# Geometry and Global Stability of Higher Dimensional Monotone Maps

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We discuss a new notion of monotonicity for maps on $\mathbb{R}^k$, called normal monotonicity, that has recently been introduced in [1] and builds on previous work of the authors in [2]. The new definition of monotonicity extends the classical notion of competitive planar maps of Smith [3] and geometrically captures the dynamics of a competitive system in higher dimensions. Namely, a map $F : \mathbb{R}^k \to \mathbb{R}^k$ is monotone at $p$ if for any hypersurface $\Gamma$ containing $\mathbf{p}$ with $\eta_\Gamma(\mathbf{p}) > 0$, we have $\eta_{F(\Gamma)}(F(\mathbf{p})) > 0$. Here $\eta$ denotes the normal vector at a hypersurface. Our main result is to show global stability for monotone maps that have a unique coexistence fixed point.

[1] Balreira, E.C., Elaydi, S., and Luis, R., Geometry and Global Stability of Higher Dimensional Monotone Maps, *Preprint*.

[2] Balreira, E. C., Elaydi, S., and Luis, R., Local stability implies global stability for the planar Ricker competition model *Discrete and Continuous Dynamical Systems - Series B*, 19(2) (2014), 323–351.

[3] Smith, H., Planar competitive and cooperative difference equations, *J. Differ. Equations Appl.*, 3 (1998) 335–357.

[1]Joint work with Saber Elaydi and Rafael Luís.