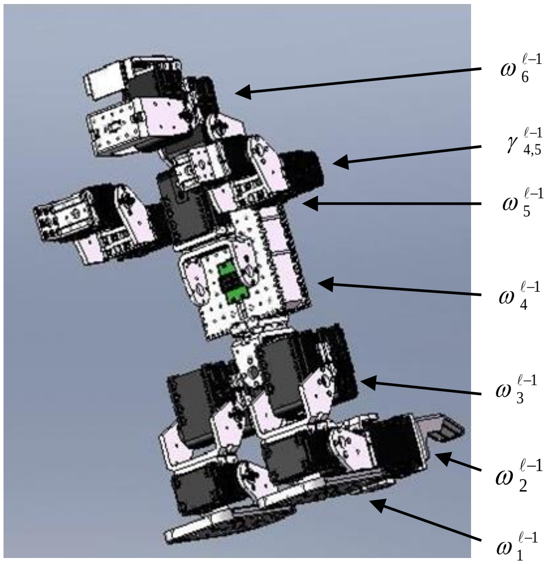


Fig. 6.1. Structure σ^{ℓ} of Dinosaur robot, $\omega_i^{\ell-1}$ are structural elements, ${}_{\sigma}\gamma_{4,5}^{\ell}$ is structural connection of $\omega_4^{\ell-1}$ and $\omega_5^{\ell-1}$ robot elements

For example, common part of the arm $\omega_5^{\ell-1}$ and the body $\omega_4^{\ell-1}$ is their structural connection, i.e. shoulder ${}_{\sigma}\gamma_{4,5}^{\ell}$ which is the structural element of a lower level of Dinosaur robot. The shoulder is constructed from servo motor Dymanixel AX-12 [66] presented in Figure 6.3. Being the electromechanical subsystem of robot mechatronic system the servo motor is describe in $(\overline{\rho},\overline{\varphi})$ form at the Conceptual Design phase. $(\overline{\rho},\overline{\varphi})$ conceptual description is transformed to State Space equations at the Detailed Design phase (Fig. 6.2) and can be concretized and presented in the following form (6.2):

$$\dot{x} = \begin{bmatrix} \dot{i}_a \\ \dot{\omega}_m \end{bmatrix} = \begin{bmatrix} \frac{-L_a}{R_a} & \frac{K}{R_a} \\ \frac{-K}{B} & \frac{-K}{B} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} + \begin{bmatrix} \frac{-1}{R_a} \\ 0 \end{bmatrix} E$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} + 0 \tag{6.2}$$

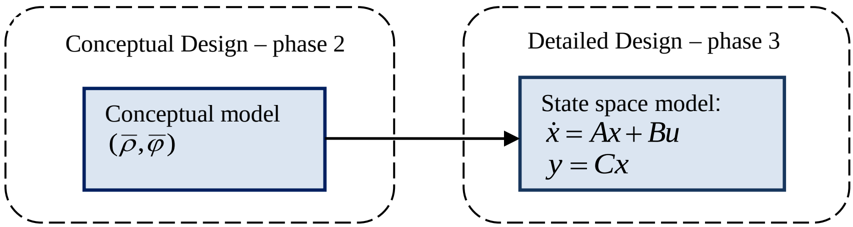


Fig. 6.2. Conceptual model transformation at the design phases

In this case first state equation of (6.2) corresponds to $\overline{\varphi}$ function, and output equation corresponds to reaction $\overline{\rho}$ of $(\overline{\rho}, \overline{\varphi})$ representation. State space equations (6.2) can be transformed at DD phase to the following transfer function (6.3) if necessary:

$$G(s) = \frac{\omega_m(s)}{e_a(s)} = \frac{K_i}{s^2 JL_a + sJ_m R_a + K_i K_b} \tag{6.3}$$

The meaning of the variables given in (6.2), (6.3) and Figure 6.3.are explained below.

a) b)

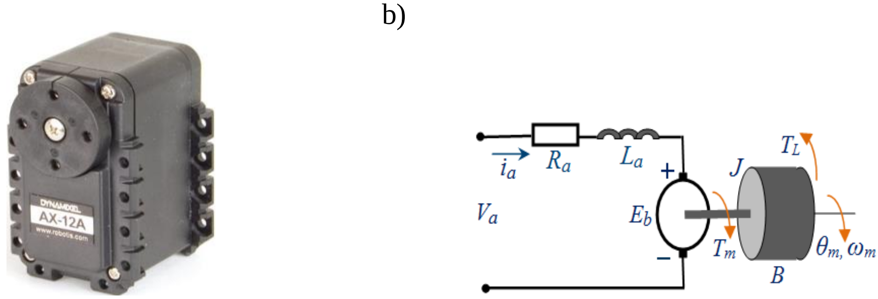


Fig. 6.3. Dymanixel AX servo motor a), and its equivalent electro-mechanical scheme b)

In Figure 6.3b and equations (6.2) and (6.3), i_a is armature current, L_a is armature inductance, R_a is armature resistance, $V_a(t)$ is input voltage, E_b is back emf, K_b is voltage constant, T_L is load torque, T_m is motor torque, θ_m, ω_m are motor angular change and velocity respectively, K_i is moment constant, J is motor moment of inertia, B is friction constant, K is a constant.

Coordinator S_0^{ℓ} of robot ${}_{o}S^{\ell}$ is presented in the form of (3.15) (see Section 3.1) and controls the process of robot motion on its strata of selection, learning and self-organisation. Coordinator control functions on selection stratum are realised in this case by

Bioloid CM-5 control unit (Fig. 6.4) placed on the Dinosaur robot body $\omega_4^{\ell-1}$. Coordinator learning and self-organization strata are realized by the design system. Motion process ${}_{o\pi}S^{\ell}$ performed by robot ${}_{o}S^{\ell}$ in its environment ${}_{\varepsilon}S^{\ell}$ is presented as a change of robot's interconnections ${}_{\omega}\gamma^{\ell}$ with elements S_i^{ℓ} ($i \in I^{\ell+1}$) of environmental system ${}_{\varepsilon}S^{\ell}$ and described according to $(\overline{\rho},\overline{\varphi})$ model (3.4). Contact state ${}_{\omega}\gamma_1^{\ell}$ of robot's base leg with a pathway surface ${}_{\varepsilon}S_1^{\ell}$ is regarded as robot's interconnection ${}_{\omega}\gamma_1^{\ell}$ with sub-system ${}_{\varepsilon}S_1^{\ell}$, i.e. the element of robot's environment ${}_{\varepsilon}S^{\ell}$.

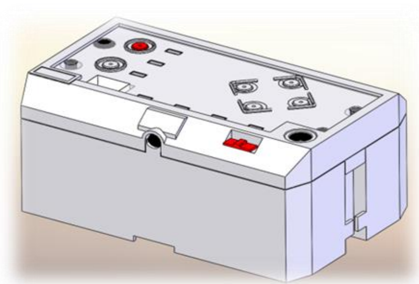


Fig. 6.4. Design model of Bioloid CM-5 control unit created in SolidWorks system environment

All the metrical characteristics of robot's bio-kinetic apparatus (relational angles, length of limbs, etc.) and motion process (displacement, velocity, acceleration) are described in the form of numerical characteristics of HS conceptual model, similar to the case of human motion design (see Chapter 7).

6.2. Robot motion design

Motion design of Dinosaur robot in described in the frames of the suggested conceptual model. In this model, it is possible to present the *measures of motion* (velocity, acceleration) as a change of measures of the states of dynamic systems in elementary time in metrical, structural and boundary (geometric forms) spaces [55,72], see Chapter 4. For Dinosaur robot, the velocity, for instance, is defined as change Δ of its parts interconnections ${}_{\sigma}\gamma_{tt'}$ in elementary time interval *tt', t'>t*. The velocity is directly proportional to the changes of the interconnections ${}_{\sigma}\gamma_{tt'}$. The dependence of the interconnections change and the kinds of robot motions is presented as follows:

$\Delta_{\sigma}\gamma_{tt'} = 0; \quad \rightarrow$ standing phase,
$\Delta_{\sigma}\gamma_{tt'} = const; \rightarrow$ motion with constant velocity,
$\Delta_{\sigma}\gamma_{tt'} \neq const; \rightarrow$ motion with acceleration.

The measures of motion have their directions dependent on the elements' addresses in the environment, according to which the present process is considered. One of these elements from robot environment ${}_{\varepsilon}S^{\ell}$ is the pathway ${}_{\varepsilon}S_1^{\ell}$.

Computer program construction for design of Dinosaur robot motion is presented according to HS conceptual model (see Fig. 3.1, Chapter 3). Coordinator S_0^{ℓ} is presented in the form of control program, subsystems $\bar{S}^{\ell-1}$ play the role of subprograms, process ${}_{\pi}S^{\ell}$ describes the calculation process, inputs and outputs are correspondent interactions of subprograms and calculation processes. One of the subsystems of the robot environment ${}_{\varepsilon}S^{\ell}$ is operator ${}_{\varepsilon}S_i^{\ell}$ which observes the design process on computer and changes program construction if necessary.

According to the conceptual HS presentation, all parts of body construction of Dinosaur robot are presented in the form of the correspondent program units $\omega^{\ell-1}$. Their connections ${}_{\sigma}\gamma^{\ell}$ are realized as programs' interactions and are presented in the form of parameters, i.e. common program variables changeable in calculation process. Control program which coordinates the design process is presented in the form of coordinator (3.15). Design process is described as coordinator process ${}^{\beta}_{o\pi}S_0^{\ell}$ (3.22), see Section 3.1.

The coordinator's task in the process of design robot movements is to provide a coordinated motion of robot in its environment and deformations of its elements, i.e. ${}_\sigma\gamma^\ell$ changing. ${}_\sigma\gamma^\ell$ in geometrical sense are angles between robot parts (body, foot, leg). So, the coordinator's task is to continuously change the angles to provide the required locomotion process of robot. Formally, the task performing is described as reaction ρ_0^ℓ of the coordinator (6.4). At the same time, it is described according to $(\overline{\rho},\overline{\varphi})$ model as state transition function $\varphi_{tt'}^\ell$ of the system (robot bio-kinetic apparatus) in the form (6.5) as follows:

$$\rho_0^\ell : C_0^\ell \times \sigma_t^\ell \to \sigma_{t'}^\ell, \tag{6.4}$$

$$\varphi_{tt'}^\ell : \sigma_t^\ell \times {}_\sigma\gamma_{tt'}^\ell \to \sigma_{t'}^\ell \ , \tag{6.5}$$

where σ_t^ℓ is the state of robot bio-kinetic apparatus structure at the moment of time *t*, ${}_\sigma\gamma_{tt'}^\ell$ is the change of angles during the period of time *tt'*, $\sigma_{t'}^\ell$ is the state of robot bio-kinetic apparatus structure at the moment of time *t'* and *t'*>*t*.

As stated above, the connections ${}_\sigma\gamma^\ell$ are presented as program variables on computer. In motion design the coordinator (control program) changes continuously the angles, i.e. parameters which are common for two correspondent program units, and in this way realizes the required motion process ${}_\pi S^\ell$ of robot.

6.3. Assembly of dinosaur robot parts

HS conceptual design model is presented in this Section for the design of Bioloid robot assembly operations. In the basis of the conceptual model described above, the robot being assembled is presented in the form of (6.1), and the assembly operations are presented as a particular case of synthesis operations considered in Chapter 4, and partially described in [73]. The difference between the synthesis and the assembling operation is that the common part (structural element) of one of the two united objects is an "imaginary" geometric element. For instance, a cylindrical hole can be regarded as a geometric object – cylinder – marked with a "minus". Such imaginary elements have their structural addresses given in numeric codes in (4.12) form.

The exemplary assembling process of Bioloid Dinosaur robot foot from its parts, i.e. its two frames, screws and nuts is presented as follows.

At the *first stage* the robot foot parts (geometric objects ${}_gS_i^{\ell}$), which are of different constructive dimensions, are not connected objects: ${}_gS_1^3$ and ${}_gS_2^2$ are frames F1 (3D object) and F3 (conditional surface) respectively, ${}_gS_3^1$ and ${}_gS_4^0$ are screws S1 (conditional lines) and nuts N1 (conditional points), Fig. 6.5. Coordinator defines the sequence of the parts being assembled and structural addresses of their common parts (links). The addresses of the interconnections are defined in a numeric positional code in the form similar to the connections defects ξ^{ℓ} (4.12).

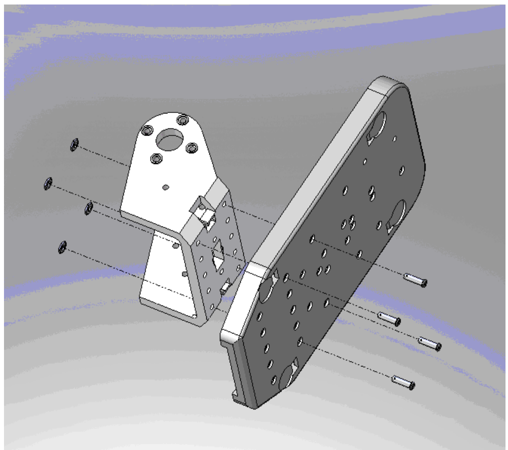


Fig. 6.5. Constructive dimensions of foot parts being assembled: **1.** $\delta_{\omega 1}^{\ell} = 1\_0\_0\_0$ (one frame F1); **2.** $\delta_{\omega 2}^{\ell} = 0\_1\_0\_0$ (one frame F3); **3.** $\delta_{\omega 3}^{\ell} = 0\_0\_4\_0$ (four screws S1); **4**. $\delta_{\omega 4}^{\ell} = 0\_0\_0\_4$ (four nuts N1)

At the *second stage*, coordinator executes consecutive connections of the foot parts. Simultaneously, both the basic element (state C_1: part ${}_gS_2^2$) and the input element X_1 (part ${}_gS_1^3$) are indicated. Output (Y_1) is a construction of two assembled parts ${}_gS_{1,2}$ (Fig. 6.6), which is the basic element (C_2) for the next assembling step. States dynamics of object ${}_gS^{\ell}$ (dinosaur robot) in the assembling process is described by (6.5). The reaction is:

$$\{{}_o\rho_1 : C_1 \times X_1 \to Y_1 \ \& 1 \in T\}^{\ell}. \tag{6.6}$$

Structure σ^{ℓ} of the assembled Dinosaur robot foot is presented according to (3.14). Structural elements of a lower level are subsystems ${}_gS_1^3$ and ${}_gS_2^2$, i.e. frames F1 and F2 respectively. Connections ${}_\sigma\gamma_{1,2}$ are presented by elements ${}_g\bar{S}_3^1$ and ${}_g\bar{S}_4^0$, i.e. screws S1 and nuts N1 respectively (Fig. 6.5). After creation of the robot foot (Fig. 6.6), the following steps of the assembly process are realized. Two of the steps are presented in Figure 6.7. Resultant assembled Dinosaur robot is presented in Figure 6.1.

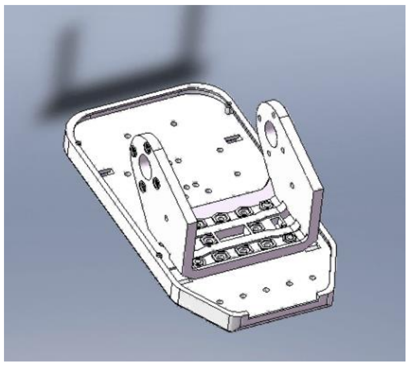


Fig. 6.6. Constructive dimension of resultant assembled robot foot is $\delta_\sigma = 1\_1\_4\_4$

Figures 6.1 and 6.4-6.7 are the results of a computer design of Dinosaur Bioloid robot and its assembling operation in SolidWoks (Dassault Systèmes SolidWorks Corp.) program system environment [74].

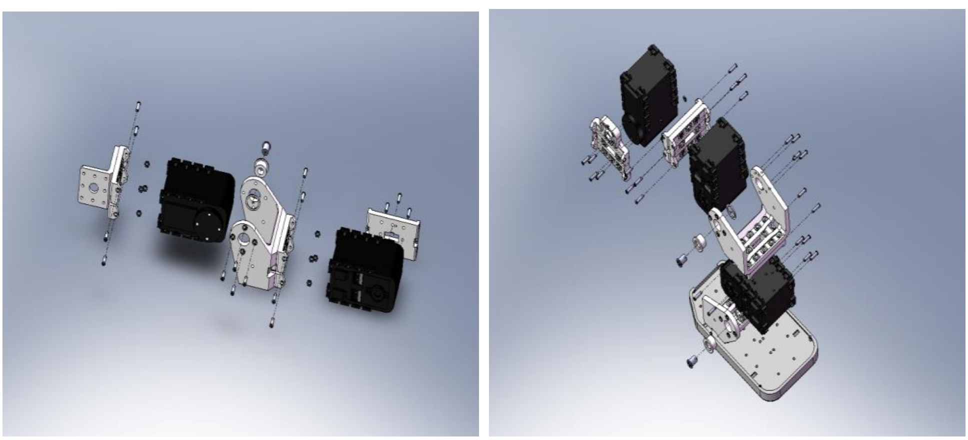


Fig. 6.7. Two exemplary steps of the assembling process of Dinosaur robot leg

Preliminary conclusions

The conceptual model of Dinosaur Bioloid robot created in theoretical basis of HS is presented in this Chapter. In comparison with traditional methods, the formal construction of the suggested conceptual model contains connected descriptions of the Dinosaur robot structure, its aggregated dynamic representation as a unit in its environment, part of the environment model. All the descriptions are connected by the coordinator which performs the design and control tasks on its selection, learning and self-organization strata. Besides, numeric and geometric characteristics of the robot being designed are presented in the frames of HS conceptual model in the form of numeric positional system. It allows the model to fulfill the general requirements of design and control systems, see Chapters 1 and 3, and CAD systems in particular.

Therefore, hierarchical systems technology is applied in the Chapter in the exemplary tasks of conceptual description and mechatronic design of Dinosaur Bioloid robot, design of robot's assembly operations and motion design. The examples of computer realizations of the design tasks in the frames of SolidWorks program system are presented. Mechanical, electromechanical and program computer subsystems of general robot mechatronic system are partially described in common formal bases of the conceptual model suggested. The model dynamics while transferring from the conceptual to detailed design phases is shown for the case of the robot servo-motor.

7. HUMAN MOTION DESIGN USING HIERARCHICAL SYSTEMS METHOD

Realization of Hierarchical Systems (HS) design method and conceptual model in design and control of human motion is presented in this Chapter. Design and control of human motion requires consideration of man's connected levels and their dynamics. Advanced design methods are required for effective problems solving. At present there are a lot of models and methods, which are applied in tasks of coordination (design and control) of bio-mechanical systems and human motion in particular [75-78]. The overview of theories concerning human bipedal walking is well presented by Vaughan in [79]. All the methods differ with reference to the type of movements they describe and kind of problem being solved. The most popular mathematical models which are used for biomechanical motion description, such as differential equations (DE), integral-differential equations (IDE) are single level by nature and useful enough for modeling of mechanical kind of physical motion.

In comparison with physical type of movements which are caused by action of external forces only, the bio-mechanical one executed by man is characterized by structural changes (deformation) of human body elements – muscles compressing and limbs angular change – which lead to the man movement in its environment. So, the biomechanical movements in general and human motion in particular are hierarchical by nature. To design and control human motion it is necessary to take into account several levels: 1) human body structure, 2) human dynamic representation in its environment, 3) the environment structure. Traditional one-level formal models can-not describe human body construction and motion process on multi-levels. It is also impossible to regard the task of connected motion and deformation within the frames of one common formal model. The most popular Finite Elements Method (FEM) is usually used for the deformation tasks performing and Integral-Differential Equations (IDE) are used for motion description.

That is why the construction of hierarchical systems (HS), its coordinator technology and the design methods described in Chapters 3,4 are applied in this Chapter as a theoretical basis for coordination problems solving and conceptual design of human motion. The main aim of the study presented here is the development of a new method for human motion design with the use of coordination technology, Hierarchical Systems and Dynamic Systems approach (see Chapters 3 and 4). The method takes into account all the levels mentioned above and provides a common theoretical framework for a formal expression of both the human motion and design procedures. The aim of the experimental study is to *answer the question* if the computer design method developed corresponds to the real processes of human motion, in our case to human sitting-standing-up movements, and if it is possible to predict and coordinate (design and control) human motion using the HS design method developed.

