Explicit Euler methods for reproducing blow-up and finite-time stability in stochastic differential equations

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Explosions in solutions of stochastic differential equations (SDEs), or solutions reaching equilibria in finite time, can typically happen only when these differential equations are highly nonlinear (in the sense that drift or diffusion coefficients do not obey linear bounds). In each case this is captured by the existence of a finite (random) time T at which the solution tends to infinity or to the equilibrium.

Due to rapidly changing solutions, and the randomness of the time T, the analysis of the continuous–time behaviour as $t \uparrow T$ is challenging. Accordingly recovering in numerical solutions the presence of an explosion, as well as the asymptotic behaviour as (numerical) explosion is approached presents difficulties. Moreover, any numerical algorithm should be able to distinguish between extremely rapid growth or decay in solutions, and explosions or finite–time stability. The algorithm should not falsely create explosions in the numerical schemes when they are not present in the original SDE.

In this talk, we summarise some results from the continuous–time theory, and show how the above requirements for the numerical method can be achieved by considering appropriate co–ordinate changes in the original SDE, and then choosing non–uniform and state–dependent step sizes in the new co–ordinate system to recover the salient properties of the solution. Our methods can sometimes be shown to be optimal, in the sense that if less computational effort is expended, important properties of the solution of the SDE can be lost.

The talk relies jointly on work with Brian Colgan (DCU).