An ergodic theory approach to chaos

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We present some results concerning chaotic behaviour of dynamical systems in infinite-dimensional spaces. Such dynamical systems are often generated by transformations in Banach spaces and in the case of continuous time dynamical systems (semiflows) by evolution equations. First, we present an introduction to an ergodic approach to infinite dimensional chaotic systems [1]. We construct an invariant measure under a dynamical system having strong ergodic and analytic properties (e.g. mixing, positivity on open sets). There are a few methods of construction of invariant measures. The first one is based on the Krylov-Bogolubov theorem on the existence of invariant measure for continuous transformations on topological compact spaces. In the second method we use measures induced by Gaussian processes (Wiener measures) and isomorphism of the original dynamical system with a translation transformation (or semiflow) on a properly chosen space. This method allows us to prove that the system is topological mixing and its trajectories are turbulent in the sense of Bass. We illustrate general mathematical results by showing chaotic properties of the differentiation operator on the space of entire functions. We also mention about applications to some biological models: an age-structured model of a semelparous population; the evolution of maturity of blood cells in the bone marrow; and a size-structured model of cells reproduction system.
It should be noted that most of the recent papers concerning chaos for linear operators are based on studying spectral properties. This approach seems to be easier than ours. But, in our opinion, the approach based on the isomorphism with translation operators and using invariant measures reveals why such dynamical system are chaotic. The second advantage of the ergodic theory approach is that we can prove much stronger results concerning chaos.

[1] R. Rudnicki, An ergodic theory approach to chaos, *Discrete and Continuous Dynamical Systems* 35 (2015), 757–770.