To reach the aims: 1) a theoretical basis of the design method proposed is described using hierarchical and dynamic systems [9,10] hierarchical mathematics, and *aed* (ancient Greek word) coordination technology of HS [6-8], see Chapters 3, 4; 2) coordination (design and control) task is formulated, ways of human motion design and method steps are defined in this Chapter in Section 7.1, 3) systems movements and deformations are defined within the frames of theoretical basis of the HS technology in Section 7.2; 4) an algorithm of computer design and steps of the method realization for human sitting-standing-up motion are given in Section 7.3; 5) computer motion design, and 6) an experimental motion study are performed and described in Sections 7.3 and 7.4 respectively; 7) comparison of the results of both in vivo laboratory and computer experiments is performed, and design method analysis is given in Sections 7.4 and 7.5 of this Chapter.

7.1. Coordination task formulation

In the coordination process (design and control) of human motion it is necessary to consider several connected levels of the object under consideration. All connected levels, i.e. 1) human body structure, 2) human dynamic representation as a unit in its environment, 3) the environment structure are taken into account within the frames of *aed* coordination method of HS, described in Chapters 3,4 and sections 7.1-7.3 of this Chapter.

The main design tasks, i.e. synthesis and analysis, are performed by coordinator of hierarchical system (HS), which connects human body structural change with its behavior in the environment and in this way predicts various kinds of human movements taking into account the environment construction.

Coordinator of HS is a multilayer system (see Chapter 3). The tasks of various layers (strata) in within the frames of general coordination task under conditions of any knowledge uncertainty, are as follows:

– creation or improvement of the system construction & technology, to make its activity in higher level system most coordinated with the desired environment states on all its levels; (selection stratum);

– change the ways of main design & control task carrying out when designed constructions & technologies are multiplied and knowledge uncertainty is removed; (learning stratum);

– change the selection and learning strata as new (higher level) knowledge constructions & technologies are created (self-organization stratum).

Human motion design task is carried out by the coordinator on its selection stratum. In this process coordinator S_0^{ℓ} creates the desired geometrical construction of the object under consideration (human bio-kinetic apparatus) according to the required object's representation (form) in its environment (*synthesis task*). In its turn, the coordinator partly changes the structure of the resultant object and analyses the object's behavior in the environment (*analysis task*). All the tasks are performed by the means of realization of one of the coordinator's strategies [3, 80].

The immediate performance of the task is realized at the fourth stage of HS design method (see Chapter 4). The main general steps of HS method realization in the process of human motion design are the following.

1) Description of the object being designed (human body construction and its motion process) in the form of *aed* (two-level system with dynamic units) – standard block of hierarchical systems.

2) Representation of basic geometric elements of human bio-kinetic apparatus in the hierarchical form as conditional objects – conditional point, line, surface, 3D object.

3) Definition of metric and structural characteristics of geometric construction of human body – constructive dimension of body segments, addresses and directions of the segments (limbs) connections (body segments structural connections) – in the codes of numeric positional system (see Chapters 3, 4).

4) Realization of the design process of human motion – synthesis and analysis – as the coordination process of HS.

Fourth stage of the method realization contains two design sub-tasks, i.e. synthesis of geometric construction of human bio-kinetic apparatus and analysis. Analysis tasks are two connected tasks of deformation and motion. Deformation in our case means partial change of human body structural connections (joints angles, velocities, limbs lengths). Motion is considered to be a change of human interactions with the elements of the environment system (human movements in its environment) caused by the structural deformations of human bio-kinetic apparatus. An example of the change of human interactions with its environment is the change of the contact of human feet with the road while walking. Both synthesis and analysis tasks are formulated and described below within the frames of the presented HS design method (see Sections 7.3). Theoretical basis of the HS method of human motion design – *aed* formal model, coordinator model, numeric and geometric characteristics – is described above in Chapters 3, 4.

7.2. Definition of human movements and deformations

The design task of object movements and deformation is defined as a coordination task of the hierarchical system. The motion process ${}_{o\pi}S^{\ell}$ of man ${}_{o}S^{\ell}$ in its environment ${}_{\varepsilon}S^{\ell}$ is presented using HS *aed* technology as changing human interconnections ${}_{\omega}\gamma^{\ell}$ with elements S_i^{ℓ} ($i \in I^{\ell+1}$) of the environment system ${}_{\varepsilon}S^{\ell}$. Interconnections ${}_{\sigma}\gamma^{\ell}$ changing of the elements of human bio-kinetic apparatus structure σ^{ℓ} is considered to be *deformations*. Coordinator S_0^{ℓ} controls the process of human motion by changing structural connections ${}_{\sigma}\gamma^{\ell}$ (joint angles) of human structure. Continuous changes in joint angles cause continuous changes of external connections ${}_{\omega}\gamma^{\ell}$ of man in its environment ${}_{\varepsilon}S^{\ell}$, i.e. human motion. All the tasks are performed by coordinator S_0^{ℓ} on its layers.

It is possible to define different kinds of movements and deformations by introducing corresponding conditions. For instance, the following relation corresponds to physical movement (object's coordinates changing) in time interval $\overline{T}_{tt'}^{\ell}$:

$$ {}_{\sigma}\gamma^{\ell} \mid \overline{T}_{tt'}^{\ell} = const. \tag{7.1}$$

Here structural connections ${}_{\sigma}\gamma^{\ell}$ are not changed (in this time period) and structure σ^{ℓ} state is not changed either. Such movement is conditioned by environment process ${}_{\varepsilon\pi}S^{\ell}$ and described in the following form:

$$ {}_{\varepsilon\pi}(\overline{\rho},\overline{\varphi})^{\ell}: \qquad {}_{\varepsilon\pi}\overline{\rho}^{\ell} = \{{}_{\varepsilon\pi}\rho_t^{\ell} : {}_{\varepsilon}X_t^{\ell} \times {}_{\varepsilon}C_t^{\ell} \to {}_{\varepsilon}Y_t^{\ell} \;\&\; t \in T^{\ell}\}, \tag{7.2}$$

$$ {}_{\varepsilon\pi}\overline{\varphi}^{\ell} = \{{}_{\varepsilon\pi}\varphi_{tt'}^{\ell} : {}_{\varepsilon}X_t^{\ell} \times {}_{\varepsilon}C_{tt'}^{\ell} \to {}_{\varepsilon}X_{t'}^{\ell} \;\&\; t,t' \in T^{\ell} \;\&\; t' > t\}.$$

For the bio-mechanical movement the following relation is true:

$$({}_{\tau\varepsilon}X^{\ell} \times {}_{\tau\varepsilon}Y^{\ell})\Big|\overline{T}^{\ell}_{tt'} = {}_{\tau\omega}\gamma^{\ell}\Big|\overline{T}^{\ell}_{tt'} = const, \tag{7.3}$$

in other words the external connections ${}_{\omega}\gamma^{\ell}$ in time interval $|\overline{T}^{\ell}_{tt'}$ depend on structural interconnections changes ${}_{\sigma}\gamma^{\ell}\ |\overline{T}^{\ell}_{tt'}$ only. This movement is conditioned by process ${}_{o\pi}S^{\ell}$ of the object and is described in the following (7.4) form:

$$\begin{aligned} &{}_{o\pi}(\overline{\rho},\overline{\varphi})^{\ell}: \quad {}_{o\pi}\overline{\rho}^{\ell} = \{{}_{o\pi}\rho^{\ell}_{t} : X^{\ell}_{t} \times C^{\ell}_{t} \rightarrow Y^{\ell}_{t} \,\&\, t \in T^{\ell}\}, \\ &{}_{o\pi}\overline{\varphi}^{\ell} = \{{}_{o\pi}\,\varphi^{\ell}_{tt'} : X^{\ell}_{t} \times C^{\ell}_{tt'} \rightarrow X^{\ell}_{t'} \,\&\, t,t' \in T^{\ell} \,\&\, t' > t\}. \end{aligned} \tag{7.4}$$

Geometric and physical values of biomechanical objects, e.g. man, are defined in state spaces: M^{ℓ}, Ξ^{ℓ}, Ω^{ℓ}, and T^{ℓ} which are metrical, structural, dynamical and time characteristics of systems respectively (see Section 4.1). All calculations are carried out in these spaces with codes of numeric positional system L^{S}. The ways of human motion design depend on the characteristics (metrical characteristics $\tilde{\mu}^{\ell}$, connection defect $\tilde{\xi}^{\ell}$, constructive dimension $\tilde{\delta}^{\ell}$) of both structural σ^{ℓ} and aggregated dynamic ω^{ℓ} representations. The ways correspond to coordination strategies ${}_{\tau}\lambda^{\ell}_{o} \in \Lambda^{\ell}_{o} = \{{}_{\psi}\lambda^{\ell}_{o}, {}_{\chi}\lambda^{\ell}_{o}, {}_{\varphi}\lambda^{\ell}_{o}, {}_{\lambda}\lambda^{\ell}_{o}\}$ for different levels of coordinator S^{ℓ}_{0} information uncertainty indicated by $\psi, \chi, \varphi, \lambda$ indexes.

7.3. Computer design of human motion

Motion process ${}_{o\pi}S^{\ell}$ of man (or human-like robot) ${}_{o}S^{\ell}$ in its environment ${}_{\varepsilon}S^{\ell}$ is presented within the frames of HS *aed* technology as changing the man's interconnections ${}_{\omega}\gamma^{\ell}$ with elements S^{ℓ}_{i} ($i \in I^{\ell+1}$) in its environment system ${}_{\varepsilon}S^{\ell}$. The design task of human motion is defined as coordination task of Hierarchical System using *aed* technology. The process of human motion is presented according to the formal description given in Section 7.2 and Subsection 7.3.1. The design task is performed using *aed* conceptual design method (see Chapters 3 and 4).

7.3.1. Algorithm of human motion design

The design algorithm of human movements is realized according to the steps of HS design method (see Chapters 3 and 4). These steps are as follows.

1. *Synthesis of the geometrical construction of human bio-kinetic apparatus.*

This process corresponds to the calculations of the resultant characteristics $\{\Omega, \Gamma, \Sigma\}$ of the geometrical construction of bio-kinetic apparatus. $\{\Omega, \Gamma, \Sigma\}$ is the set of parameters which describe the geometric construction of mechatronic object according to *aed* presentation (4.7), see Section 4.1 of Chapter 4. $\Omega = \{\omega_i : i \in I_\Omega\}$ is the description (with the help of metrical characteristics μ_i^ℓ which are the coordinates, length, squares, volumes, angles) of the state of geometric construction of human bio-kinetic apparatus presented in aggregated dynamic form ω^ℓ; $\Gamma = \{\gamma_i : i \in I_\Gamma\}$ is the description of the human body interconnections ${}_\omega\gamma^\ell$ with other objects (boundary of the human body defined by the coordinator); $\Sigma = \{\sigma_i : i \in I_\Sigma\}$ is the description of human bio-kinetic apparatus (body) structure σ^ℓ. So, human body construction as well as any geometric construction of mechatronic object being synthesised can be presented in the form of relations $S \subset \Omega \times \Gamma \times \Sigma$ on the Cartesian product of variables Ω, Γ, Σ.

Within the frames of the conceptual design method proposed (see Chapter 4), synthesis of human bio-kinetic apparatus structure σ^ℓ of level ℓ from lower level elements $\overline{\omega}^{\ell-1}$ (body segments) is realised by coordinator S_0^ℓ on its selection stratum (see 2.2.3) according to the following algorithm. In the synthesis process coordinator performs the following actions:

a) defines the constructive dimension of the resultant object, i.e. geometric construction of human bio-kinetic apparatus (chooses the required structure class $\sigma^0, \sigma^1, \sigma^2, \sigma^3$ from Σ);

b) defines kinds of changes of initial standard element of the defined structure which are necessary to perform the constructing of human bio-kinetic apparatus S_τ^ℓ;

c) sets the sequence of the changing actions from the determined classes;

d) performs the actions defined in c), – this process corresponds to the calculations of the resultant bio-kinetic apparatus characteristics $\{\Omega, \Gamma, \Sigma\}$.

Formally, this synthesis process is presented as follows:

$$S_0^{\ell} : \{\overline{\omega}^{\ell-1}, {}_{\sigma}\gamma^{\ell}\} \rightarrow \sigma^{\ell} \tag{7.5}$$

where S_0^{ℓ} is coordinator, σ^{ℓ} is structure of human bio-kinetic apparatus of level ℓ, $\overline{\omega}^{\ell-1}$ are lower level ℓ-1 elements, i.e. bio-kinetic apparatus segments, from which structure σ^{ℓ} is synthesized, ${}_{\sigma}\gamma^{\ell}$ are their structural connections (joints) (see Fig.7.1).

As a result of this *synthesis step*, structure σ_{τ}^{ℓ} of human bio-kinetic apparatus S_{τ}^{ℓ} is synthesised as well as structure $\sigma^{\ell+1}$ of environment $S^{\ell+1}$ (${}_{\varepsilon}S^{\ell}$) where man S_{τ}^{ℓ} performs his motion. The process of synthesis goes in such a way that elements of structures $\sigma^{\ell+1}$ and σ_{τ}^{ℓ} obtain common parts ${}_{\sigma}\gamma^{\ell+1}$ and ${}_{\sigma}\gamma_{\tau}^{\ell}$, and the connections ${}_{\sigma}\gamma^{\ell+1}$ include the interconnections ${}_{\omega}\gamma_{\tau}^{\ell}$ of human bio-kinetic apparatus S_{τ}^{ℓ} with other elements S_i^{ℓ} $(\tau \neq i)$ of environment system $S^{\ell+1}$.

Relations between the basic geometrical structures are presented in the form of procedures from the given classes. So, it became possible to choose a class of procedures for the synthesis of a required output object (human bio-kinetic apparatus construction) from the given input objects. Both the states of structure and form of human bio-kinetic apparatus are continuously changed. The process dynamics is described by dynamic system $(\overline{\rho},\overline{\varphi})^{\ell}$ (3.5). As an example, the result of synthesis process of the human body geometric construction is given in Figure 7.3.

2. *Changing interconnections of segments of human bio-kinetic apparatus.*

Usually, the data of the relative joint angles, velocities, trajectories, etc. for a definite kinds of human movements is obtained from ‘in vivo’ experiments, e.g. using motion capture system. Design of a similar motions is executed by changing input parameters, such as limb length, mass, etc. Prediction of another kind of movements can be conducted by intuitive setting of arbitrary parameters, such as joint angles. The results of computer motion design, for instance obtained trajectories and velocities of various points of body segments, can be compared with the experimental results, i.e. trajectories and velocities of various points of markers, placed on the human body segments.

3. *Realization of human motion (the change of human interactions with the elements of its environment) by changing the body structural parameters (partial structural deformation) on the predefined time interval.*

The process is realised in the following way.

For *biomechanical kind of human motion* the changes take place inside the human bio-kinetic apparatus structure σ_τ^ℓ without the influence of external effects (not taking into account gravitation forces); structural connections ${}_\sigma\gamma_\tau^\ell$ are weakened in one place and become stronger in another one, the general metric characteristic ${}^0\mu_\tau^\ell$ of human bio-kinetic apparatus S_τ^ℓ remains the same; the change of structural connections ${}_\sigma\gamma_\tau^\ell$ causes the change of external interconnections ${}_\omega\gamma_\tau^\ell$ (the change of human body S_τ^ℓ location in environment structure $\sigma^{\ell+1}$, i.e. human motion); the consecutive changes of structural connections ${}_\sigma\gamma_\tau^\ell$ on time interval simulate the deformation evolution in human bio-kinetic apparatus structure σ_τ^ℓ and correspondent human movement in its environment system, caused by the deformations of structure σ_τ^ℓ.

The relations between the variations of human structure σ_τ^ℓ changes and the kinds of human motion are set by the coordinator. So, the task of human motion design – coordination task of the connected human movements and its bio-kinetic apparatus deformations – is reduced by the proposed conceptual design method to the coordination task of standard *aed* block of hierarchical systems. Formally, human motion task performing is presented with the help of $(\overline{\rho},\overline{\varphi})$ functions as follows:

$$\rho_0^\ell : C_0^\ell \times \sigma_t^\ell \to \sigma_{t'}^\ell \;\&\; \sigma_{t'}^\ell = Y_0, \sigma_t^\ell = X_0, \tag{7.6}$$

$$\varphi_{tt'}^\ell : \sigma_t^\ell \times {}_\sigma\gamma_{tt'}^\ell \to \sigma_{t'}^\ell,$$

i.e. in the form of coordinator reaction ρ_0^ℓ, and in the form of state transition function $\varphi_{tt'}^\ell$ of the human system, where σ_t^ℓ and $\sigma_{t'}^\ell$ are the states of human bio-kinetic structure at moments of time *t* and *t'* respectively, *t'>t;* ${}_\sigma\gamma_{tt'}^\ell$ is the change of structural connections (joint angles) during the period of time *tt'*; C_0^ℓ coordinator states, i.e. the states of computer control program. The exemplary task of computer design of human sitting-standing-up motion is described below.

7.3.2. Computer design of human sitting-standing-up movements

The following design steps of human sitting-standing-up movements are coordinated with the general algorithm of human motion design presented in Subsection 7.3.1.

At the first step, the construction of human bio-kinetic apparatus σ^{ℓ} is synthesized by coordinator from the apparatus elements $\overline{\omega}^{\ell-1}$ of lower level $\ell-1$ (see Fig. 7.1). This process is executed according to the first step of the algorithm presented above in Subsection 7.3.1. During the synthesis process the human 3D apparatus is constructed as a complex of its details $\overline{\omega}^{\ell-1}$ of δ^{ℓ}=0010 constructive dimensions, i.e. conditional lines – limbs, which are created independently from conditional points, i.e. cubes (Fig. 7.2). The details have incoherent scales (sizes) and locations. Computer HS technology of synthesis in the given case only demands the indication of common (in the resultant bio-kinetic construction) details (joints) of the units, i.e. limbs, being united. The other details are restored from the indicated ones thanks to their connections defined before. As a result of the uniting technology realization, coordinator creates a connected unit of human bio-kinetic apparatus with coherent scales and locations of its details. This unit can be reconstructed by any of its details.

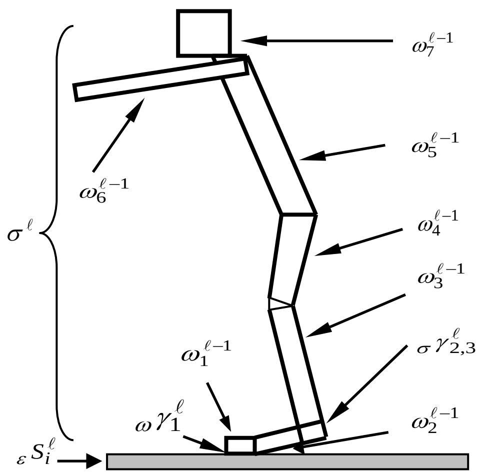


Fig. 7.1. The structure σ^{ℓ} of human body (bio-kinetic apparatus), $\omega_i^{\ell-1}$ are structural elements, $_{\omega}\gamma^{\ell}$ external and $_{\sigma}\gamma^{\ell}$ structural connections, part of environment system $_{\varepsilon}S_i^{\ell}$ is a base-surface

Synthesized human bio-kinetic apparatus structure σ^{ℓ} contains aggregated representations $\overline{\omega}^{\ell-1}$ of lower level elements (thigh $\omega_4^{\ell-1}$, shank $\omega_3^{\ell-1}$, foot $\omega_2^{\ell-1}$, fingers $\omega_1^{\ell-1}$, body $\omega_5^{\ell-1}$, arm $\omega_6^{\ell-1}$, head $\omega_7^{\ell-1}$), and their connections $_{\sigma}\gamma^{\ell}$ – joints (Fig. 7.1). For example, the common part of the foot $\omega_2^{\ell-1}$ and the shank $\omega_3^{\ell-1}$ is their structural connection, i.e. heel $_{\sigma}\gamma_{2,3}^{\ell}$. One element of environment $_{\varepsilon}S_i^{\ell}$ is the pathway on the base surface. The external connection $_{\omega}\gamma_1^{\ell}$ of man is the contact place of his foot with the pathway on the base surface.

