One generalization of the Euler-Maclaurin formula

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We consider the problem of the multiple summation namely the problem of finding the sum of the form $\sum\limits_{\|t\|\leq x} \varphi(t)$, where $\varphi(t) = \varphi(t_1, ..., t_n)$ is a given function and $\|t\| = t_1 + ... + t_n$, t_j are non-negative integer numbers. Sometimes this summation is referred "triangle"' summing. When φ is the function of one variable, then this sum can be written as

$$S(x) = \sum_{\|t\|\leq x} \varphi(x - \|t\|).$$

It is shown that it satisfies the difference equation $(\delta - 1)^n S(x) = \varphi(x + n)$, where $\delta f(x) = f(x + 1)$, that makes it be possible to solve the problem of summation. Analogue of the Euler-Maclaurin formula for the sum $S(x)$ will be written the following way

$$S(x) = \sum_{\mu=0}^{n-1} \frac{B_\mu(m)}{\mu!} P_{n-\mu}\varphi(x + n) + \sum_{\mu=0}^{\infty} \frac{B_{\mu+n}(m)}{(\mu + n)!} D^\mu \varphi(x + n),$$

where $P_{n-\mu}\varphi(x + n)$ is primitive of order $n-\mu$ of the function $\varphi(x + n)$, and $B_\mu(m)$ are generalized Bernoulli numbers.

[1] Shishkina, O.A., The Euler-Maclaurin Formula and Differential Operators of Infinite Order, *Journal of Siberian Federal University, Mathematics & Physics,* 8(1) (2015), 86–93.