

Topological conjugacies for normal forms

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Homeomorphisms allowing us to prove topological equivalences between parameter dependent families of maps undergoing the same bifurcation are constructed in this paper. This provides a solution for a classical problem in bifurcation theory that was set out three decades ago by Arnold [1] (p.285-286), affirmed in [2] without proof and remained unexpectedly unpublished [3] until now.

Moreover, we prove topological conjugacies between prototype truncated and general non-truncated (or even highly degenerated) normal forms of bifurcations. The main results of this work state that increasing homeomorphisms with the same number of fixed points with the same sequence (or reversed sequence) of stabilities are topologically conjugated and that decreasing homeomorphisms with one fixed point and at most a period two orbit with the same stabilities are topologically conjugated.

For differentiable families the construction provides differentiable topological conjugacies if the maps of the original families have the same slopes at the fixed points, if it is not the case the conjugacies are only differentiable in the open intervals connecting the fixed points.

Joint work with Francisco Balibrea and José Carlos Valverde.

[1] Arnold, V.I., *Geometrical Methods in the Theory of Ordinary Differential Equations*, Grundlehren der mathematischen Wissenschaften, 250 (A Series of Comprehensive Studies in Mathematics). Springer-Verlag: New York, Heidelberg, Berlin, 1983.

[2] Arnold, V.I., Afraimovich, V., Il'yashenko, Y. & Shil'nikov, L., *Bifurcation theory, in V. Arnold, ed., 'Dynamical Systems V. Encyclopaedia of Mathematical Sciences'*, Springer-Verlag, New York, 1994.

[3] Kuznetsov, Y.A. , *Elements of Applied Bifurcation Theory*. 2nd Edition. Appl. Math. Sci., 112. Springer-Verlag: New York, 1998.