Basin of attraction of triangular maps
with applications to difference equations

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In the papers [1,3] we found counterexamples to the so called continuous and discrete Markus-Yamabe conjectures. These counterexamples were constructed by using triangular vector fields or maps. In this talk we will recall them as a motivation to study the dynamics of planar triangular maps

$$x_{n+1} = f_0(u_n) + f_1(u_n)x_n, \quad u_{n+1} = \phi(u_n).$$

These maps preserve the fibration of the plane given by $\mathcal{F} = \{\phi(u) = c, c \in \text{Image}(\phi)\}$. We assume that there exists an invariant attracting fiber $\{u = u_*\}$ for the dynamical system generated by ϕ and that on this fiber the system contains either a global attractor, or it is filled by fixed or 2-periodic points. Then we study the limit behavior of all the points that lie in the basin of attraction of this invariant fiber. We apply our results to a variety of examples, from particular cases of triangular systems to some planar quasi-homogeneous maps, and some difference equations. Some concrete examples are the multiplicative and additive difference equations of types $x_{n+2} = x_n g(x_n x_{n+1})$ and $x_{n+2} = -bx_{n+1} + g(x_{n+1} + bx_n)$. The results presented have been published in [2].

[1] Cima, A., Essen, A., Gasull, A. Hubbers, E., Mañosas F., A polynomial counterexample to the Markus-Yamabe conjecture, *Adv. Math.* 131 (1997) 453–457.

[2] Cima, A., Gasull, A., Mañosa, V., Basin of attraction of triangular maps with applications, *J. Difference Equ. Appl.* 20 (2014) 423–437.

[3] Cima, A., Gasull, A., Mañosas, F., A polynomial class of Markus-Yamabe counterexamples. *Publ. Mat.* 41 (1997) 85–100.