At the second step, sitting-standing-up motion parameters are defined as the input data of the computer program. In this process the external connection $_{\omega}\gamma_1^{\ell}$ of man (contact place of its foot with the base surface) is constant value, i.e. $_{\omega}\gamma_1^{\ell}$ =const. Velocity of interaction change (angular velocity of knee joint) is greater than ones of heels and other joints. So, joints angular velocities ω are defined in the program with the help of weights of joint angels change as follows:

$a1(i)=a1(i)+w1*a0$,

where: $a0$ is a minimal angular change on each i step (discrete time moment), $w1$ is weight, $a1(i)$ is a meaning of the value of angle a of 1 joint at i discrete moment of time. Acceleration ε is presented in the program as a change of velocity $w1(i)$, i.e. weight, at each time step. The inertia moment of the bio-kinetic apparatus n segment (see Fig. 7.2) is defined as follows:

$$I_x = \Sigma m_i h_i^2 \text{ or } I_x = \int_V \rho h^2 dV , \tag{7.7}$$

where ρ is mass density and V is the volume of a segment element, i.e. conditional point.

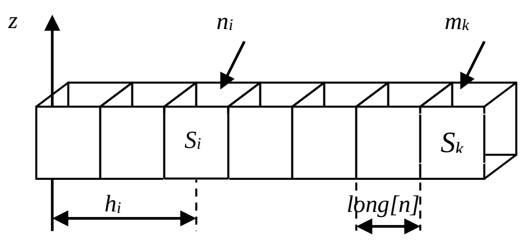


Fig. 7.2. Structure of human body *n* segment (conditional lines) synthesized from conditional points S_i (cubes); m_i is the mass of *i* conditional point S_i; $h_i = (i-1)*long[n]+0,5*long[n]$ is S_i distance from *z* axis, $k=lip[n]$ is the number of the conditional points in structure of the body segment *n*

In the computer program the inertia moment is defined as follows:

```
for i:=1 to lip[n] do
begin
write('enter mass of I element of n body segment?:');
readln(mas[i,n]);
end;
    mis[n]:=mas[1,n]*0.5*long[n]*0.5*long[n];
    for i:=1 to lip[n] do begin
    mis[n]:=mis[n]+(i-1)*mas[i,n]*((i-1)*long[n])*((i-1)*long[n]);
```

where lip[n] is a number of elements (conditional points) in the construction of *n* segment of the bio-kinetic apparatus, long[n] is the length of the edge of each conditional point of *n* segment, mas[i,n] is a mass of *i* element of *n* body segment, mis[n] is inertia moment of apparatus segment *n*.

The torque of each segment of bio-kinetic apparatus is defined in the standard form:

$$I_x\varepsilon=M_z \tag{7.8}$$

and in the computer program:

```
Torque:=w1(i)*mis[n].
```

The other parameters, i.e. way of interconnections (joint angles) changing, velocities, trajectories of segments of human body are also defined. This input data for sitting-standing-up kinds of human movements necessary for computer motion design was obtained from experiments (see Fig. 7.6), using Optotrak 3020 motion capture system (Northern Digital Inc., Canada).

At the *third step*, a human sitting-standing-up motion for one cycle is realized. One cycle contains the change of bio-kinetic apparatus structural parameters, i.e. joint angles, from their initial to maximal values (sitting) and vise versa (standing-up).

Design of similar sitting-standing-up motions is executed by changing input parameters, such as limb length, mass, etc. Design process is visualized, so it is possible to make visual control of design process by observing the computer movie. Prediction of another kind of movements is conducted by setting of arbitrary parameters, such as limbs lengths, weights and joint angles. The predicted results of computer motion design, e.g. trajectories and velocities of various points of bio-kinetic apparatus segments, are compared with the experimental laboratory results, i.e. trajectories and velocities of various points of markers placed on the subject’s body.

The above algorithm is realized as a computer program and is used in the computer experiment of human sitting-standing-up movements design. The computer experiment contains the following particular steps:

1) definition of the program input parameters: length of human body segments (lip[n]*long[n]; lip[n] is a number of elements (conditional points) in the construction of the body segment *n*, long[n] is length of the edge of each conditional point of segment *n*); mass of the segments elements (lip[n]*m[i] is a mass of the body segment *n*, m[i] is elementary mass of *i* conditional point of segment *n*); coefficients of joints angels change wi in the elementary time interval;

2) realization of motion design, i.e. simulation of human motion and visualization of the process;

3) calculation of the trajectories of definite points of human body 3D segments (hand, arm, leg, head), correspondent displacements, and velocities – analogous of a forward kinematics task;

4) calculation of relative angular velocities of joints of human bio-kinetic apparatus;

5) program simulation of various angular accelerations of joints and calculation of correspondent inertia moments and toques;

6) comparison of the computer design data with the experimental results obtained from motion capture system in the laboratory environment;

7) model validation;

8) computer design of human sitting-standing-up motion for another input data;

9) computer design and prediction of other kinds of human motion by defining other conditions and arbitrary parameters of motion based on previous skills.

The first step corresponds to the coordinator (model) *learning* process when real parameters obtained from in vivo laboratory experiment are introduced to the computer model. The last four steps 6-9 presented above correspond to the stages of model modification (adaptation) on the *self-organization* strata of the coordinator. The other steps of the computer design experiment are the *selection* strata functions (see Chapters 3 and 4).

The results of human sitting-standing-up movements design, i.e. four images *a*)-*d*) of the exemplary four states of geometric construction of human bio-kinetic apparatus obtained from a computer monitor in the process of motion design, are presented in Figs. 7.3. The forward kinematics task is performed as accompanying detailed design task of general task of motion design within the frameworks of one common computational model. The data of human arm end-point trajectory is presented in Fig. 7.4 as a result of exemplary forward kinematics tasks performance. In comparison with the traditional forward kinematics tasks the motion parameters can be calculated for any point of 3D construction of human body.

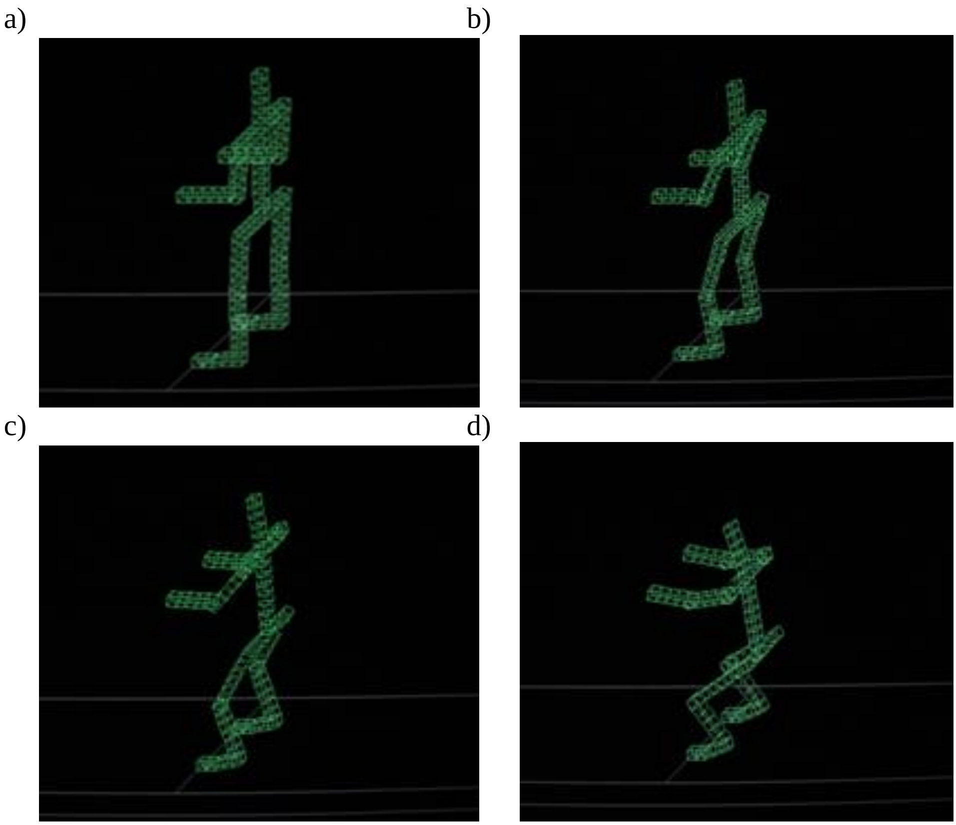
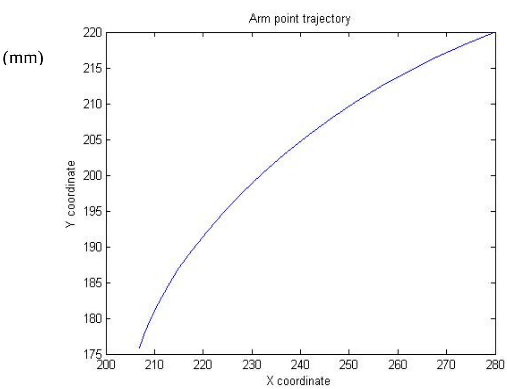


Fig. 7.3. Four states a), b), c) and d) of geometric construction of human bio-kinetic apparatus in sitting-standing-up motion – obtained from computer monitor in process of motion design



x=[280, 273.946, 268.011, 262.224, 256.613, 251.202, 246.018, 241.082, 236.416, 232.039, 227.969, 224.222, 220.812, 217.748 215.041, 212.696, 210.717, 209.107, 207.864, 206.985]

y=[220, 218.356, 216.543, 214.57, 212.451, 210.198, 207.828, 205.356, 202.802, 200.187, 197.532, 194.86, 192.194, 189.56, 186.98, 184.48, 182.087, 179.82, 177.72, 175.8]

Fig. 7.4. Data obtained in a computer motion design experiment – the analogous of the classic forward kinematics task: *x* and *y* are coordinates (mm) of human arm end-point for discrete time moments and corresponding trajectory plot for the exemplary motion

The graphic user interface (GUI) of the developed computer program is presented in Fig. 7.5. All computer program modules were developed by Delphi 7 program system using Pascal language. MATLAB system is used for data analysis and graphic presentation of the experimental results. The analysis conducted at the last stages of the study shows the coherence of the results of computer motion design and the data obtained in a laboratory experiment of human sitting-standing-up movements registration using Optotrak 3020 motion capture systems, i.e. coherence of joints angular changes, trajectories, and velocities of various points of human bio-kinetic apparatus. It shows the validity of both the conceptual and computer models developed. The description of the experiment of the human movement fixation and analysis using motion capture system is presented below.

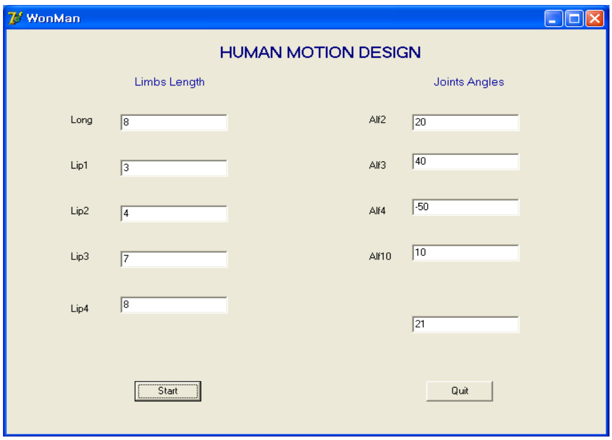


Fig. 7.5. GUI window of human motion design program; the input parameters are human limbs lengths *Lip*(*i*) given by number of elementary length units (*long*) of each limb, and joints angular changes *Alf*(*i*) (*deg*); the program output parameters are the trajectories and velocities of various limbs points; the output parameters are written in corresponding program data files; human motion design and visualization processes are accompanied by the performing of forward kinematics task

7.4. Description of the laboratory experiment

The main aim of the laboratory experiment on human motion is to obtain the real motion data which can be used in computer design and learning process as input data of a computer model. The data is also necessary for model modification (self-organization) purposes. Therefore, the experiment is performed to obtain the coordinates of trajectories of the human body points selected (indicated by markers) during the process of sitting-standing-up motion of different groups of subjects –

people of various height, mass, limb length, definition of relative joint angles change, i.e. angular coefficients, which allow calculation of displacements, angular velocities and accelerations, and comparison of the data obtained with the results of computer human motion design.

Protocol of the laboratory experiment contains the following steps.

1. Choice of the criteria (characteristics are limb length, height, mass, number of persons – subjects) for selecting an experimental group.

At this step the criteria for selecting groups of subjects being examined are formulated. The following groups G1 and G2 of subjects were chosen according to the predefined height criteria: 165-170 cm. for G1, and 175-180 cm. for G2. There are 3 subjects in each group.

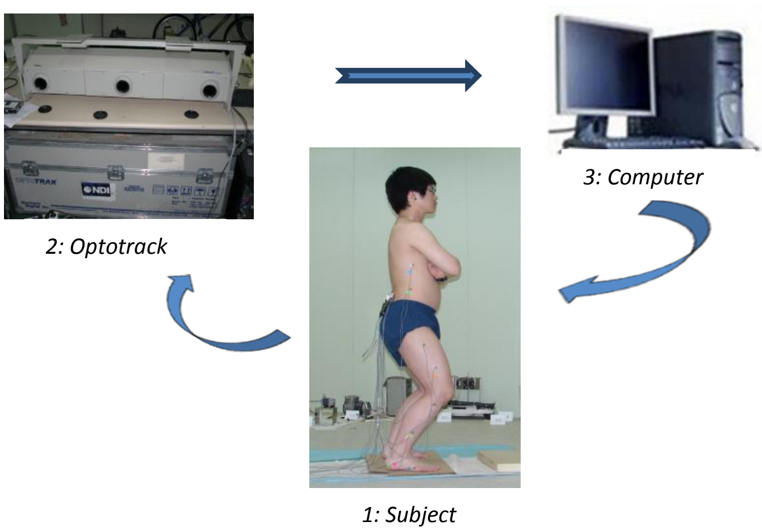


Fig. 7.6. Experiment scheme, where 1 is a subject – person being examined with markers placed on his body, 2 is Optotrack camera, 3 is a computer, arrows are interconnections; *measured values* are coordinates of markers; *calculated ones:* displacements, angles, velocities, accelerations

2. Definition of the standard sitting-standing-up posture.

The way of sitting-standing-up motion (way of legs, arms, and back positions changing) performed by the subjects being examined was defined. All the experiments were conducted with subjects standing in their initial position upright on the floor with heels slightly apart (see Figs. 7.6-7.8) and parallel and elbows held close to the torso. Their back was straight vertical. The arms were bent in the elbow joints. Sitting-standing-up process was executed uniformly with little delay in lower down position. Heels were taken off the floor. Body was slightly bent forward.

3. Definition of the experiment conditions.

The following conditions of the experiment were defined: time of the sitting-standing-up process (regulated during the registration process with the help of a metronome) is 20 s.; the values being measured are coordinates (trajectories) of markers placed on the predefined body points (Fig.7.9); scales of the measurements: discrete moments on the time interval of 1/30 sec., scale of the measured space is 3x3 m.; number of observations for each subject is 3.

4. Calibration of the Optotrack 3020 motion capture system.

Optotrack motion capture system was set according to the predefined experiment scheme (see Figs. 7.6 and 7.7).

Fig. 7.7. One state of the laboratory experiment process: subject, i.e. person being examined, markers placed on the shank, and camera of Optotrack motion capture system

5. Choice of the places of markers.

First, markers number 10, 11, and 12 were placed to define sagittal plane and floor base surfaces (Fig. 7.8). After that, the other markers were placed in the definite points of human body to execute further measurements: two markers on foot, and two on each limb, i.e. shank, thigh, leg, subject's body side.

6. Execution of the measurements and data obtaining.

Data acquisition of markers coordinates is performed at each predefined moment of time. The markers trajectories are built after that. The data collected have been given to MATLAB program system environment for further processing and graphic presentation. Various markers coordinates in numeric format (Fig. 7.9) and their trajectories in graphic format (Fig. 7.10) are plotted in MATLAB system.

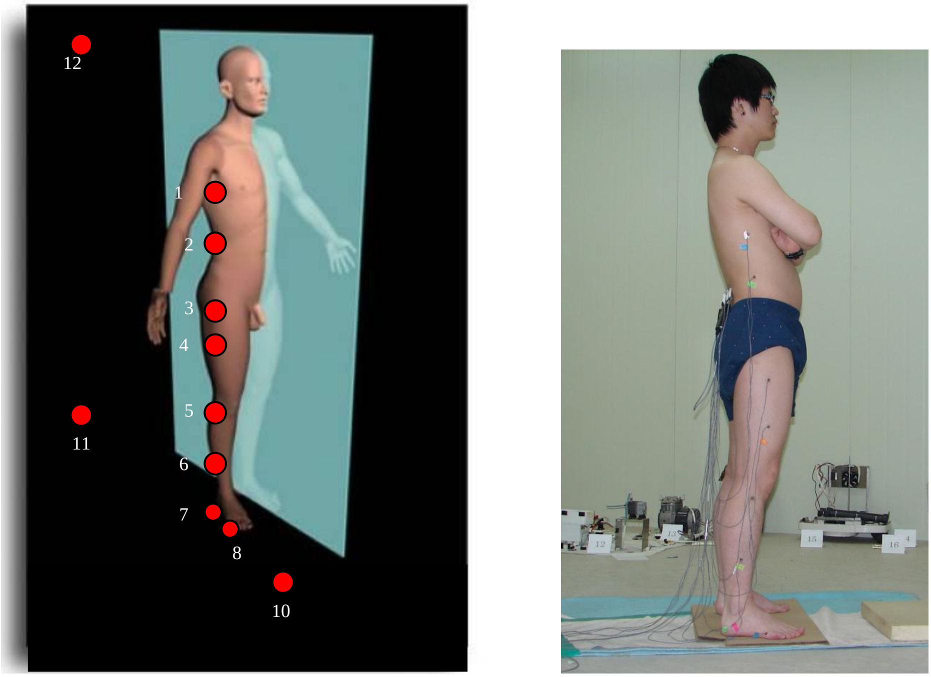


Fig. 7.8. Places of markers and subject being examined

7. Measurement results processing.

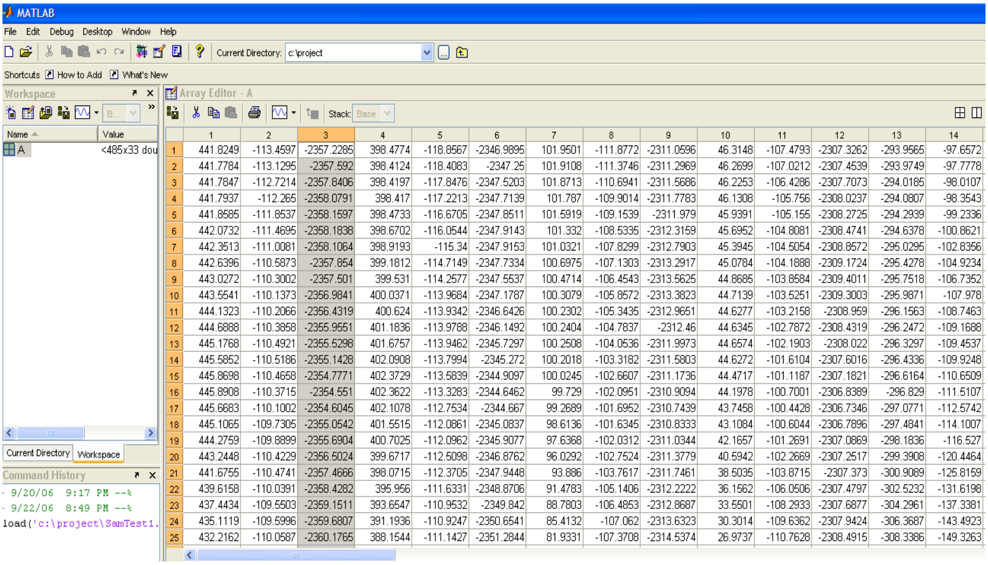
The representative data is calculated using the dependence:

$$X_j(t)=\Sigma X_j(t)i/n, \tag{7.9}$$

where $X_j(t)$ is X coordinate change of j marker, $X_j(t)i$ is the X coordinate change of j marker obtained in i observation, n is the number of observations.

8. Calculation of the joint rotation centers, limbs lengths as the distance between the joints rotation centers of correspondent limbs.

This procedure is executed by using the known mathematical relations. First, the rotation centers are defined as the point of intersection of two lines each of which is defined by two markers. After that, the limbs lengths are defined as a distance between the rotation points.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	441.8249	-113.4597	-2357.2285	398.4774	-118.8567	-2346.9895	101.9501	-111.8772	-2311.0596	46.3148	-107.4793	-2307.3262	-293.9565	-97.6572
2	441.7784	-113.1295	-2357.592	398.4124	-118.4083	-2347.25	101.9108	-111.3746	-2311.2969	46.2699	-107.0212	-2307.4539	-293.9749	-97.7778
3	441.7847	-112.7214	-2357.8406	398.4197	-117.8476	-2347.5203	101.8713	-110.6941	-2311.5686	46.2253	-106.4286	-2307.7073	-294.0185	-98.0107
4	441.7937	-112.265	-2358.0791	398.417	-117.2213	-2347.7139	101.787	-109.9014	-2311.7783	46.1308	-105.756	-2308.0237	-294.0807	-98.3543
5	441.8585	-111.8537	-2358.1597	398.4733	-116.6705	-2347.8511	101.5919	-109.1539	-2311.979	45.9391	-105.155	-2308.2725	-294.2939	-99.2336
6	442.0732	-111.4695	-2358.1838	398.6702	-116.0544	-2347.9143	101.332	-108.5335	-2312.3159	45.6952	-104.8081	-2308.4741	-294.6378	-100.8621
7	442.3513	-111.0081	-2358.1064	398.9193	-115.34	-2347.9153	101.0321	-107.8299	-2312.7903	45.3945	-104.5054	-2308.8572	-295.0295	-102.8356
8	442.6396	-110.5873	-2357.854	399.1812	-114.7149	-2347.7334	100.6975	-107.1303	-2313.2917	45.0784	-104.1888	-2309.1724	-295.4278	-104.9234
9	443.0272	-110.3002	-2357.501	399.531	-114.2577	-2347.5537	100.4714	-106.4543	-2313.5625	44.8685	-103.8584	-2309.4011	-295.7518	-106.7352
10	443.5541	-110.1373	-2356.9841	400.0371	-113.9684	-2347.1787	100.3079	-105.8572	-2313.3823	44.7139	-103.5251	-2309.3003	-295.9871	-107.978
11	444.1323	-110.2066	-2356.4319	400.624	-113.9342	-2346.6426	100.2302	-105.3435	-2312.9651	44.6277	-103.2158	-2308.959	-296.1563	-108.7463
12	444.6888	-110.3858	-2355.9551	401.1836	-113.9788	-2346.1492	100.2404	-104.7837	-2312.46	44.6345	-102.7872	-2308.4319	-296.2472	-109.1688
13	445.1768	-110.4921	-2355.5298	401.6757	-113.9462	-2345.7297	100.2508	-104.0536	-2311.9973	44.6574	-102.1903	-2308.022	-296.3297	-109.4537
14	445.5852	-110.5186	-2355.1428	402.0908	-113.7994	-2345.272	100.2018	-103.3182	-2311.5803	44.6272	-101.6104	-2307.6016	-296.4336	-109.9248
15	445.8698	-110.4658	-2354.7771	402.3729	-113.5839	-2344.9097	100.0245	-102.6607	-2311.1736	44.4717	-101.1187	-2307.1821	-296.6164	-110.6509
16	445.8908	-110.3715	-2354.551	402.3622	-113.3283	-2344.6462	99.729	-102.0951	-2310.9094	44.1978	-100.7001	-2306.8389	-296.829	-111.5107
17	445.6683	-110.1002	-2354.6045	402.1078	-112.7534	-2344.667	99.2689	-101.6952	-2310.7439	43.7458	-100.4428	-2306.7346	-297.0771	-112.5742
18	445.1065	-109.7305	-2355.0542	401.5515	-112.0861	-2345.0837	98.6136	-101.6345	-2310.8333	43.1084	-100.6044	-2306.7896	-297.4841	-114.1007
19	444.2759	-109.8899	-2355.6904	400.7025	-112.0962	-2345.9077	97.6368	-102.0312	-2311.0344	42.1657	-101.2691	-2307.0869	-298.1836	-116.527
20	443.2448	-110.4229	-2356.5024	399.6717	-112.5098	-2346.8762	96.0292	-102.7524	-2311.3779	40.5942	-102.2669	-2307.2517	-299.3908	-120.4464
21	441.6755	-110.4741	-2357.4666	398.0715	-112.3705	-2347.9448	93.886	-103.7617	-2311.7461	38.5035	-103.8715	-2307.373	-300.9089	-125.8159
22	439.6158	-110.0391	-2358.4282	395.956	-111.6331	-2348.8706	91.4783	-105.1406	-2312.2222	36.1562	-106.0506	-2307.4797	-302.5232	-131.6198
23	437.4434	-109.5503	-2359.1511	393.6547	-110.9532	-2349.842	88.7803	-106.4853	-2312.8687	33.5501	-108.2933	-2307.6877	-304.2961	-137.3381
24	435.1119	-109.5996	-2359.6807	391.1936	-110.9247	-2350.6541	85.4132	-107.062	-2313.6323	30.3014	-109.6362	-2307.9424	-306.3687	-143.4923
25	432.2162	-110.0587	-2360.1765	388.1544	-111.1427	-2351.2844	81.9331	-107.3708	-2314.5374	26.9737	-110.7628	-2308.4915	-308.3386	-149.3263

Fig. 7.9. Coordinates of various marker presented in the MATLAB table in numeric format

9. Calculations of correspondent displacements, angular velocities and angular coefficients for each group of subjects being examined.

To perform the calculations, special program functions were written in MATLAB. One of the functions for joints angles calculation is presented in the following form:

```
for i=1:593
alphaSL(i)=atan((A(i,10)-A(i,7))/(A(i,11)-A(i,8)));
betaSL(i)=atan((A(i,16)-A(i,13))/(A(i,17)-A(i,14)));
fiSL(i)=alphaSL(i)+betaSL(i);
end;
plot(fiSL)
fiSL
```

where i is the number of registrations of markers coordinates placed on the subject's body; $A(i, j)$ are fields of markers coordinates written in correspondent data file while performing the laboratory experiment; *fiSL* is a correspondent ankle joint angle between two limbs of human bio-kinetic apparatus – foot and leg in this case. The results of relative angles calculation using the function presented above and other functions are presented in Fig. 7.10.

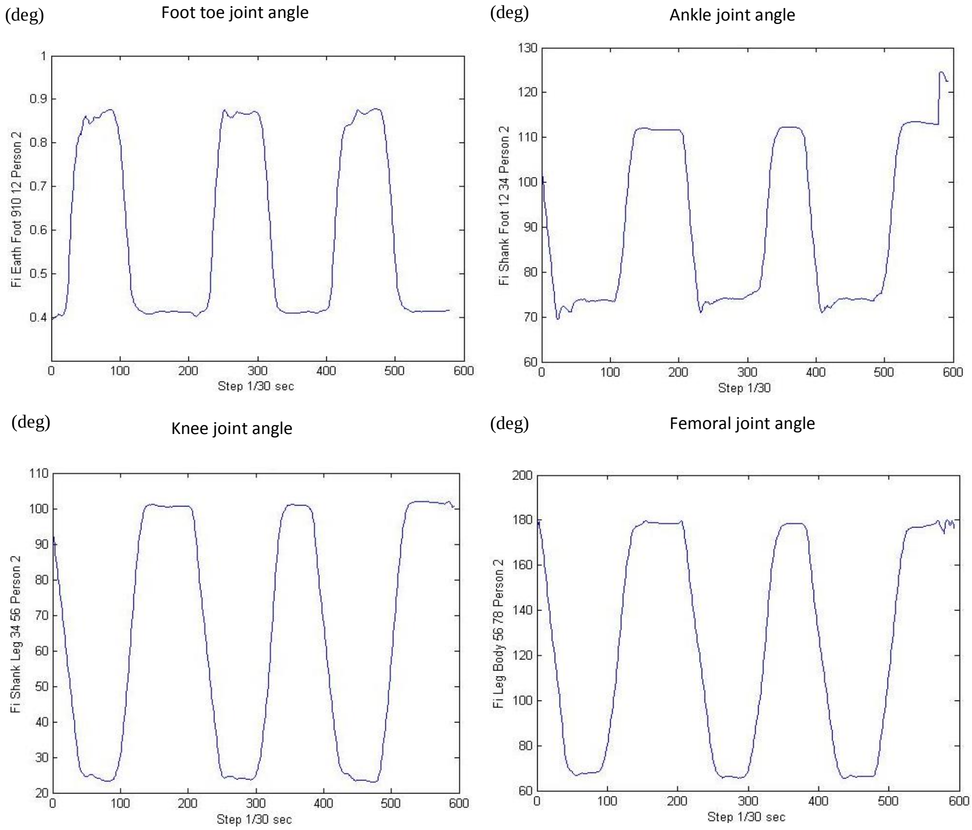


Fig. 7.10. Human joints angular change – while performing sitting-standing-up motion – calculated as a result of experimental data processing by program functions developed in MatLab system environment; values of *Fi* angular change for each joint are given in degrees

10. Comparison of experimental and computer design data and prediction, i.e. design, of other kinds of human sitting-standing-up motion.

The results of computer motion design and experimental data obtained by Optotrack system for human sitting-standing-up motion were compared and analyzed. Prediction of motion is conducted for virtual subject with other limbs parameters. The results of prediction are compared with the data obtained from the laboratory experiment with real subject with the same parameters (see Fig. 7.11). Positive results of the analysis allows the validity to be stated of both conceptual and computer models developed within the frameworks of the suggested HS design method.

a) b)

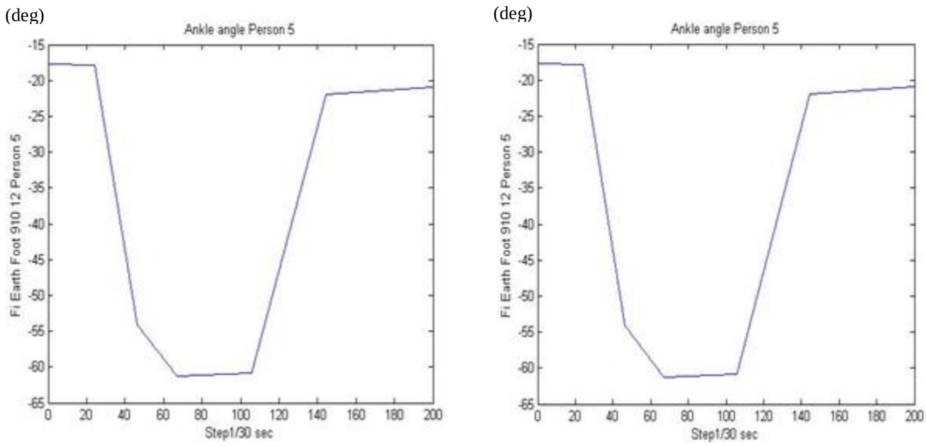


Fig. 7.11. Data analysis results: a) results of computer motion design for virtual subject with predefined limbs parameters – program output data of exemplary ankle joint angular change of human bio-kinetic apparatus, and b) data of ankle joint angular change obtained from laboratory experiment with real subject with the same limb parameters – are given in both plots a) and b) after interpolation procedure; values of *Fi* angular change are given in degrees

Preliminary conclusions and results

The main aim of the task described in this Chapter was HS conceptual model and design method application in the design and control tasks of human motion. As a result, a new HS method for human motion design was developed. The method presents all the connected levels, i.e. human body structure (lower level), human body dynamic representation as a unit in its environment, the environment structure (higher level), in common HS formal basis and takes into account geometric, numeric, biomechanical and physical characteristics. The method developed allows performing of the motion design task more sufficiently in comparison with traditional methods because of the functional abilities of the HS design method, such as reconstructing the whole object being designed from its any structural element, coordination of human movements and bio-kinetic apparatus deformations within the frameworks of one common formal model, prediction of human motion, learning and others. The aim of the experimental study was to answer the question if the computer design method developed corresponds to the real process of human motion, in our case to human sitting-standing-up movements, and if it is possible to predict and coordinate (design and control) the human motion using the developed program modules.

To reach these aims, theoretical basis of the method suggested was described in the work using hierarchical dynamic systems and *aed* coordination technology. Coordination task of human motion was formulated, ways of motion design and the method steps were defined, systems movements and deformations were described within the frameworks of the suggested HS theoretical basis. Algorithm of computer design and steps of the method realization for human sitting-standing-up motion were given. Experimental study of human motion (using motion capture system) and computer design with the motion prediction were performed. Comparison of experimental and computer simulation results as well as the method analysis were given.

The computer motion design was performed using a geometric model of human bio-kinetic apparatus constructed from conditional geometric objects (see Chapter 4). The objects were presented according to HS *aed* model. The design process of human sitting-standing-up motion was performed using initial experimental data of one subject with definite parameters – limbs length and weights. Prediction of motion was conducted for a virtual subject with other limbs parameters. The results of computer prediction were compared with the data obtained in the laboratory experiment with real subject with the same parameters. Coherence of computer design and laboratory experiment results shows the efficiency of the suggested HS design method. Exemplary analogous task of the traditional forward kinematics task have been performed as the accompanying task within the frameworks of the suggested method as a result of human sitting-standing-up motion design and simulation.

As a result, new conceptual formal model and HS design method of human motion coordination – design and control – have been created. The design method has the following characteristics:

1) describes a human bio-kinetic apparatus as a hierarchical system; it allows the design and control task of human motion to be performed as interconnections coordination of coordinated structural and dynamic representation of man in its environment;

2) makes possible designing (synthesis) of different kinds of human movements, predicts the motion and obtains correspondent motion data using only the geometrical or biomechanical data such as limbs length and weights; consecutive interactive performing of the analysis design task, i.e. changing human structural interactions (joint angles) and its subsystems characteristics – limbs length and mass, and observation of the change of motion process being designed executed by man in its environment (higher level system) allows obtaining the desired characteristics of this process, i.e. trajectories coordinates, displacements, velocities, etc.;

3) connects three levels in common formal basis, such as the presentation of human structure (bio-kinetic apparatus) and structural deformations (limbs motion and joints angular change), description of human motion in environment (change

of human interactions with subsystems of environment), and construction of environment (set of subsystems of higher level);

4) performs the connected tasks of human body deformations and human motion in its environment cased by these deformations within the frameworks of one common HS *aed* model not using traditional approaches, i.e. Finite Elements method and Integral Differential equations; allows performing of motion synthesis and analysis tasks within the frameworks of one common computer model - software construction.

5) allows presentation and processing of information of biomechatronic subsystems of various nature – mechanical, electronic, biomechanical, computer – in frames of one common HS *aed* formal basis, what is very important in the case of bio-mechanical, robotics, medical, rehabilitation and other systems coordination and allows avoiding of modelling conflicts of various subsystems of different nature;

6) makes possible the modification of the developed conceptual and computer models on the self-organization strata of coordinator, and motion prediction using controlled experimental data;

7) presents the parts of human body in computer design as connected 3D objects due to the operations with HS models of conditional geometric objects (see Chapter 4);

8) allows performing geometric design and forward kinematics tasks as a common computational task owing to the description of geometric construction of bio-kinetic apparatus, coordination (design and control) procedures and kinematic characteristics within the frameworks of one common conceptual model.

Computer program created for human motions design has been constructed according to HS structure, and the program functioning corresponds to the actions of *aed* coordinator of HS. The design tasks are realized using commercial software Delphi (Embarcadero Technologies Inc., CA, USA) and Matlab (The MathWorks Inc., MA, USA) systems. All calculations effectively executed on computer as operations with systems quantitative characteristics are presented in numeric positional code. During the motion design, the dependences of functional parameters (kinds of motions) on the structural ones (sizes, angles) of a human body segments were determined.

Creation of the HS theoretical basis, experimental study and computer simulation of human motion are parts of the development process of *aed* conceptual model and HS method of human motion design. Theoretical constructions made possible the development of computer program modules which realize the technology of the method. Experimental measurements obtained by the O*ptotrack* motion capture system and the results of simulation (computer experiment) of sitting-standing-up human motion allowed the data comparison and the model validation. The results of the conducted experimental study show the correspondence of the HS design method and computer model developed to real processes of human motion and wide abilities of HS design method in motion design.

8. DESIGN AND TESTING OF ELECTRONIC CIRCUIT BOARDS

The conceptual model for design of an electronic Printed Circuit Boards (PCBs) and HS technology of PCB quality testing is presented in this Chapter. The processes of the PCB conceptual model creation and PCB quality testing take place at the design and production phases of the general technological process of PCB life cycle (see Fig. 1.2, Chapter 1). PCB conceptual design is realized in the industrial PCB production process by human-computer CAD system coordinated by design engineer. The quality of circuit boards is usually tested by a visual system – the system of machine vision. To perform the PCB design and testing tasks, various methods of PCBs synthesis and analysis are used nowadays for each particular case [81-87].

The processes of synthesis and visual testing (analysis) of geometric constructions of PCBs conductive pathways is described in this book using the proposed conceptual design method using *aed* formal model and HS technology (see Chapters 3 and 4). The task is also partially described in [88].

In the technological process of PCB production it is a usual situation for PCB surface to have topological damages, such as conductive pathways breaks or joints. So, the final stage of the technological process is the analysis of the PCBs quality. Usually, the testing is realized as a video analysis, which contains the following stages:

1) PCB graphic (standard) image creation,
2) obtaining a video-pattern of a tested PCB in one of graphic formats,
3) comparison of the obtained pattern with the standard image,
4) identification of PCB quality in the case of correspondence of a graphic image of PCB to its standard and vice versa.

This approach, being based on standard methods of presentation and processing of video-information (graphic images), has several drawbacks. The main of them is the lack of coincidence between a graphic image of the board and the PCB standard, which can occur during the third stage and is not caused by the quality of a produced PCB. The reason is the changes of coordinates and the scale of the graphic image of PCB obtained at the second stage, which is sometimes accidentally caused by:

– the change of product's (PCB) orientation in relation to the video camera;

– the change of the scale, i.e. distance between the video camera and the tested PCB.

In each of these two cases the images of the standard and PCB being tested do not coincides. As a result, a lot of PCBs of a good quality are recognized (at the fourth stage) as defective goods.

The proposed conceptual design method is implemented in the form of a computer program and allows the mentioned drawbacks to be avoided. It has the following steps:

1) geometric synthesis of standard of PCB – using HS conceptual design method – with the formation of its graphic image (this stage is the part of the PCB design phase as well),

2) obtaining a video-pattern – graphic image of the PCB being tested – in one of graphic formats,

3) matching of the standard to the obtained image of PCB – coordination of the PCBs orientation and scale,

4) comparison of the obtained image with PCB standard and pathways quality testing,

5) PCB quality assessment.

Each of the above steps differs from the steps of a wide-spread technologies of the task performing. For example, the task of geometric synthesis of the PCB standard (1st step) is a part of the general PCB design task and corresponds to the phase of geometric design of circuit boards.

Computer program realization of the PCB conceptual design method was used in tasks of circuit boards design and testing at Horizont enterprise (Minsk, Belarus). The scheme of the mechatronic unit – technological stand with machine vision system – for quality testing of conductive pathways of PCBs is presented in Figure 8.1.

8.1. Geometric construction of circuit board

Geometric synthesis of the construction of PCB standard is the PCD design task which is performed at both the conceptual design (CD) and detailed design (DD) phases of the PCB life cycle (see Fig.1.2, Chapter 1**).** At the same time, it (geometric synthesis of PCB standard) is the first stage of the HS design method realization for visual analysis – quality testing – of conductive pathways of electronic printed circuit boards (PCBs).

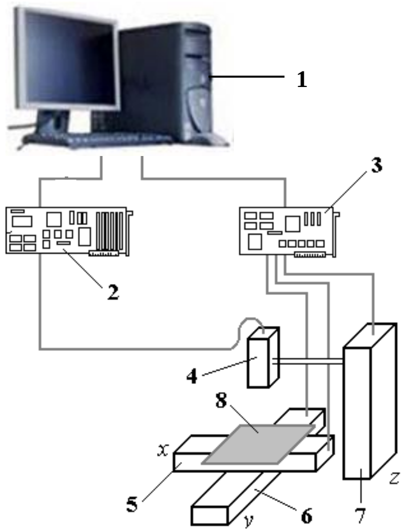


Fig. 8.1. Technological stand with mashine vision unit for PCB quality testing: 1 – personal computer; 2 – video camera control board; 3 – controller of coordinates; 4 – video camera; 5 – drive of coordinate *X*; 6 – drive of coordinate *Y*; 7 – drive of coordinate *Z*; 8 – circuit board being tested

Geometric construction of conductive pathways of PCBs is presented in accordance with (4.1) in the following (8.1) form:

$$ {}_g S^{\ell} = {}_g \{\omega^{\ell}, S_0^{\ell}, \sigma^{\ell}\}, \tag{8.1}$$

where: ω^{ℓ} – aggregated description of the construction as a unit in its environment,

σ^{ℓ} – structure of interconnections,

S_0^{ℓ} – coordinator, which connects σ and ω, and therefore the higher and lower levels of the PCB construction,

g – index, which shows the geometrical nature of the mechatronic system under consideration.

Structure σ^{ℓ} is defined according to (4.3) as follows:

$$\sigma^{\ell} = \{S_0^{\ell}, \{\overline{\omega}^{\ell-1}, {}_{\sigma}\gamma^{\ell}\}\}, \tag{8.2}$$

and contains aggregated representations $\overline{\omega}^{\ell-1}$ of PCB geometric elements of the lower level, their interconnections ${}_{\sigma}\gamma^{\ell}$ and coordinator S_0^{ℓ} which sets relations between the elements.

Coordinator is presented according to (3.15) in the following (8.3) form:

$$S_0^{\ell} = \{\omega^{\ell}, S_0^{\ell}, \sigma^{\ell}\}_0, \qquad (8.3)$$

and in this case is implemented by a computer program as a law of interlevel connections and as rules in accordance with which the geometric elements were connected in geometric constructions of PCBs on different levels.

Quantitative and metrical characteristics of elements $\overline{\omega}^{\ell-1}$, i.e. conductive pathways, and the addresses of their connections ${}_{\sigma}\gamma^{\ell}$ are defined in codes of the numeric positional system (4.9-4.14).

Geometric construction of PCB conductive pathways is created by S_0^{ℓ} using one of its coordination strategies (see Chapters 3 and 4). According to (4.4), coordinator synthesizes the structure of pathways from subsystems $\overline{\omega}^{\ell-1}$ of three levels:

$$S_0^{\ell} : \{\overline{\omega}^{\ell-1}, {}_{\sigma}\gamma^{\ell}\} \to \sigma^{\ell}, \qquad (8.4)$$

where $\ell = \ell$ -2, ℓ -1, ℓ .

On level ℓ -2 the parts of conductive pathways are constructed from conditional points $\omega^{\ell-3}$ presented in the form of quadrates (Fig. 8.2). Number n and sizes $\mu_1^0 = w$ of ℓ -3 level elements define width $\mu_1^1 = w$ and length μ_2^1=l=wn of ℓ -2 level segment of pathway.

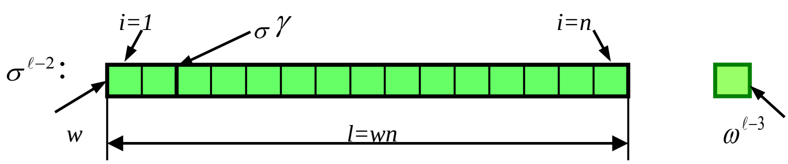


Fig. 8.2. Structure $\sigma^{\ell-2}$ of a pathway segment constructed from conditional points $\omega^{\ell-3}$

This process is described at conceptual design (CD) phase as follows:

$$\ell = \ell -2, \qquad S_0^{\ell-2} : \{\overline{\omega}^{\ell-3}, {}_{\sigma}\gamma^{\ell-2}\} \to \sigma^{\ell-2}. \qquad (8.5)$$

Structure $\sigma^{\ell-1}$ of each conductive pathway of the PCB standard is constructed from conditional lines – bands $\varpi^{\ell-2}$ of different length $\mu_2^1 = l = wn$ (Fig. 8.3).

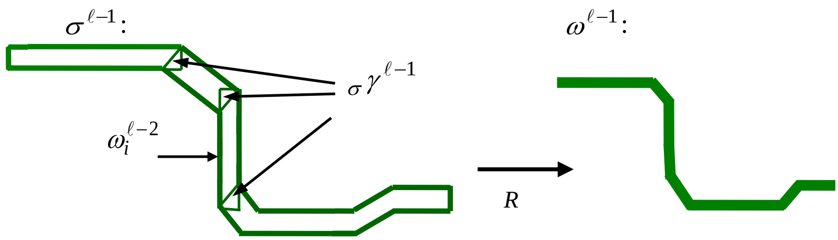


Fig. 8.3. Structure $\sigma^{\ell-1}$ of PCB pathway $\omega^{\ell-1}$ constructed from conditional lines $\omega_i^{\ell-2}$

The process is presented in the form of coordinator $S_0^{\ell-1}$ mapping as follows:

$$\ell = \ell - 1, \qquad S_0^{\ell-1} : \{\varpi^{\ell-2}, {}_\sigma\gamma^{\ell-1}\} \to \sigma^{\ell-1}. \tag{8.6}$$

Mapping R is carried out by coordinator of level $\ell - 1$, $S_0^{\ell-1} : \sigma^{\ell-1} \to \omega^{\ell-1}$, and realized as a transition from structure $\sigma^{\ell-1}$ to aggregated pattern $\omega^{\ell-1}$. Similar transitions take place on each level during the synthesis of the PCB pathways structure. On level ℓ general structure σ^{ℓ} (Fig. 8.4) is formed from set of PCB conductive pathways $\varpi^{\ell-1}$:

$$\ell = \ell, \qquad S_0^{\ell} : \{\varpi^{\ell-1}, {}_\sigma\gamma^{\ell}\} \to \sigma^{\ell}. \tag{8.7}$$

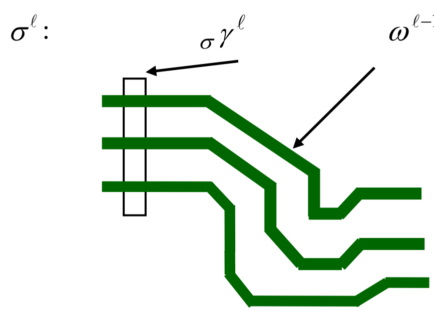


Fig. 8.4. A pathways fragment of general PCB construction σ^{ℓ}

While performing the synthesis task at the detailed design (DD) phase, the geometric characteristics of each conductive pathway are interactively defined in program realization and written in typical computer files. The pathways are also defined as a set of parameters which are presented in typical files in the form of corresponding numeric codes. While any of the pathway elements is changed, the others are automatically reconstructed by coordinator – presented in the form of computer control program – with simultaneous change of corresponding program codes. That is possible thanks to the availability of the connections between the pathways elements – conditional geometric objects (see Figs. 8.2 and 8.3). It allows easy correction of the geometric structure of PCB conductive pathways at any stage of its synthesis and reconstruction processes.

The availability of mechanism of information aggregation $\sigma^{\ell} \to \omega^{\ell}$ allows the reduction of computer memory volumes in the process of transition to a higher level of the PCB geometric structure during its formation. The basic result in comparison with well known PCB design and testing approaches is that the graphic image of PCB is obtained in the process of construction of PCB geometric representation. The availability of transition from the graphic image to geometric representation and vice versa allows effective adaptation of the PCB geometric structure at testing stages 2-5, in particular at the stage of adaptation of the standard to the received graphic image, when coordinates and scales are in process of changing.

8.2. Stages of PCB interconnections quality testing

Structure of information in the process of PCB testing is also presented in *aed* form (3.1). But in this case we have a system for which the construction of geometric representation ${}_{g}S^{\ell}$ (stage 1) is structural element σ^{ℓ}, and ω^{ℓ} is the graphic image of interconnections. PCB image ω^{ℓ} is presented as follows:

$$\omega^{\ell} = \{\omega_S, {}_{\omega}\gamma, \omega_V\}^{\ell}, \tag{8.8}$$

where: ω_S is graphic representation of standard; ω_V is received video-image of PCB, ${}_{\omega}\gamma$ – connections of ω_S and ω_V (for instance, three points of the images, by which the coordination of images is determined).

PCB testing process corresponds to 2-5 stages described above, which are executed in a technological process of PCB production. In comparison with the known approaches, *stage 2* of the proposed method is almost the same as for known technologies and realized by using standard video cameras, e.g. Chameleon

[89] or Grasshopper [90] cameras. The exemplary PCB image obtained while task performing is presented in Fig. 8.5.

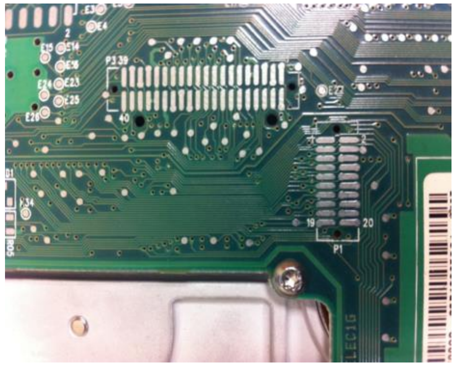


Fig. 8.5. The result of obtaining a PCB video image

a) b)

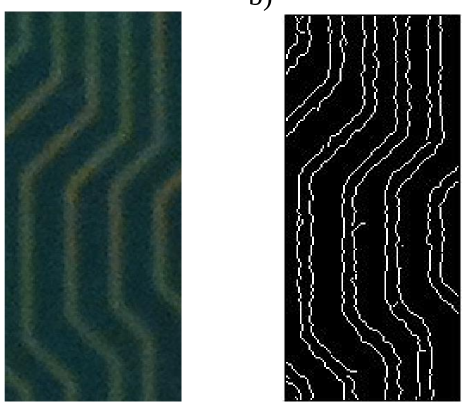


Fig. 8.6. The results of the PCB image processing: a) *imcrop* function application, b) the result of *rgb2gray* and *edge* functions application

The obtained PCB image is preprocessed to be fit for the next steps of the PCB quality testing. The results of the image processing – the fragment acquisition and edges extracting of conductive pathways are presented in Fig. 8.6 a), and Fig. 8.6 b) accordingly.

For the PCB fragment acquisition the Matlab *imcrop* function is used. To extract the edges of the conductive pathways the *edge* function of the Matlab Image Processing Toolbox is applied.

The matching process of the PCB standard (*stage 3*) is usually not presented in widespread testing technologies. At this stage in frames of the proposed method, the PCB standard is built on to PCB image by its three basic points, which allow to coordinate the orientation and sizes of the standard and the received graphic image of PCB thanks to the availability of its HS geometric representation.

a) b) c) d)

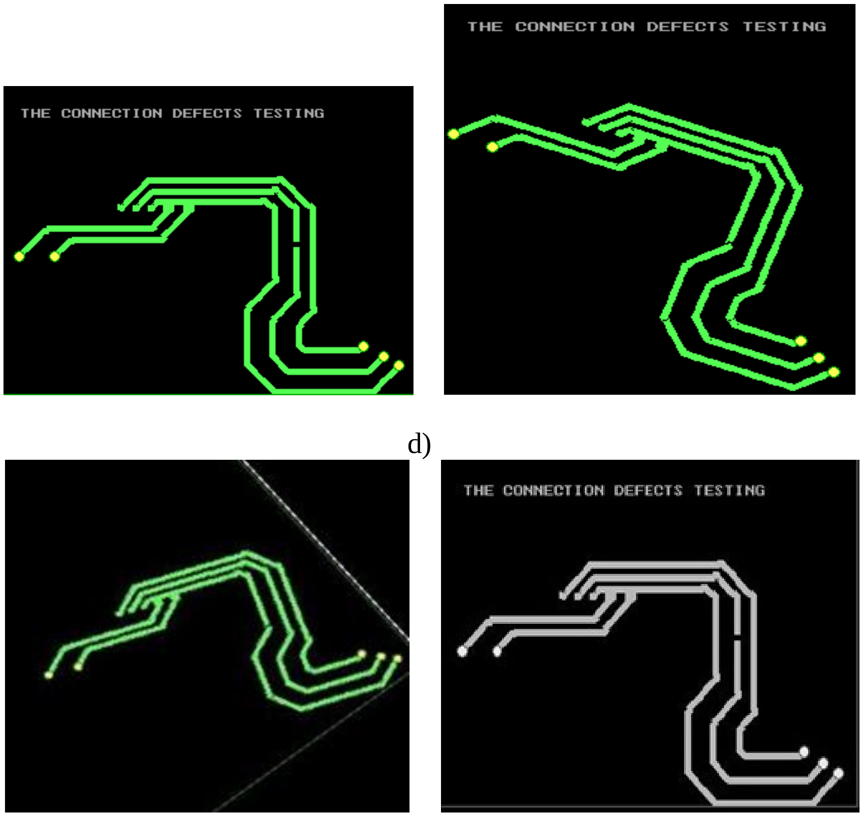


Fig. 8.7. Images a)-c) of PCB conductive pathways of different orientation and scales obtained from computer monitor; d) is the image of the pathways presented in *grey* format

It permits to avoid disparity between the standard and the image which is caused not by damages of PCB but by changes of PCB scale, i.e. camera distance from it, and orientation (Fig. 8.7). The matching process of graphic images is realized after definition of matching control points using Matlab functions of Image Processing Toolbox and Computer Vision System Toolbox [91]. The results of the matching process of the obtained PCB fragment and PCB standard images is presented in Fig. 8.8.

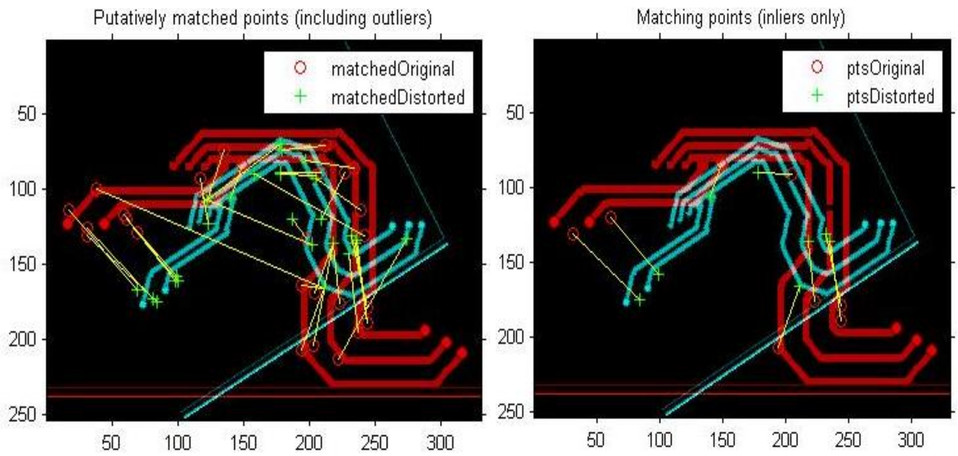


Fig. 8.8. Matching of PCB fragment images of different orientation and scale, axes units are pixels

Standard Matlab procedures allow realization of images fitting by using of *imrotate()* and *imresize()* functions, detecting features applying in both images to be matched *detectSURFFeatures() function,* and extract feature descriptors using *extractFeatures()* function. Matching features by using their descriptors is realized with help of *matchFeatures()* function. This function also allows computing the transformation from the obtained PCB fragment image to the original PCB standard image. The matching process of the two PCB images, i.e. original (o) image – red, and distorted one (+) – blue, is presented in Fig. 8.8.

PCB image obtaining is performed using standard digital cameras to match the image of PCB standard. Exemplary fragment of pathways structure of PCB standard (Fig. 8.7) was synthesized using Pascal programming language. The images of PCB conductive pathways of different orientation and scales presented in Fig. 8.7 are the screenshots obtained from a computer monitor.

At the *fourth stage* of PCB quality testing, comparison and detection of PCB defects is carried out with regard to the geometric representation of PCB standard by checking the conductive pathways for the gaps (Fig. 8.9a) – not by continuous scanning of image lines, as in usual technologies, but by scanning along the conductive pathways structures. It allows the time of comparison process and volumes of processed information to be reduced.

In the process of pathways soldered joints test, the aperture moves along the imaginary construction (Fig. 8.9b) which corresponds to the construction of *i* interconnection (*i, i+1* – the neighbouring pathways).

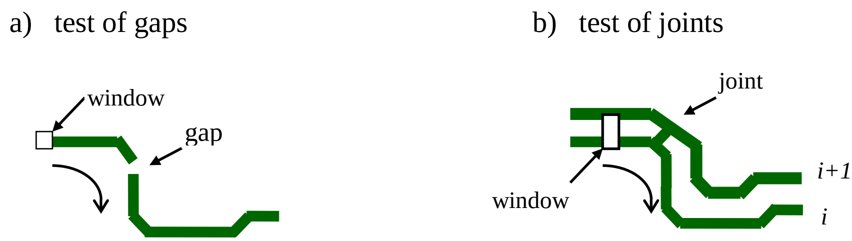


Fig. 8.9. Schemes of a) gaps and b) soldered joints testing processes

The *fifth stage* of the PCB quality assessment is a simple logic operation over the results of the fourth stage.

Preliminary conclusions and results

The introduced PCB conceptual design method and HS technology of PCBs conductive pathways quality testing yields the following main results:

– a graphic image of PCB standard is a result of formation of geometric representation of PCB in the process of synthesis of PCB pathways construction;

– availability of geometric representation allows easy correction of pathways topology at each phase of PCB constructing, including conceptual design (CD) and detailed design (DD) phases;

– the mechanism of information aggregation permits to avoid the growth of computer memory volumes during the growth of the PCB construction level in the design and production processes;

– the PCB testing technology gained new qualities: the possibility of scales and orientation of PCB pathways geometric construction to change (during PCB standard rebuilding on to the obtained graphic image of PCB) allows to avoid the identification of PCB quality as defective in a technological process, when the PCB orientation and the distance in relation to the video camera are changed;

– the standard and obtained PCB images are compared not by continuous scanning of image lines as in usual technologies, but only by scanning along the conductive pathways with regard to the pathways geometric representation, which allows the reduction of the time of comparison and computer memory volumes.

9. DESIGN AND CONTROL OF MCM CUTTING MACHINE

In this Chapter, the conceptual model, design (synthesis and analysis) and motion planning tasks of MCM machine are described. MCM – Manhole Cutting Machine – is used in production environment to perform technological operations of pipes cutting and welding [92-94]. At the Conceptual Design (CD) phase of MCM life cycle, the systemic formal models of MCM elements and coordination (design and control) processes were presented in HS theoretical basis (see Chapters 3,4). At the Detailed Design (DD) phase, much attention is paid to the MCM analysis – kinematics tasks, and synthesis – the task of MCM construction creation. Different concrete elements which potentially could be used in MCM structure were analyzed to perform the synthesis task. Construction and technologies of the control subsystems, i.e. Mobile Panel of 5MP050.0653-03 type, ARNCO soft CNC system, Automation Studio program environment [95], as well as actuators, e.g. ACOPOS servo drivers of B&R [96] were analyzed. Systemic formal models of these systems were also given at the CD phase before realizing synthesis task at the DD phase.

According to the definition of the conceptual model given in Chapters 3 and 4, the developed conceptual model of MCM machine presented below contains formal descriptions of MCM structure and its environment, dynamic models of structural elements and their connections, functional model of MCM machine and its coordinator. The coordinator is presented as a system which realizes the design and control processes and connects in this way structure and functions of MCM machine. The suggested conceptual model of MCM machine is also partially described in [94].

9.1. MCM conceptual model

Conceptual model of MCM machine (Manhole Cutting Machine) is given in the formal basis of HS using its standard block *aed* (3.1), see Chapter 3, and is presented as follows:

$$S^{\ell} \leftrightarrow \{\,\omega^{\ell},\, S_0^{\ell},\, \sigma^{\ell}\,\}, \tag{9.1}$$

where ω^{ℓ} is functional dynamic description of MCM machine in its technological environment ${}_{\varepsilon}S^{\ell}$, σ^{ℓ} is MCM structure, S_0^{ℓ} is coordinator, ℓ is index of level, $\ell \in L^s$, L^s is numeric positional system. According to *aed* scheme (see Fig. 3.1), formal description S^{ℓ} contains the object (MCM) model ${}_{o}S^{\ell}$ and models of MCM sub-systems ${}_{o}S_i^{\ell}$, production environment ${}_{\varepsilon}S^{\ell}$ model, models of coordinator S_0^{ℓ} (design and control system of MCM) and technological process ${}_{\pi}S^{\ell}$ performed by MCM, i.e. cutting and welding.

Functional descriptions of MCM system ${}_{o}S^{\ell}$ is presented according to (3.2) in the following form of dynamic system $(\overline{\rho},\overline{\varphi})$:

$$
\begin{aligned}
&{}_{o}(\overline{\rho},\overline{\varphi})^{\ell}:\ {}_{o}\overline{\rho}^{\,\ell} = \left\{{}_{o}\rho_t : C_t \times X_t \rightarrow Y_t \;\&\; t \in T\right\}^{\ell}, \\
&{}_{o}\overline{\varphi}^{\,\ell} = \left\{{}_{o}\varphi_{tt'} : C_t \times X_{tt'} \rightarrow C_{t'} \;\&\; t,t' \in T \;\&\; t' > t\right\}^{\ell},
\end{aligned} \tag{9.2}
$$

whre T^{ℓ} is time of ℓ level; C^{ℓ}, X^{ℓ}, Y^{ℓ} are state, input and output of MCM ${}_{o}S^{\ell}$ system respectively; $\overline{\varphi}$ is state transition function, $\overline{\rho}$ is the reaction.

Structure σ^{ℓ} of MCM is defined according to (3.13) as the following formal system:

$$\sigma^{\ell} \leftrightarrow \{\, S_0^{\ell}, \{\,\overline{\omega}^{\,\ell-1}, {}_{\sigma}\gamma^{\ell}\,\}\} \leftrightarrow \{\, S_0^{\ell}, \tilde{\sigma}^{\ell}\,\}, \tag{9.3}$$

where S_0^{ℓ} is coordinator of MCM, $\overline{\omega}^{\,\ell-1}$ are dynamic models $(\overline{\rho},\overline{\varphi})$ of MCM subsystems $\overline{S}^{\,\ell-1} = \{S_i^{\ell-1} : i \in I^{\ell}\}$, ${}_{\sigma}\gamma^{\ell}$ are structural connections.

Structure of MCM is created while performing *synthesis task* at both CD and DD phases. In our case, structure of MCM was created within the frames of SolidWorks system environment [74], see Fig. 9.1. After that, MCM model was placed into RobWork program system environment [97, 98] to perform *analysis tasks* – forward and inverse kinematics tasks (Fig. 9.3). The process of analysis tasks performance is observed on a computer screen thanks to RobWork system interface which illustrates the simulation process – technological process of pipe cutting – running within the frameworks of RobWork system environment (Fig. 9.3).

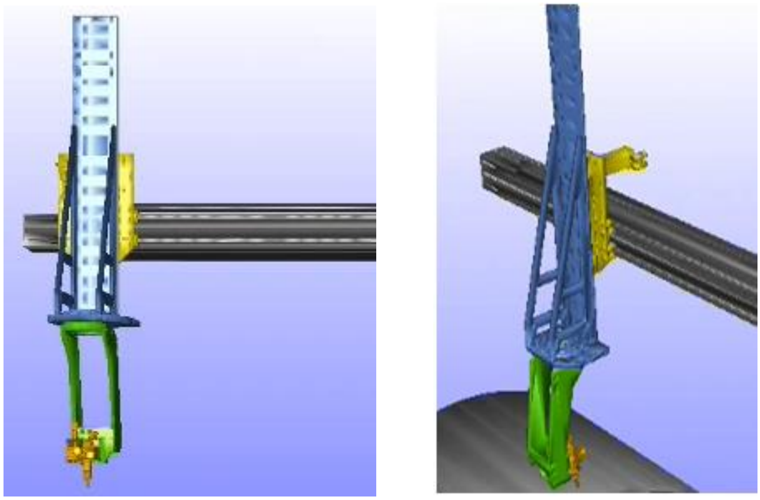


Fig. 9.1. Structure of MCM created within the frames of SolidWorks system – two a) and b) views

Being the elements of MCM structure σ^{ℓ} (see Fig. 9.2) the following subsystems $S_i^{\ell-1}$ presented in $\omega_i^{\ell-1}$ aggregated dynamic form are:

$\omega_1^{\ell-1}$: machine base;

$\omega_2^{\ell-1}$: MCM arm X with servo-motor which realizes left/right motion;

$\omega_3^{\ell-1}$: machine arm Z with servo-motor which realizes up/down motion;

$\omega_4^{\ell-1}$: arm of the cutting torch which makes possible 360° rotation;

$\omega_5^{\ell-1}$: torch holder which allows its inclination;

$\omega_6^{\ell-1}$: bracket connecting the machine base with arm X.

Structural connections ${}_{\sigma}\gamma^{\ell}$ (9.3) of MCM subsystems $\omega_i^{\ell-1}$ are as follows:

${}_{\sigma}\gamma_{1,6}^{\ell}$: bearing and the drive mechanism which allow the rotation of the entire support;

${}_{\sigma}\gamma^{\ell}_{1,2}$: rack begin the connection which allows the displacement of X arm relative to MCM base;

${}_{\sigma}\gamma^{\ell}_{2,3}$: rack begin the connection which allows the displacement of Z arm relative to X arm;

${}_{\sigma}\gamma^{\ell}_{3,4}$: bearing which makes possible 360^{o} rotation of torch arm;

${}_{\sigma}\gamma^{\ell}_{4,5}$: bevel gear used for the cutting torch inclination.

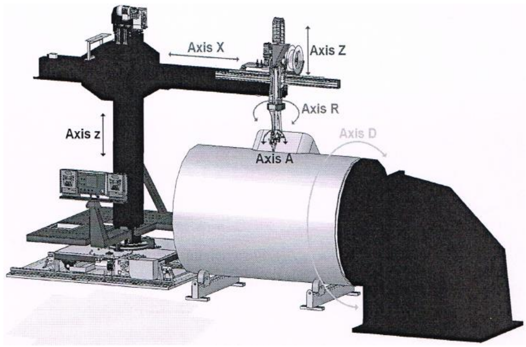


Fig. 9.2. Axis and structure of MCM machine

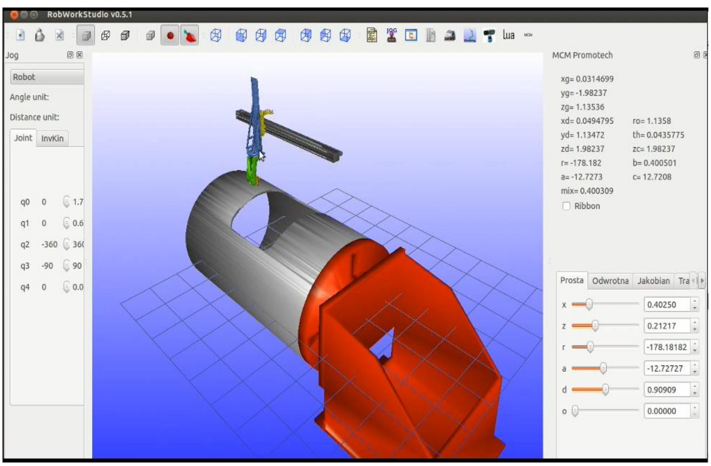


Fig. 9.3. MCM design model placed into RobWork simulation system environment to perform *analysis* – forward and inverse kinematics – tasks while simulating technological process of pipe cutting; the MCM environment elements – pipe (grey color) with hole being cut and support (red color) – are elements of the technological module

Environment ${}_{\varepsilon}S^{\ell}$ of MCM machine contains the other technological units and a man-operator. Elements of environment ${}_{\varepsilon}S^{\ell}$ structure ${}_{\varepsilon}\sigma^{\ell}$ which are in direct contact with MCM machine are a pipe being cut ${}_{\varepsilon o}S_{1}^{\ell}$ and support ${}_{\varepsilon o}S_{2}^{\ell}$. The support allows the piper rotation in the technological cutting process (see Fig. 9.3). Pipe being cut ${}_{\varepsilon o}S_{1}^{\ell}$ is colored grey in Figure 9.3 and the support ${}_{\varepsilon o}S_{2}^{\ell}$ is colored red.

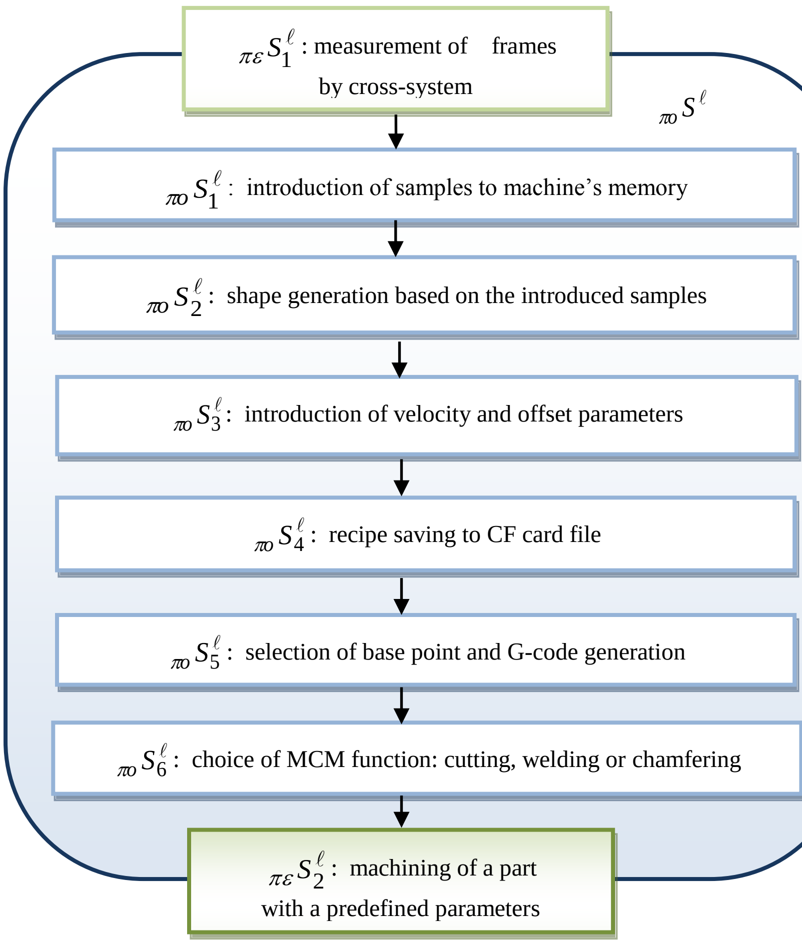


Fig. 9.4. General ***process*** ${}_{\pi o}S^{\ell}$ and sub-processes ${}_{\pi o}S_{i}^{\ell}$ performed by MCM; ${}_{\pi\varepsilon}S_{1}^{\ell}$ and ${}_{\pi\varepsilon}S_{2}^{\ell}$ are sub-processes executed in MCM environment

General *process* ${}_{\pi o}S^{\ell}$ and sub-processes ${}_{\pi o}S_i^{\ell}$ which are executed by MCM while performing pipe cutting operations are presented in Figure 9.4. Each of the sub-processes ${}_{\pi o}S_i^{\ell}$ is described by $(\bar{\rho},\bar{\varphi})^{\ell}$ dynamic model at CD phase. The descriptions are concretized at DD phase.

Within the frameworks of the suggested MCM conceptual model the coordinator is presented in the form of the design and control system of MCM machine. MCM coordinator S_0^{ℓ} is formally described by (3.15) equation. S_0^{ℓ} performs the design and control tasks on its selection, learning and self-organization strata.

Dynamic realizations of coordinator strata are presented by canonic models $\hat{\omega}_0^{\ell} \leftrightarrow \{\ (\bar{\hat{\varphi}},\bar{\hat{\lambda}})_0^{\ell}, S_0^{\ell}\}$ (see Chapter 3) in the following form:

$$\bar{\hat{\varphi}}_0^{\ell} = \{\hat{\varphi}_{0tt'}^{\ell} : C_0^{\ell} \times X_{0tt'}^{\ell} \to C_0^{\ell}\ \&\, t,t' \in T^{\ell}\ \&\, t' > t\}, \tag{9.4}$$

$$\bar{\hat{\lambda}}_{0t}^{\ell} = \{\hat{\lambda}_{0t}^{\ell} : C_0^{\ell} \times \hat{\tilde{X}}_0^{\ell} \to \hat{\tilde{Y}}_0^{\ell}\ \&\, t \in T^{\ell}\}.$$

Canonic model [94] is constructed using the concept of the multilayer (multi-strata) system, where states transition functions $\{\bar{\hat{\varphi}}_0^{\ell}\}$ correspond to learning and self-organization strata, and output functions $\{\bar{\hat{\lambda}}_0^{\ell}\}$ correspond to the strategies of the selection strata.

The developed block-scheme of elements ${}_2S_0^{\ell}, {}_2S_0^{\ell+1}$ of coordinator S_0^{ℓ} being the control system of servomotor ${}_2S^{\ell}$ of MCM arm is presented in Figure 9.5, where:

G^{ℓ} is control signal *u_set[v]=e(t)* which is realized with the help of B&R technology [96] and goes from ACOPOS controller ${}_2S_0^{\ell}$ to servomotor ${}_2S^{\ell}$;

W^{ℓ} is the feedback from controller ${}_2S_0^{\ell}$, realized in the form of impulses *s_act[inc]*;

$G^{\ell+1}$ is the control signal from PC panel ${}_2S_0^{\ell+1}$ to controller ${}_2S_0^{\ell}$ presented in the format {position (*s*), velocity (*v*), acceleration (d^2s/dt^2), delay ($-d^2s/dt^2$)};

$W^{\ell+1}$ feedback to PC panel $S_0^{\ell+1}$ presented in the format {status (*enable/disable*), position (*s*), velocity (*v*), error (*error*)};

X_2^{ℓ} is the servomotor input – electromotive force *E(t)* (Fig.9.5);

Y_2^{ℓ} is the servomotor output – torque $\tau_m(t)$.

Canonic model of the controller ${}_2S_0^{\ell}$ in the form of (9.4) is built using objects of inputs $X_0^{\ell}=\{G^{\ell+1}, W^{\ell}\}$, outputs $Y_0^{\ell}=\{G^{\ell}, W^{\ell+1}\}$ and states C_0^{ℓ} of coordinator ${}_2S_0^{\ell}$, where C_0^{ℓ} is state of Acopos ${}_2S_0^{\ell}$ controller.

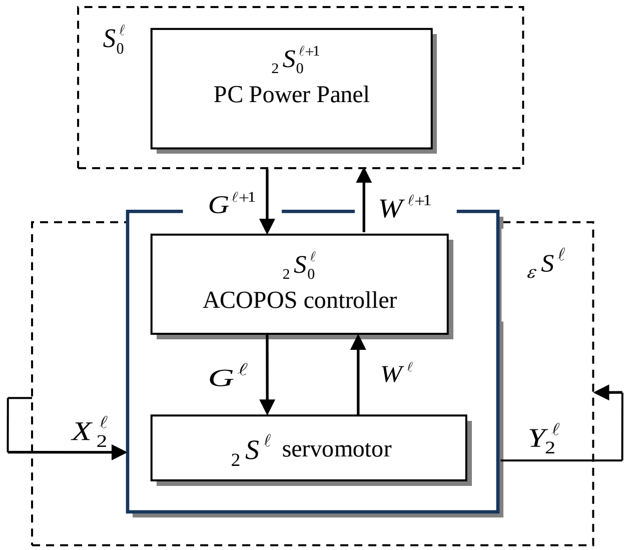


Fig. 9.5. Conceptual scheme of control system of MCM arm servomotor presented in coordinator form

Canonic model of the controller ${}_2S_0^{\ell}$ in the form of (9.4) is built using objects of inputs $X_0^{\ell}=\{G^{\ell+1}, W^{\ell}\}$, outputs $Y_0^{\ell}=\{G^{\ell}, W^{\ell+1}\}$ and states C_0^{ℓ} of coordinator ${}_2S_0^{\ell}$, where C_0^{ℓ} is state of Acopos ${}_2S_0^{\ell}$ controller.

At the *detailed design* phase, control system is created by the selection of concrete elements and realization of their structural connections. In this way the synthesis design task is performed. Formal description of this task (9.8) is given below. Concrete system based on ACOPOS servo-motors control and synthesized at DD phase for MCM arms motion control is presented in Figure 9.6. Here the elements, i.e. ACOPOS servo drives, X67 input-output units, X20 control and remote I/O systems, visualization and operation system, are connected by the network using CAN bus.

The following stuctural elements are presented In Figure 9.6. ACOPOS 1045 is an intelligent servodrive [99] with very short scan times and communication cycles of 400 µs, which equal to 50 µs in the control loop. Intelligent ACOPOS servodrives work with AC three-phase synchronous motors 8JS, 8LS from B&R. X67 system is a safe digital and analog I/O module [100] with IP67 protection. In addition to its normal I/O tasks, it monitors the signal line as well as the connected sensors and actuators. Because X67 systems are integrated within the entire system, they initially function in the standard application as normal modules. They deliver input signals and process output commands. System X20 is a segments based remote I/O system. It is a complete control solution. Depending on the application requirements, the X20 system makes it possible to combine the exact components necessary. Visualization and operation system – Power Panel 65 – provides maximum flexibility. It is equipped with 2 USB interface and a Fast Ethernet port for exchanging data with higher-level systems.

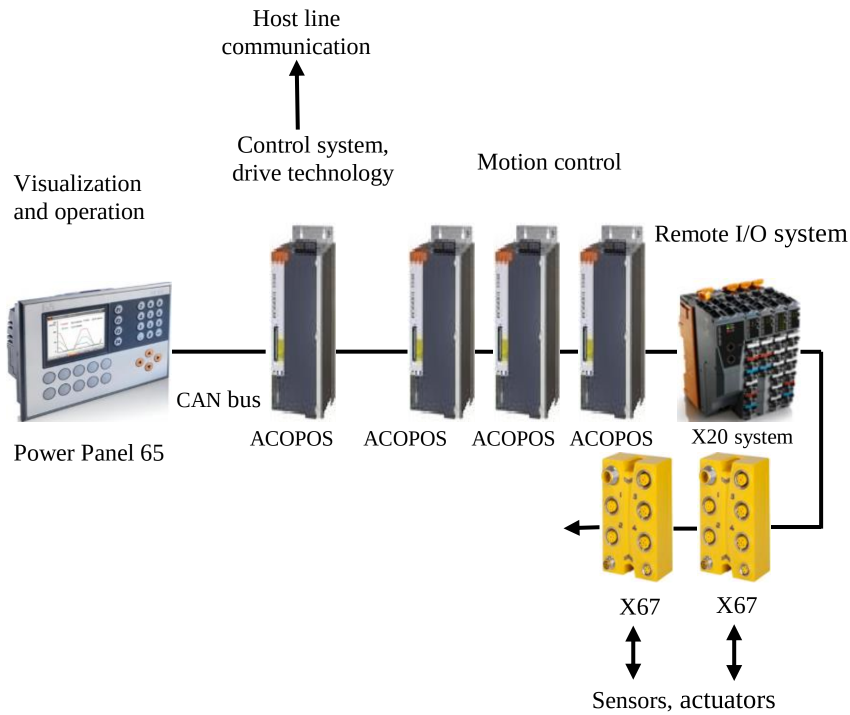


Fig. 9.6. Concrete system synthesized for MCM motion control

The developed conceptual model of MCM in the formal basis of Hierarchical Systems allows easy transfer form Conceptual Design phase to Detailed Design phase in the life cycle of MCM machine [94] .

While transferring from conceptual to detailed design [94], the systemic models of MCM subsystems are concretized. For example, $\left(\overline{\rho},\overline{\varphi}\right)$ model of AC servo motor ${}_2S^{\ell}$ (Fig. 9.7) being the element of the machine arm is presented in the following form (9.5) of DE:

$$a_0 \frac{dy}{dt} + a_1 y + a_2 = u \tag{9.5}$$

which is concretized and presented at the DD phase as follows [101]:

$$\begin{aligned}
&\frac{d}{dt} i_a = \frac{1}{3L_s}(2v_{ab} + v_{bc} - 3R_s i_a + \lambda p \omega_m(-2\Phi'_a + \Phi'_b + \Phi'_c)) \\
&\frac{d}{dt} i_b = \frac{1}{3L_s}(-v_{ab} + v_{bc} - 3R_s i_b + \lambda p \omega_m(\Phi'_a - 2\Phi'_b + \Phi'_c)) \\
&\frac{d}{dt} i_b = -(\frac{d}{dt} i_a + \frac{d}{dt} i_b) \\
&T_e = p\lambda(\Phi'_a \cdot i_a + \Phi'_b \cdot i_b + \Phi'_c \cdot i_c) \\
&\frac{d}{dt}\omega_m = \frac{1}{J}(T_e - T_f - F\omega_m - T_m) \\
&\frac{d}{dt}\theta = \omega_m
\end{aligned} \tag{9.6}$$

where equations (9.6) correspond to the states transition functions $\overline{\varphi}$ of the conceptual model ($\overline{\rho}, \overline{\varphi}$), (see Fig. 9.8). DEs (9.6) are further transformed at the DD phase to the transfer function (9.7):

$$G(s) = \frac{\omega_m(s)}{T_{em}(s)} \tag{9.7}$$

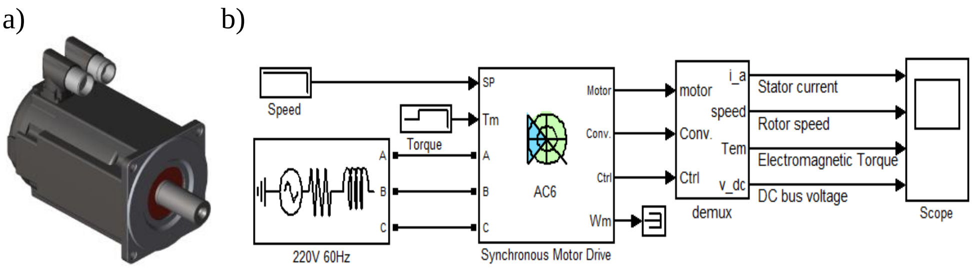


Fig. 9.7. CAD model of AC 8BJ motor of B&R (a) [99], and its Matlab simulation scheme (b) [101]

In Figure 9.8 and equations (9.6) and (9.7), the variables are as follows: L_s is the inductance of the stator; R_s is a resistance of the stator; i_a, i_b, i_c are currents of *a*, *b* and *c* phases respectively; Φ_a', Φ_b', Φ_c' are electromotive forces of *a*, *b* and *c* phases respectively; v_{ab}, v_{bc} are *ab* and *bc* phase to phase voltages respectively; ω_m is angular velocity of the rotor; λ is the amplitude of the flux induced by the permanent magnets of the rotor in the stator phases; p is number of pole pairs; T_{em} is electromagnetic torque; J is combined inertia of rotor and load; F is combined viscous friction of rotor and load; θ is rotor angular position; T_m is shaft mechanical torque; T_f is shaft static friction torque; ω_m is angular velocity of the rotor – mechanical speed.

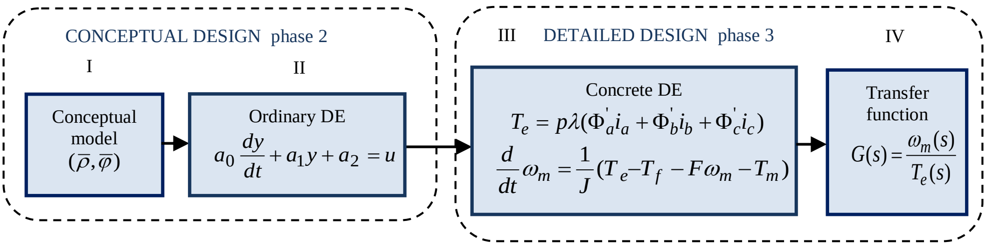


Fig. 9.8. Steps of conceptual model $(\overline{\rho},\overline{\varphi})$ concretization of AC servo-motor subsystem $\omega_i^{\ell-2}$ at the design phases of MCM life cycle; conceptual model is transformed to ordinary DE and concretized at DD phase by transfer function $G(s)$

9.2. Design tasks

The main design tasks – *synthesis & analysis* – are performed at both conceptual and detailed design phases. The first basic design task which is performed by MCM coordinator is the *synthesis task*. In the basis of the conceptual model described above and the general formulation of the synthesis task within the frameworks of HS technology, the structure of the MCM machine being synthesized is presented in form (9.8), and the synthesis operations are described by $(\overline{\rho},\overline{\varphi})$ models of coordinator processes ${}_{\pi}S_0^{\ell}$. In the process of performing a *synthesis task* in SolidWorks system environment the robot structure σ^l is synthesized from its elements ω^{l-1} by realizing structural connections ${}_{\sigma}\gamma^l$. The task is described at CD phase within the frameworks of HS conceptual model as coordinator S_0^{ℓ} task as follows:

$$S_0^{\ell}:\{\overline{\omega}^{\ell-1},{}_{\sigma}\gamma^{\ell}\}\rightarrow\sigma^{\ell}. \qquad (9.8)$$

Geometric parameters of the MCM structure itself are changeable in the synthesis process. At CD phase the parameters are described by constructive dimension and connections defect, introduced in Section 4.2 by numerical positional system. At DD phase the concrete sizes as well as physical characteristics of MCM parts are defined.

In the synthesis process, MCM structure dynamics is described by reaction ρ of (ρ,φ) model:

$$\overline{\rho}^{\ell} = \{\rho_t : C_t \times X_t \rightarrow Y_t \;\&\; t \in T\}^{\ell} \tag{9.9}$$

where: C_i is the initial MCM part at $t=i$ moment of time (or composition of previously connected parts in the synthesis process), X_i is a part to be added, Y_i is the union (composition) of C_i and X_i parts, $t=i$ – moment of time T. The exemplary MCM machine construction synthesized within the frameworks of SolidWorks system and placed in RobWork system environment is presented in Figs. 9.3 and 9.11.

At the detailed design (DD) phase the concrete MCM machine parts were chosen, e.g. the ACOPOS servo drivers of B&R [99], Mobile Panel of 5MP050.0653-03 type, ARNCO soft CNC system, Automation Studio program system [95], etc. Parts of the MCM mechanical arm were produced in Promotech enterprise (Bialystok, Poland). MCM arm parameters were defined after stress and materials analysis performing. The results of the synthesis for the cases of MCM mechanical construction and MCM arms motors control system are given in Figs. 9.2-9.3, and Fig. 9.6 respectively.

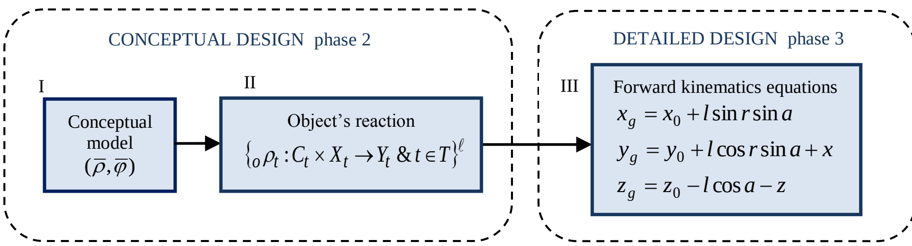


Fig. 9.9. Steps of conceptual model concretization at the design phases of MCM life cycle; conceptual model is transformed to object's reaction ${}_o\rho_t$ and concretized at DD phase by kinematics equations

The *analysis design tasks* are kinematics and dynamics tasks. *To perform the kinematics task,* kinematic scheme of MCM is developed (see Fig. 9.10c) at the CD phase, kinematic model of MCM is presented in $(\overline{\rho},\overline{\varphi})$ form and transformed to the form of reaction ${}_o\rho_t$ model. At the DD phase, a *mathematical model of*

MCM kinematics is created as concretization of the conceptual model and presented in the form of forward kinematics equations [46,93], see Figure 9.9.

The scheme presented in Figure 9.9 shows the dynamics of the object's kinematic model – MCM machine – in its design process. Transformation of the model of MCM dynamics is realized in the similar way. The construction and kinematic scheme of the MCM machine are given in Figure 9.10, where the pipe being cut, MCM axis and both global "G,O" and detail "D" coordinate systems are presented as well.

The aim of the *kinematics task* performed at the DD phase is the definition of the locations and orientations of MCM tool – operating point of the cutting torch. The equations created at DD phase (Fig. 9.9) which define the MCM end effecter (TCP – tool center point) position in global "G,O" coordinates are as follows:

$$\begin{aligned} x_g &= x_0 + l \sin r \sin a \\ y_g &= y_0 + l \cos r \sin a + x \\ z_g &= z_0 - l \cos a - z, \end{aligned} \tag{9.10}$$

where

$$\begin{aligned} x_0 &= 0,03068 \\ y_0 &= -2,40975 \\ z_0 &= 1,45775 \\ l &= 0,113 + \text{offset}, \end{aligned} \tag{9.11}$$

x_0, y_0, z_0 are parameters connected with the machine geometry (*m*); *l* is the effective tool length (m); *x, z, r, a, d* are the locations of each machine axis (m).

Position of the end effecter in detail coordinate system „D" is described by the following equations:

$$\begin{aligned} x_d &= x_g \cos d + z_g \sin d \\ y_d &= -x_g \sin d + z_g \cos d \\ z_d &= -y_g \end{aligned} \tag{9.12}$$

where x_d, y_d, z_d are coordinates of the end effecter (TCP) in the detail coordinate system „D"; x_g, y_g, z_g are coordinates of the end effecter in the global coordinate system „ O".

To perform the forward kinematics task the following algorithm was developed [93].

Input:
Actual axes configuration $\mathbf{q} = \{\mathbf{x}, \mathbf{z}, \mathbf{r}, \mathbf{a}\ \mathbf{d}\}$

Output:
Actual global position $\mathbf{p_g} = \{\mathbf{x_g}, \mathbf{y_g}, \mathbf{z_g}\}$
Actual position in the detail coordinate system $\mathbf{p_d} = \{\mathbf{x_d}, \mathbf{y_d}, \mathbf{z_d}\}$

Calculate effective tool length:
$l \leftarrow l_0$+offset

Calculate position in the global coordinate system "0":
$x_g \leftarrow x_0 + l \sin \mathbf{r} \sin \mathbf{a}$
$y_g \leftarrow y_0 + l \cos \mathbf{r} \sin \mathbf{a} + \mathbf{x}$
$z_g \leftarrow z_0 - l \cos \mathbf{a} - \mathbf{z}$

Calculate position in the detail coordinate system "D":
$x_d \leftarrow x_g \cos \mathbf{d} + z_g \sin \mathbf{d}$
$x_d \leftarrow - x_g \cos \mathbf{d} + z_g \cos \mathbf{d}$
$z_d \leftarrow - y_g$

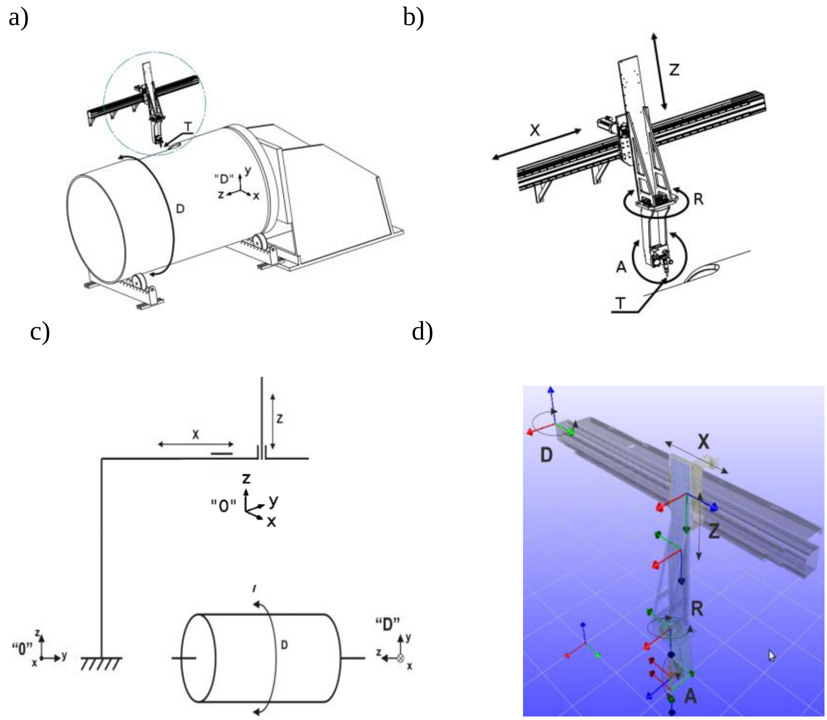


Fig. 9.10. Coordinate systems a) and d) – d) is a computer screen-short, MCM axis b) and d), and kinematic scheme c) of MCM, where **D** is the coordinates system connected with the pipe detail, **O** is the global coordinates system, **R** is the reference frame of the instrument holder rotation joint, **A** is the reference frame of the tool rotation joint

Simulation models were also implemented in program environment of RobWork system [93, 98].

9.3. MCM motion design and simulation – Robwork Studio integration

MCM 3D design model was created at DD phase to simulate and control the positioning process during pipe cutting technological operations performed by MCM machine. Correspondent virtual technological module was synthesized at DD phase in the form of a workcell for the use with RobWork Studio software [98]. Figure 9.11 presents the mutual interconnections of simulation program files of MCM workcell within the framework of RobWork system environment. At CD phase the construction of the MCM workcell and its files interconnections are presented according to *aed* diagram (Fig. 3.1) described in Section 3.1, where the whole system S^l is realized by MCM.model.wc.xml unit, subsystems S_i are presented by MCMarm.model.wc.xml and Tool.xml files, subprocesses ${}_{\pi}S_i$ and general process ${}_{\pi}S^l$ are simulation processes, coordinator S_o contains ProximitySetup.xml, EasyBotCollisionSetup.xml and Device.xml files, environment ${}_{\varepsilon}S$ is presented by Environment.wc.xml program unit. All the files are described below and connected at DD phase by scheme given in Figure 9.11.

In Figure 9.11 the blue line shows that the given file is contained in the given folder. The black line shows the direct dependence of the connected blocks, i.e. files. File MCM.model.wc.xml is the main file which describes the given technological environment, i.e. the scene in which simulation takes place. Workcell file MCMarm.model.wc.xml describes the MCM mechanical arm program model which is inserted in simulation scene. Device.xml file contains the descriptions of interconnections, movements and reference frames for MCM arm. Tool.xml file contains the descriptions of interconnections, movements and reference frames for MCM end-effector – tool. Workcell file Environment.wc.xml contains the description of MCM technological environment. ProximitySetup.xml and EasyBotCollisionSetup.xml files contain the descriptions which allow the collision avoidance of the MCM mechanical arm and other parts of MCM including MCM technological environment.

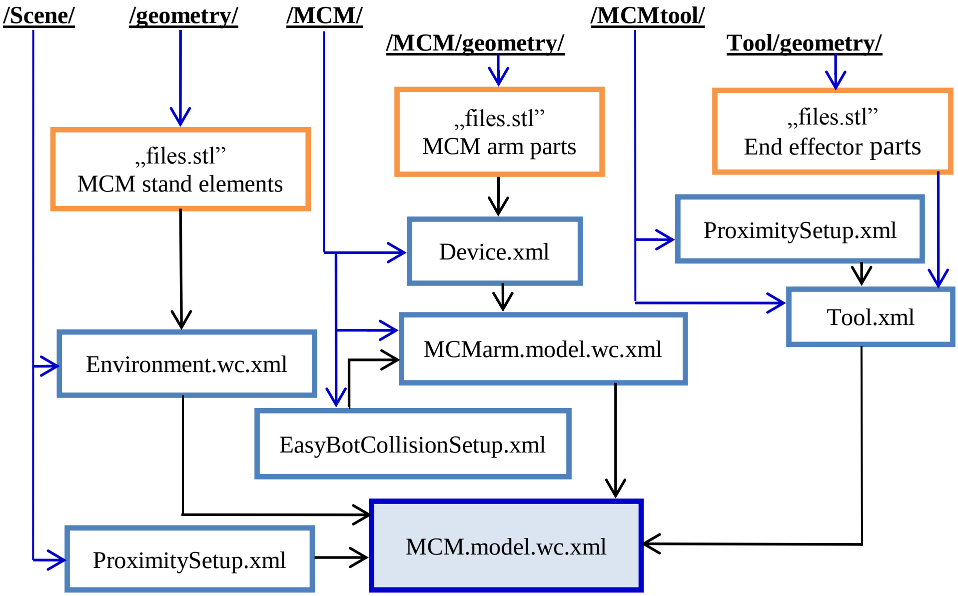


Fig. 9.11. Interconnections of simulation program files of MCM workcell in RobWork system

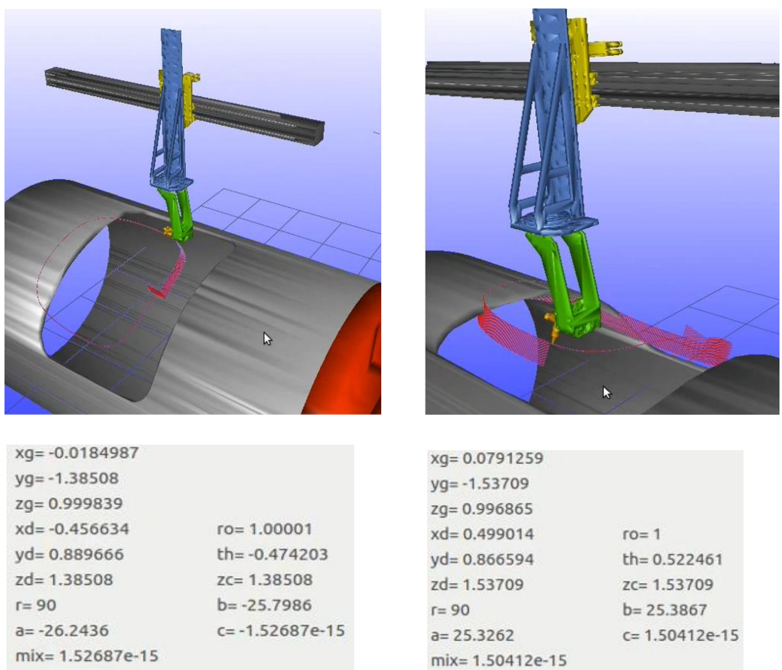


Fig. 9.12. Two states of hole cutting process simulated in RobWork system environment and meanings of correspondent parameters

Furthermore, to test the obtained kinematics solutions a plugin implemented in C++ language was developed. The plugin allows visualization of linear interpolation with the use of the Jacobean inverse method as well as forward and inverse kinematics calculations [46,93,102]. The developed MCM design model synthesized within the framework of CAD SolidWorks system [74] with correspondent technological module placed into RobWork simulation environment and the plugin being in action are shown in Figure 9.3. Two states of hole cutting process simulated in RobWork system and meanings of correspondent parameters are given in Figure 9.12.

Positioning process of MCM machine contains the motion of the tool – cutting torch – along the specified linear edge with predetermined angles *r* and *a*. The input parameters are predefined goal position of tool and velocity of the tool motion. Velocities of each MCM joint are calculated while performing interpolation according to equation (9.13). It allows obtaining smooth linear cutting path.

To calculate the joint rates necessary to perform a linear interpolation with constant translational and rotational velocities the following equation (9.13) is used:

$$
\begin{aligned}
&\dot{x} = -\dot{z}_d \\
&\dot{r} = \dot{r}' \\
&\dot{a} = \dot{a}' \\
&\begin{bmatrix} \dot{z} \\ \dot{d} \end{bmatrix} = J^{-1} \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix}
\end{aligned}
\tag{9.13}
$$

where $\dot{r}' = (\omega_{\max} / \omega) \cdot (r_2 - r_1)$ and $\dot{a}' = (\omega_{\max} / \omega) \cdot (a_2 - a_1)$ are presented in “D” reference frame, and the Jacobian is invertible.

Jacobean from (9.13) is presented in the following form (9.14):

$$
J = \begin{bmatrix} \dfrac{\partial x_d}{\partial z} & \dfrac{\partial x_d}{\partial d} \\ \dfrac{\partial y_d}{\partial z} & \dfrac{\partial y_d}{\partial d} \end{bmatrix} = \begin{bmatrix} -\sin d & -x_g \sin d + z_g \cos d \\ -\cos d & -x_g \cos d - z_g \sin d \end{bmatrix}. \tag{9.14}
$$

Algorithm of linear interpolation is presented for MCM tool motion along the edge. As an example, the edge between positions $p_1 = [\rho = 1\ \varphi = 0\ z = 0\ r=0\ a = 0]$ and $p_2 = [\rho = 0{,}9\ \varphi = 45^{o}\ z = 1\ r = 25^{o}\ a = 45^{o}]$ was taken under consideration. Simulation result is presented in Figure 9.13. The figure shows the variations of cylindrical coordinates during the tool motion along (p_1, p_2) edge.

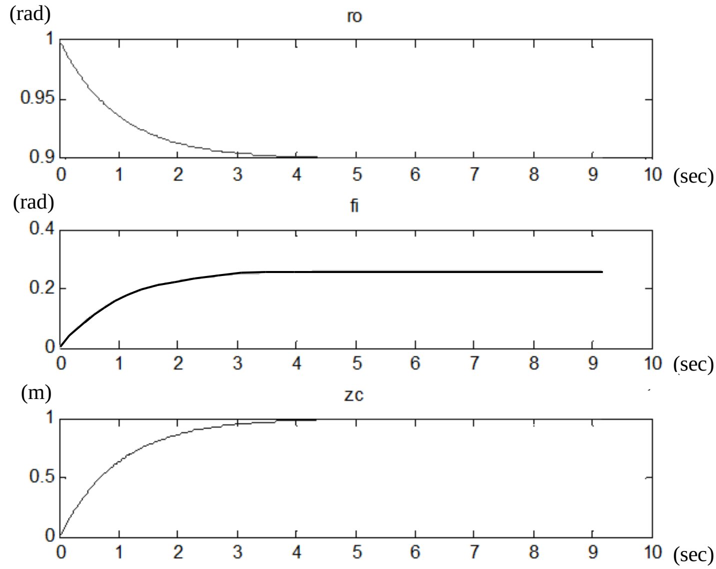


Fig. 9.13. MCM tool position in cylindrical coordinate system of detail

Variations of velocities and coordinates of MCM machine axes while performing linear displacement of tool along (p_1, p_2) edge calculated during Jacobian interpolation process are presented in Figure 9.14.

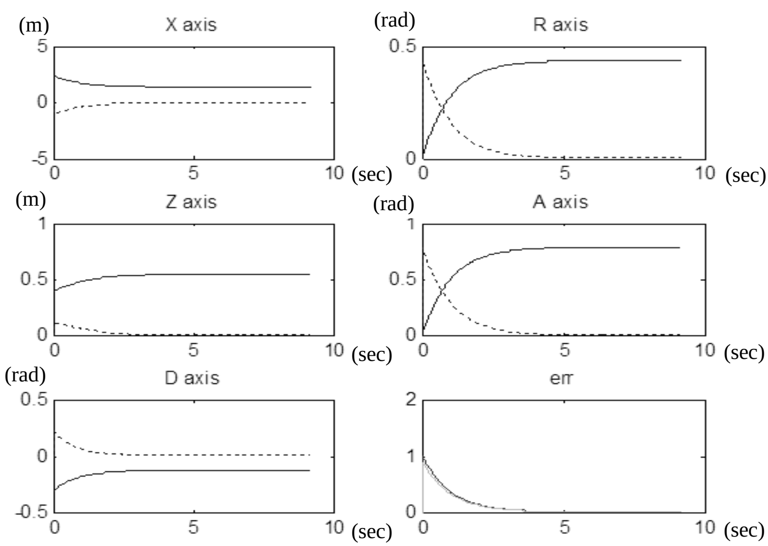


Fig. 9.14. Coordinates of joints (solid lines) and velocities (dashed lines) calculated in interpolation process

In our case, due to the presented MCM machine design, i.e. the choice of joints axes, the joint velocities calculation is simplified in a marked degree. Tool orientation is directly correlated to axes motions, and also the motion along z axis in 'D' coordinate system is governed by the x axis translation. According to these considerations, it is only necessary to calculate Jacobian relating (x_d, y_d) coordinates to z and d joint positions.

Preliminary conclusions

The use of Hierarchical Systems (HS) method in design and control of MCM (Manhole Cutting Machine) is presented in this Chapter. Being mechatronic by nature, MCM system contains human – man-operator, computer control, visual informatics, electronic – sensors, electromechanical – servo drives, and executive mechanical (MCM tool, arm and its joints) subsystems. The presented conceptual model of the MCM system being designed contains connected systemic models of the above mentioned subsystems presented in the general formal basis. *Aed* (standard block of HS) model, see Chapter 3, was chosen as the formal basis of MCM conceptual model. The MCM system structure, its dynamic presentation as the unit in its technological environment, MCM environment – other objects and technological units of the production process, and MCM control unit – ACOPOS controller – were also described in this basis. MCM design and control functions are taken into account owing to HS coordinator availability. Coordinator performs coordination (design and control) tasks on its selection, learning and self-organization strata.

The presented conceptual formal model of MCM machine is coordinated with traditional systems of information presentation in mechatronics [103-112]: numerical, graphic and natural language forms [8, 127]. Being the formal basis of MCM conceptual model, *aed* technology and HS method are coordinated with general requirements of design and control systems and allow easy transfer from Conceptual Design to Detailed Design phase in MCM life cycle. It brings new possibilities to creating a formal language for MCM and similar technological machines conceptual design. The process of MCM conceptual model creation as well as MCM design and control processes were executed within the framework of the R&D project maintained by the agreements RO-210.0610/25/2013 – U/WM/7/2013 and U/WM/7/2014 – between Bialystok University of Technology and Promotech enterprise (Bialystok, Poland).

CONCLUSION

The new hierarchical systems (HS) method – theoretical and practical computer means – of conceptual design of mechatronic systems based on HS theory and oriented towards usage in automation systems including CAD, automatic control, robotics and mechatronics is described in the given book. The presented process of the method development contains the following phases.

First, the theoretical basis for the design method, i.e. *aed* theoretical construction of HS and formal model of its coordinator, was constructed.

Second, after creating the mathematical model, the HS coordinator was presented in this book in the form of theoretical construction of *aed*-processor – the cybernetic system oriented to numerical and geometric information presentation and processing while performing design tasks. It was shown that the processor is able to execute operations with real, natural, complex and hyper-complex numbers in the frames of one common formal construction. This processor is oriented towards the geometric information processing presented in numerical code.

The theoretical construction of the core element of design systems, i.e. *aed* model created, and its coordinator symbol construction have the following characteristic features:

aed includes the dynamic structure and aggregated models of any system being coordinated – designed and controlled – and its environment, and allows the main design task to be carried out with the help of coordination strategies for the different levels of knowledge uncertainty;

coordination strategies are the states of a design and control system, their dynamics (like the process of uncertainty removal) reflects the dynamics of that system;

the ability of *aed* for self-organization (with the following alteration of any designed system model and coordination strategies) is ensured thanks to the availability of higher strata of the coordinator;

the basic informational constructions, i.e. numerical and geometric systems, are presented in formal basis of *aed* symbol construction; computer program realizations of *aed*-processor effectively work with number codes and changeable graphic images and allow design and control tasks, which are complicated or irresistible for one-level theoretical construction, to be performed.

Theoretical basis of the conceptual HS design method, i.e. *aed* formal model, presented in this book in Sections 3 and 4, was improved in comparison with its previous version described in [140], and has got the following characteristic features:

1) general scheme of *aed* (two-level system) has improved so that system S^{ℓ} block became included in its environment ${}_{\varepsilon}S^{\ell}$;

2) all elements of *aed* model were presented in the form of systems with correspondent indexes from the set ${}_{k}L \leftrightarrow \{o, o\pi, \pi\varepsilon, \varepsilon\}$, which indicate object (o), environment (ε), and their processes $(o\pi)$ and $(\pi\varepsilon)$;

3) symbol presentation of systems connections was changed from ${}_{\sigma}U^{\ell}$ to ${}_{\sigma}\gamma^{\ell}$ what made it agreed with presentation of coordination signals γ_0^{ℓ} generated by coordinator while performing systems structures synthesis in design process by arranging subsystems structural connection.

4) fundamental *aed* law of systems levels grows was modified by the law of systems outlook increasing in the levels space, i.e. increasing of number τ of system levels $\ell \pm \tau$ being coordinated 'up' and 'down' in regard with current level ℓ of system under consideration.

5) symbols set of levels of system organization ${}_{\Psi}L$ and correspondent basic system ${}_{\Psi}\tilde{T}^{\ell}$ (which indicate levels of system organization and coordinator strata) were changed to ${}_{\Psi}L \leftrightarrow \{\psi, \chi, \varphi, \lambda\}$ and improved in such a way, that they become coordinated with coordinator canonic model $(\hat{\varphi}, \hat{\lambda})$ and its reactions λ and functions φ performed on selection and learning strata respectively.

Furthermore, modified *aed* model presented in Chapters 3 and 4 was further improved. New *aed* formal model for systems design and control was presented and described by Novikava and Miatluk in [8], and used after that by Miatluk and Kim in [80].

Therefore, HS method and the *aed* formal construction of coordinator (and its realization as *aed*-processor) are coherent with the above-defined requirements to conceptual model of mechatronic systems being designed as well as the requirements of the general design and control systems (see Chapter 1).

Third, the conceptual design method of hierarchical mechatronic objects has been worked out basing on the theoretical means developed and presented above (Chapters 3, 4). The main phases of the design method realization – while performing both the conceptual and detail design tasks – were defined and described. After that, the basic geometric elements of mechatronic objects (MO) were presented in the hierarchical form as conditional geometric objects. Furthermore, the geometrical and numeric characteristics of MO were introduced in the numeric positional code. The main numeric characteristics of MO are constructive dimension and connections defect.

Later, coordinator procedures of systems design were defined and algorithms of hierarchical MO synthesis and analysis were presented. Finally, the exemplary computer realizations of the method for performing the basic design tasks were described for the synthesis of 3D objects, and analysis – design of caterpillar and snake like robots movements (see Chapter 4).

The HS design method presented meets the requirements of conceptual model of mechatronic systems, and all the requirements of the CAD systems (Chapter 1) are satisfied concerning the strata of selection, learning and self-organization of *aed* coordinator. Requirements of optimal execution of the geometric design tasks, i.e. GO synthesis and analysis, object transference in the space, the change of its metrical characteristics etc., are met by the actions of coordinator strategies with geometric elements of MO which are described in Section 4.4.

Fourth, implementations and assessments of the conceptual design method developed in the exemplary design tasks in mechatronics, robotics and automation systems were performed. Practical examples of the method realization are presented in Chapters 5-9 of this book.

The conceptual design process of the *surgical robot biomechatronic system* (SRS) was described in Chapter 5. Conceptual models of SRS subsystems of different nature – mechanical, electromechanical, electronic, human-computer, and biological, i.e. robot manipulator, navigation, computer-control and pre-operating planning systems, surgeon-operator and bone being operated – were described in the common *aed* formal basis of HS. All the descriptions are connected by SRS coordinator presented in the form of the design and computer control system (CCS). Exemplary control loop of coordinator is presented in this chapter in the form of Multi Motion Controller (MMC), which realizes coordination of AS3 robot motion. Model of SRS human-computer design system is described in the form of a higher level coordinator which performs the design tasks on its strata: selection, learning and self-organization.

Hierarchical systems technology was applied in Chapter 6 in the exemplary tasks of conceptual model creation of *biologically inspired robot (Bioloid Dinosaur robot)* in *aed* theoretical basis, design of Bioloid robot's assembly operations and robot motion coordination. Formal construction of the conceptual model suggested contains connected descriptions of the Bioloid robot structure, its aggregated dynamic representation as a unit in its environment, and part of the environment model. All the descriptions are connected by the coordinator which performs the design and control tasks on its selection, learning and self-organization strata. Numeric and geometric characteristics of Bioloid robot designed were presented in the frames of the developed conceptual model in the form of numeric positional system. Mechanical, electromechanical and program computer subsystems of general robot mechatronic system were partially described in common *aed* formal basis of the model proposed as well. It allows the conceptual model to fulfill the general requirements of design and control systems and mechatronic systems in particular (see Chapter 1).

The robot model dynamics while transferring from the conceptual to detailed design phases was shown for the case of the robot mechatronic subsystem of electromechanical nature, i.e. servo-motor. The examples of computer realizations of Bioloid robot design tasks in the frames of SolidWorks program system were also presented in Chapter 6.

The dependence between the systemic characteristics and physical parameters of motion, i.e. displacement, velocity, acceleration, was established for the case of the *human motion design* (Chapter 7). The main purpose of the task described in this Chapter was the application of the developed HS design method in human motion design. As a result, new conceptual formal model and HS design technology of human motion coordination – design and control – have been created. The method has the following characteristics:

1) describes a human bio-kinetic apparatus as a hierarchical system, i.e. takes into account the connected levels of human body structure (lower level), human body dynamic representation as a unit in its environment, and the environment structure (higher level); it allows the design and control task of human motion to be performed as interconnections coordination of cohered structural and dynamical representation of man in its environment;

2) makes possible the design (synthesis) of different kinds of human movements, predicts the motion and obtains correspondent motion data using only geometric or anatomic data such as limbs length and weight; consecutive interactive performing of the analysis design task, i.e. changing of the human structural interactions (joint angles) and subsystems characteristics (length, mass) and observation of the change of process being designed (motion) executed by system (man) in its environment (higher level systems), allows obtaining the desired characteristics of this process (trajectories coordinates, displacements, velocities, etc.);

3) performs the connected tasks of human body deformations and human motion in its environment cased by these deformations in the frames of one common HS *aed* model not using traditional approaches, i.e. Finite Elements method and Integral Differential Equations; allows solving of human motion synthesis and analysis tasks in the frames of one common computer model;

4) allows presentation and processing of information about systems of various nature – engineering, computer, social, biological, physical – in the frames of one common HS *aed* formal model, which is very important in the case of bio-

mechanical, medical, rehabilitation and other biomechatronic systems coordination and allows avoiding modelling conflicts between various subsystems of different nature;

5) makes possible the modification of the model developed using controlled experimental data on the self-organization layer of coordinator and prediction of a motion;

6) presents parts of human body in computer design process as connected 3D geometric objects due to the conditional geometric objects representation (see Chapter 4);

7) allows performing – at CD and DD phases – both the geometrical design and forward kinematics tasks as the common computational task owing to the description of geometric construction, coordination (design and control) procedures and kinematic characteristics in the frames of one common computer model.

Computer program modules created for human motion design have been constructed according to HS *aed* structure and the modules functioning correspond to the actions of *aed* coordinator of HS. The design tasks are realized using commercial software Delphi (Embarcadero Technologies Inc., CA, USA) and Matlab (The MathWorks Inc., MA, USA) systems. All calculations are effectively executed on a computer as operations with systems quantitative characteristics presented in numeric positional code. It allows increasing the computational efficiency of the design procedures. During the motion design, the dependences of functional parameters (kinds of motions) on the structural ones (sizes, angles) of the human body segments were determined.

The worked out HS design method, hierarchical geometry and coordination technology of geometric elements of MO (Chapter 4) were implemented in the tasks of electronic *Printed Circuit Boards (PCB) design and testing* described in Chapter 8. The introduced HS design and testing method implemented in this task yields the following results and is characterized by the following qualities:

– a graphic image of PCB standard is a result of formation of geometric representation of PCB in the process of its conductive pathways construction synthesis;

– availability of geometric representation allows easy correction of PCB conductive pathways, i.e. interconnections topology, at each phase of plate construction including CD and DD phases;

– the mechanism of information aggregation permits to avoid the growth of computer memory volumes in the transition process to a higher level of the PCB geometric structure during its synthesis;

– the ability of scales and orientation of geometric construction to change (during PCB standard matching to the obtained PCB graphic image) allows the identification of PCB as defective one to be avoided in a technological process, when the PCB orientation and the distance in relation to the video camera are changed;

– in the process of pathways quality testing the standard and obtained PCB images are compared not by continuous scanning of PCB image lines as in the technologies already known, but only by scanning along the interconnection lines – conductive pathways – with regard to their geometric representations; it allows the reduction of both the comparison time and computer memory volumes.

Aed symbol construction of HS gave a formal basis for the descriptions of the PCB geometric structure and the processes of its design and testing presented in Chapter 8, and allowed creating the PCB conceptual model. Availability of coordinator allowed performing the decision making tasks on its strata at both PCB conceptual and detailed design phases. Algorithms of the given HS design technology was applied in PCB production process of Horizont enterprise (Minsk, Belarus).

The use of Hierarchical Systems (HS) technology in *conceptual design and control of MCM* (Manhole Cutting Machine) is presented in Chapter 9. Developed conceptual model of the MCM system contains connected systemic models of mechatronic subsystems, i.e. human – man-operator, computer control, visual informatics, electronic – sensors, electromechanical – servo drives, and executive mechanical (MCM tool, arm and its joints), presented in general *aed* formal basis of HS. The MCM system structure, its dynamic presentation as the unit in its technological environment, MCM environment – other objects and technological units of the production process, and MCM control unit – ACOPOS controller – were also described in this basis. MCM design and control functions were taken into account owing to HS coordinator availability. MCM coordinator performs coordination – design and control – tasks on its selection, learning and self-organization strata.

The conceptual formal model of MCM machine presented in Chapter 9 is coordinated with traditional systems of information presentation in mechatronics – numeric, graphic and natural language forms. It was shown, that MCM conceptual model presented in *aed* formal basis of HS is coordinated with general requirements of design and control systems and allows easy transfer from Conceptual Design phase to Detailed Design phase in MCM life cycle. It brings new possibilities in creating a formal language for MCM and similar technological machines conceptual design. Both MCM conceptual model creation and design processes described in Chapter 9 were executed in the frames of the R&D project maintained by the agreements between Bialystok University of Technology and Promotech enterprise (Bialystok, Poland).

Therefore, the conceptual model of mechatronic objects being designed, the design system and coordination – design and control – process are presented in *aed* formal basis of HS in this book. In comparison with traditional models of mathematics, artificial intelligence and widespread design methods (see Chapter 1) the developed conceptual formal model contains connected descriptions of the designed mechatronic object structure, its aggregated dynamic representation as a unit in its environment, the environment model and the design&control system. All the descriptions are connected by the coordinator which performs the design and control tasks on its strata. Besides, the suggested *aed* technology of HS coheres with traditional systems of information presentation in mechatronics: numeric, graphic and natural language forms. *Aed* technology and HS design method are also coordinated with general requirements of the design and control systems and consider mechatronic subsystems of different nature – mechanical, electromechanical, electronic, computer – in common theoretical basis.

Besides, *aed* technology and HS design method are an effective means of mechatronic objects design and conceptual models creation. These models allow an easy transfer from the conceptual design phase to a detailed design phase in the mechatronic object's life cycle and allow description of the design procedures in the form of coordination strategies of HS. At the conceptual design phase, the mechatronic subsystems of MS being designed are presented in the form of dynamic system (ρ,φ) which is generalization of DE, automata and algebra systems. So, the transition from the conceptual to the detailed design phase in the frames of the HS design method suggested is convenient and requires concretization of the abstract dynamic system only.

All the qualities mentioned above make the suggested HS design technology more efficient in design tasks performing in comparison with widespread design methods (see Chapter 1). The presented HS design method brings new informational means for the conceptual and detailed design of mechatronic systems.

The conceptual design method presented in this book has been also applied to the design and control of other engineering objects [53, 113-117], in robotics [50, 54-58, 119-125, 141-142], biomechanics [55, 118] and mechatronics [49-51, 58, 126], in design tasks of industrial knowledge networks [23, 24], written text [127] and natural language processing [8], and other tasks.

Some theoretical foundations and application results of the method presented in this book were discussed with one of the authors of the theory of hierarchical and dynamic systems Prof. Yasuhiko Takahara [2, 10], and one of the authors of mathematical and computer aided systems theory Prof. Franz R. Pichler [138, 139].

The described HS conceptual design method was also implemented in R&D projects with PROMOTECH enterprise (Bialystok, Poland), MTZ and HORIZONT enterprises (Minsk, Belarus), EU projects ICIMS-NoE, AMETMAS-NoE, DYCOMANS, ACAT Horizont 2020 project, grants and projects maintained by KOSEF and NSF (South Korea), and others.

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